

SIMULATION OF NONLINEAR DYNAMIC RESPONSE OF REINFORCED CONCRETE SCALED MODEL USING THREE-DIMENSIONAL FINITE ELEMENT METHOD

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SUMMARY

A constitutive model of reinforced concrete for cyclic or dynamic loading analysis is developed based on existing experimental results. The proposed model features hysteretic characteristics of concrete in tension, compression, shear along crack direction, and the bond between concrete and reinforcing bars. A non-orthogonal multi-directional smeared crack model is also developed for both biaxial and tri-axial stress states for realistic representation of concrete cracking under stress reversals. The applicability and efficiency of the proposed models are demonstrated through simulation analyses with various types of reinforced concrete specimens subjected to cyclic loads or seismic excitation on a shaking table.

INTRODUCTION

Recent developments in nonlinear finite element analysis of reinforced concrete are remarkable, but very few reports have attained precise simulation of dynamic responses including their inelastic behavior. Several reasons exist: concrete is characterized by its strong nonlinearity induced by cracking, softening or crushing; moreover, it exhibits a complex hysteretic stress - strain relationship under alternated reversals. Numerous attempts at hysteretic modeling of concrete have been made, but it is difficult to reproduce actual behavior with a rough model in which unloading and reloading responses are simplified to be linear with constant stiffness. Another reason is that modeling of concrete cracking under seismic conditions requires elaborate work because cracks occur in different directions because of reversed cyclic loads. Two or more cracks must be considered in a single point when simulating seismic responses. In addition, closing or reopening of those cracks must be judged appropriately in accordance with stress histories. Three-dimensional analysis requires a sophisticated crack model under tri-axial stress state. Furthermore, as widely recognized, time history response analysis of reinforced concrete is extremely sensitive to the time increment and is apt to diverge easily because of its strong nonlinearity. A stable solution scheme is indispensable for precise simulations.

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This study is intended to develop constitutive models for reinforced concrete and a nonlinear analysis method that are applicable to cyclic loading or dynamic response analysis in the practical use, and to demonstrate their applicability and accuracy through simulation analyses of past experiments.

MATERIAL MODELING

Concrete

Basic concept

Concrete is idealized using the orthotropic model based on equivalent uniaxial strain concept [1]. The model employs hypo-elastic constitutive relationships (nonlinear elasticity). The axes of orthotropy coincide with the current principal direction before occurrence of cracking. After cracking, the material axes are fixed to the crack direction.

Failure criteria

In plane stress analysis, Kupfer-Gerstle's criterion [2] is applied for failure in biaxial compression and in tension-compression. Uniaxial tensile strength is used for judging cracks under uniaxial and biaxial tension. Degradation of compressive strength after cracking is incorporated. A compressive degradation model has been derived from in-plane shear loading tests of RC panels [3]. The reduction ratio of compressive strength is defined as a function of the uniaxial compressive strength of concrete and acting normal stresses along reinforcing directions. Three-dimensional analysis employs Ottosen's four-parameter model [4] for the failure criterion under a triaxial stress state.

Stress - strain envelopes in tension and compression

In the tension zone, the stress - strain relationship is assumed to be linear up to cracking; the tension stiffening envelope after cracking is modeled on the basis of RC panel tests, as shown in Fig. 1. The tension stiffening envelope comprises two parts: a descending part and a flat part. The reinforcement ratio and the uniaxial compressive strength of concrete are determining factors for a transition point from the descending part to the flat part. The stress σ_m and strain ε_m at the transition point are given as:

where p_s is the reinforcement ratio and σ_T and σ_B are the tensile and compressive strengths of concrete, respectively. Furthermore, stress on the flat part is diminished along with degradation factor β , which is

where E_C is the tangential stiffness of concrete along the crack direction and E_0 is the elastic modulus.



Fig. 1 Tension stiffening model of concrete

Both ascending and descending curves are expressed by the following equations in the compression zone.

$$\sigma = \frac{\left(A \cdot X + (D-1.0) \cdot X^{2}\right) \cdot \sigma_{p}}{1.0 + (A-2.0) \cdot X + D \cdot X^{2}} \quad (4)$$

$$A = E_{0}/E_{p} \quad (5)$$

$$Ascending part \left(|\varepsilon| < |\varepsilon_{p}|\right)$$

$$X = \varepsilon / \varepsilon_{p} \quad (6)$$

$$D = \frac{19.6}{\sigma_{B}} \left(\frac{E_{B}}{E_{p}} - 1.0\right)^{2} \quad (\ge 1.0 - E_{0} / E_{p}) \quad (7)$$

$$Descending part \left(|\varepsilon| \ge |\varepsilon_{p}|\right)$$

$$X = \left(\varepsilon / \varepsilon_{p}\right)^{n} \quad (8)$$

$$n = 0.9 + 3.4 \left(\frac{\sigma_{B}}{98}\right)^{2} \quad (9)$$

$$D = 1.0 + \frac{177}{\sigma_{B}} \left(\frac{\sigma_{p}}{\sigma_{B}} - 1.0\right) \quad (10)$$

Therein, σ_P and ε_P are stress and strain at the peak point, respectively, σ_B is the uniaxial compressive strength, E_0 is the elastic modulus, and E_B and E_P are secant moduli corresponding to σ_B and σ_P , respectively. The above equations were proposed originally by Ahmad et al. [5]. Coefficients X, A and D are modified to express curve shape differences because of the difference in compressive strength or amount of confining stress, as shown in Fig. 2. The model represents actual behavior precisely.

Stress - strain relationship under stress reversals

Unloading and reloading response of concrete in compression is not linear. The unloading stiffness becomes lower as the strain at the unloading point exceeds the elastic limit. Unloading and reloading curves are represented using quadratic equations while considering those features, as shown in Fig. 3. We define point E on the compression envelope curve, point Z where plastic strain remains after all stress is released, and ε_E and ε_Z as corresponding strains at points E and Z, respectively. Plastic strain ε_Z is expressed by the following equation proposed by Karsan and Jirsa [9].

$$\varepsilon_{z} = (0.145 \cdot (\varepsilon_{E} / \varepsilon_{P})^{2} + 0.13 \cdot (\varepsilon_{E} / \varepsilon_{P})) \cdot \varepsilon_{P} \quad \cdots \qquad \cdots \qquad \cdots \qquad \cdots \qquad \cdots \qquad \cdots \qquad (11)$$

According to Eq. (11), the strain ε_z exceeds the unloading point strain ε_E when ε_E is larger than roughly $6\varepsilon_P$. The following equation is employed in cases where ε_E is $4\varepsilon_P$ or larger to avoid this unreasonable defect.



Fig. 2 Comparisons of stress - strain envelopes of concrete in compression



$$\varepsilon_{z} = \left[\left(\varepsilon_{E} / \varepsilon_{P} \right) - 2.828 \right] \cdot \varepsilon_{P} \quad \left(\varepsilon_{E} | \ge 4.0 | \varepsilon_{P} | \right) \quad \cdots \quad (12)$$

The unloading curve is assumed to be linear with stiffness E_E before it reaches point C, which is called the "common point" where the reloading curve crosses the unloading curve. Stress at point C is defined by the following equations proposed by Darwin et al. [10].

Stiffness E_E is assumed to be proportional to secant modulus E_{EZ} between points E and Z. It is written as:

$$E_E = \alpha_1 E_{EZ}$$
 ($\leq E_0$: Elastic modulus). (14)

Coefficient α_1 =1.5 is recommended on the basis of existent experimental results; E_E does not exceed the elastic modulus E_0 . The unloading curve starting at point *C* toward point *Z* is expressed as

where *a*, *b* and *c* are constants that are defined by the condition that the curve passes points *C* and *Z* and having stiffness E_E at point *C*. One limitation of Eq. (14) is that the stiffness at point *Z* is not less than zero:

$$E_E \leq \frac{2\sigma_E}{\varepsilon_E - \varepsilon_Z}.$$
 (16)

The turning point *R* is defined where the condition changes from unloading to reloading. Stiffness at the beginning of reloading is assumed to be α_2 times unloading stiffness at point *R*. Coefficient α_2 is 1.0 when point *R* coincides with point *C*, and $\alpha_2 = \alpha_{2z}$ at point *Z*, where α_{2z} is defined by the following equations.

Interpolating between points C and Z, α_2 is calculated as

$$\alpha_2 = \frac{(\alpha_{2Z} - 1.0)}{(\varepsilon_Z - \varepsilon_C)} (\varepsilon_R - \varepsilon_C) + 1.0. \quad (18)$$

Comparisons of the model with corresponding experimental results are shown in Fig. 4. The model reproduces observed stress - strain responses of cyclic compression tests [9].

Figure 5 shows that the unloading and reloading curves are represented in the same manner in the tension zone as in the compression zone. Points *T*, *G*, and *H* correspond to points *P*, *E* and *Z* in compression, respectively. The secant modulus E_{GH} between points *G* and *H* becomes lower as the strain at point *G* increases. E_{GH} is assumed to be proportional to the ratio of strain at point *T* to point *G*. It is

$$E_{GH} = \frac{\mathcal{E}_T}{\mathcal{E}_G} E_o.$$
(19)

The unloading curve is assumed to be linear with stiffness E_G before it reaches point L, where the reloading curve meets the unloading curve. Stress at point L is defined expediently as

$$\sigma_{r} = 0.9\sigma_{c}$$
. (20)

The unloading curve is expressed by Eq. (15). The stiffness at point G is defined as

$$E_G = \alpha_3 E_{GH} \ (\leq E_0) \ . \ (21)$$

Coefficient α_3 is set to 1.5 and does not exceed the elastic modulus E_0 in a similar way with α_1 in Eq. (14). The reloading curve starts at point *R*. The stiffness at the beginning of reloading is assumed to be α_4 times unloading stiffness just before point *R*. Coefficient α_4 is 1.0 when point *R* coincides with point *L*; $\alpha_4 = \alpha_{4H}$ at point *H*, where α_{4H} is defined by the following equation.

Interpolating the values at points L and H, the coefficient α_4 is expressed as

$$\alpha_4 = \frac{(\alpha_{4H} - 1.0)}{(\varepsilon_H - \varepsilon_L)} (\varepsilon_R - \varepsilon_L) + 1.0.$$
(23)

Comparisons of the model with test results of cyclic tension [11] are shown in Fig. 6.

When unloading continues beyond point H, the curve goes into compression regions. Figure 7 illustrates hysteretic rules between tension and compression. Point J is a transition point from unloading in tension to reloading in compression. Point K is a transition point from unloading in compression to reloading in tension. The following logarithmic equation expresses the curve from point H toward point J.

Therein, coefficients a, b, and c are determined by the condition that the curve passes points H and J and having stiffness E_H at point H.



Fig. 5 Unloading/reloading model of concrete in tension

Fig. 6 Comparisons of model with test on cyclic tension





Stress at point J is assumed to be proportional to the tensile strength σ_T . It is written as

$$\sigma_J = -\alpha_5 \sigma_T. \quad (25)$$

Coefficient α_5 is given by the next equation considering that σ_J increases with increasing strain at point *L*:

$$\alpha_5 = 1.0 + 0.02 \left(\frac{\varepsilon_L - \varepsilon_T}{\varepsilon_T} \right). \quad (26)$$

Stress at point *K* is given by the following assumption, which best describes past experimental results.

When a crack occurrs after unloading from the compression zone, as shown in Fig. 7b, the tensile strength σ_T is reduced considering the damage induced by the compression, as

$$\sigma_T = \sigma_{T_0} \frac{E_{EZ}}{E_0}, \qquad (28)$$

where σ_{To} is the tensile strength of intact concrete. Strain at point *T* is shifted in accordance with the strain at point *Z*. The line from point *Z* toward point *T* is set to be linear with stiffness at point *Z*.

Non-orthogonal multi-directional crack model

The smeared crack model with a fixed angle concept expresses cracking of concrete. Once a crack is formed in an element, a crack axis is introduced and the material axis is set to coincide with the crack axis. One set of crack axes is able to represent two orthogonal cracks in plane stress analysis and three orthogonal cracks in three-dimensional analysis at the maximum. Another crack axis is introduced in the newly cracked direction to reflect subsequent cracks that occur with an acute angle against the existent crack resulting from stress redistribution or change of loading direction. Three crack axes are introduced at the maximum; six cracks, three sets of two orthogonal cracks, are taken into account in a single point in plane stress analysis, as shown in Fig. 8a. Nine cracks, three sets of three orthogonal cracks, are considered in three-dimensional analysis, as illustrated in Fig. 8b.





Fig. 10 Comparison of model with test on cyclic shear

Shear stress - shear strain relationship after cracking

Shear transfer action is expressed by the average shear stress - shear strain relationship along the crack direction. The shear stress - shear strain envelope is determined as a function of the concrete strength, the amount of reinforcing steel crossing the cracks, and tensile strain perpendicular to the crack direction [3]. The hysteretic rule on shear stress - shear strain relationship is modeled as shown in Fig. 9. Both unloading and reloading curves are expressed as

where a and b are constants that are determined by assuming that the curve passes two points. Point U is an unloading point on the envelope, point Z is on the unloading curve when all stress is released, point Ris a turning point from unloading to reloading, and point C is a crossing point between the unloading and the reloading curves. The strain at point Z is assumed to be proportional to the unloading point strain as

$$\gamma_z = 0.5\gamma_U \ (\leq \gamma_U - \frac{4\tau_U}{G_0}) \ . \tag{30}$$

The limitation in the above parenthesis is derived from the condition that the stiffness at point U does not exceed the elastic shear modulus G_0 . After reaching point Z, the curve has no stiffness unless the shear strain keeps on decreasing. When the strain changes from decreasing into increasing, shear stress starts to increase and goes up toward point C, where shear stress is defined as

Figure 10 shows a comparison of the model with the experimental result obtained from cyclic shear loading tests on the cracked plane under constant crack width [12].

Reinforcing steel

Incremental plasticity theory is applied for steel material. The von Mises yield surface is employed to judge vielding under a multi-axial stress field along with the associated flow rule for isotropic hardening.

The stress-strain relationship under stress reversal follows Ciampi's model [13]. This model gives a good representation of actual stress - strain relationships under cyclic stresses, as shown in Fig. 11.



Fig. 11 Comparisons of Ciampi's model with test results

Bond between concrete and reinforcement

Figure 12 depicts the conceptualized unloading and reloading rules on the bond stress - slip relationship. This model is derived from past experiments focussing on bond slip behavior. The envelope curve for bond stress τ and slip *S* is expressed as

$$\tau = \frac{\tau_{\max} \left(2.0 - d \cdot S_{\max}\right)}{1.0 - d \cdot S + \left(\frac{S}{S_{\max}}\right)^2} \frac{S}{S_{\max}}, \qquad (32)$$

where τ_{max} and S_{max} are the stress and the slip at the peak point, respectively, and *d* is a coefficient which determines the shape of the curve. Differentiating Eq. (32) by the slip *S*, and substituting *S*=0.0, the initial stiffness is obtained. It is inferred to be 20.0 *K*s, which is the secant modulus toward the peak point. Therefore, the coefficient *d* is calculated using the above assumption. The unloading curve from point *E* on the envelope toward point *N*, where the stiffness of the curve becomes zero, is given as

$$\tau = a(S - S_N)^4 + \tau_N, \qquad (33)$$

where S_N and τ_N are the slip and the stress at point *N*, respectively. Morita and Kaku [16] propose τ_N as

$$\tau_{N} = -0.18 \cdot \tau_{E}. \qquad (34)$$

The slip S_N and coefficient *a* in Eq. (33) are determined from the condition that the unloading curve passes point *E* and the stiffness at point *E* equals K_0 . When the unloading curve reaches point *N*, it loses stiffness and moves with constant stress τ_N until it meets the envelope in the negative zone. The reloading curve from point *R* to point *M* is also expressed by Eq. (33), where point *M* is a transition point. The stress at point *M* is determined by Eq. (34) with τ_N instead of τ_E . The curve from point *M* to point *C* is defined in the same way. The stress at point *C* follows the proposal by Morita and Kaku [16].

When reloading starts before reaching point N, the reloading curve is assumed to be anti-symmetric with the unloading curve. It increases toward point E with initial stiffness K_0 at point R. In case reloading continues after unloading in the opposite domain, transition point M is located on the vertical axis where the slip equals zero.

Figure 13 shows comparisons between the model and experimental results [16]. The model represents the observed bond hysteretic behavior with sufficient accuracy.



Fig. 12 Unloading/reloading model of bond behavior

Fig. 13 Model verification on bond reversals

MODEL VERIFICATION UNDER CYCLIC LOADING CONDITION

All models described here are incorporated into a finite element program for concrete structures FINAL. For the first stage of verification, two reinforced concrete specimens are analyzed statically under a cyclic loading condition. One specimen requires constitutive models under a biaxial stress state; the other requires those under a triaxial stress state.

Reinforced concrete box wall subjected to cyclic horizontal force

The specimen is a box-shaped reinforced concrete wall with slabs at the top and bottom [17]. Half of the specimen is modeled using quadrilateral shell elements using the symmetric condition on the shape and loading. Top and bottom slabs are modeled by thickened elements and are assumed to behave elastically. Reinforcing bars in the wall are all replaced by equivalent layers with stiffness in the bar direction and superimposed on the shell elements.

Figure 14 shows comparison of load - displacement relationships of the test and the analysis. The analysis reproduces actual hysteresis loops not only for small displacement cycles, but also for large displacement cycles. It is particularly emphasized that the analysis shows the slight drop of the peak load after the same displacement cycle as well as the experiment does.

Reinforced concrete beam-column joint subjected to cyclic shear

As an entirely three-dimensional example, a reinforced concrete beam - column joint specimen is analyzed under the triaxial stress state. The specimen has beams connected eccentrically with the column in three directions. Testing was conducted by applying constant axial compression force on the column, then increasing shear forces at both ends of the beam. The specimen showed flexural yielding of the beam during the experiment. Concrete is modeled using hexahedral elements; the main bars in the column and the beams are explicitly modeled using truss elements. Hoops and stirrups are represented by embedded smeared reinforcements. Joint-type elements are inserted between hexahedral elements and truss elements for beam main bars for the purpose of incorporating bond slip behavior. The bond strength is determined on the basis of measured strain distributions of the beam main bars in the joint region, and the mean value 7.05 N/mm² is adopted in the analysis. Bond slip corresponding to the bond strength is assumed as 1.0 mm, referring to the study by Elmorsi et al. [18].

Relationships between shear force of the beam and story drift angle are compared in Fig. 15. Although the analytical maximum load is slightly higher than the experimental one, close agreement is obtained for hysteresis loops between the two.



Fig. 14 Comparison of load - displacement curves of box wall specimen



Fig. 15 Comparison of story shear force - drift angle relationships of beam - column joint specimen

DYNAMIC RESPONSE ANALYSIS

For the next stage of verification, simulation analyses are performed for two reinforced concrete scale models subjected to seismic excitation on a shaking table. Dynamic response analyses are conducted as the following procedure. First, initial stress is calculated statically for self-weight. Secondly, eigenvalue analysis is performed using the subspace iteration method. Thirdly, time history response analyses are conducted using Newmark- β time integration method with parameters γ =0.5, β =0.25. Internal viscous damping is assumed to be proportional to initial stiffness. Analyses are performed with uniform 1% damping for the first natural period of each model. That damping is kept constant throughout the calculation.

Reinforced concrete H-sectioned wall

The specimen is an H-sectioned wall constructed on a base slab with a top slab and additional masses, as shown in Fig. 16. The shaking table test was conducted as RUN1 through RUN5, applying horizontal excitation in a direction parallel to the web wall [19]. The specimen showed typical shear sliding failure at the lower part of the web wall during RUN5 in the test.

The finite element model is half-symmetric and assembled with shell elements for the wall and hexahedral elements for both the slabs and masses. The first eigenvalue of the analysis shows a slightly higher frequency of 13.4 Hz than the measured value of 13.2 Hz before RUN1. The calculated first eigenmode is shear-dominated deformation of the web wall, as shown in Fig. 17.

Time history analyses are performed continuously in the same sequence with the shaking table test. Time increment size of the analysis is set to 0.005 s. However, the response and residual stresses were too large when input acceleration changed drastically. Therefore, the time increment is reduced to a smaller value automatically if the input acceleration increment exceeds the prescribed upper limit. The upper limit is stipulated as 100 mm/s² per each increment in this analysis; the allowable minimum time increment is set to 0.0001 s. This time increment control contributes to stable calculation in incremental analysis of concrete materials that exhibit strong nonlinearity.

Figure 18 shows comparisons between calculated and measured acceleration responses of the top slab in RUN1, RUN4, and RUN5. Regarding RUN1, the specimen responds within the elastic range. The analysis reproduces the observed waveform, maximum acceleration, and its occurrence time quite well.







Fig. 17 Calculated eigen mode



1.5. 10 Comparisons of normalical according responses of cop

In RUN4, the waveform transforms into a long-period type because of cumulative stiffness degradation resulting from concrete cracking and steel yielding. This tendency is well represented by the analysis. Nevertheless, the latter half of RUN4 shows a difference between the analysis and test results. Presumably, the specimen underwent much damage in the analysis when it showed larger maximum acceleration than the measured value around 4.5 s.

The specimen failed at about 4.0 s in the test in the final excitation RUN5. The acceleration response changed to small amplitude and included long-period components after failure. The analysis reproduces that change precisely.

Figure 19 shows the relationships between inertial force and horizontal displacement of the top slab. Therein, the inertial force is defined as a product of the weight of the top slab, including the masses, and its acceleration response. Hysteresis loops tend to become larger as a result of progress in nonlinearity in



Fig. 19 Comparisons of hysteresis loops

RUN4. They grow remarkably large in RUN5, especially after reaching the maximum load. The maximum inertial forces and the maximum displacements of the test and the analysis show good respective correspondence.

Acceleration response spectra at the top slab are compared for the test and the analysis in Fig. 20. Regarding RUN4, slight differences appear in the peak periods. However, dynamic characteristics of response spectra agree well with each other. Several peaks, those indicating the progress of nonlinearity and failure, can be found in RUN5 for both the test and the analysis.

Non-stationary spectra of RUN4 and RUN5 are shown in Fig. 21 to elucidate the historical changes in dynamic characteristics of the specimen. This figure shows changes in the natural frequency. Dominant frequencies and historical changes mutually concur well. Moreover, gradual decreases in the dominant frequencies are revealed from the time of 2.0 s through 6.0 s. In RUN5, marked decreases from the time of 2.0 s through 4.0 s imply the specimen failure, which is represented well by the analysis.



Fig. 20 Comparisons of acceleration spectra



Fig. 21 Comparisons of non-stationary spectra

Prestressed concrete containment vessel model

The specimen is a 1:10 scaled model of a prestressed concrete containment vessel [20]. Figure 22 shows the specimen configuration. The specimen consists of a prestressed concrete cylindrical wall with two circular openings, a reinforced concrete base slab, and a top slab attached by additional masses. Steel lining plates are anchored to the inside surface of the cylindrical wall.

The shaking table test comprised 14 excitations, including horizontal and vertical excitations with and without internal pressure. The final excitation was continued until the specimen failed. Input motions to the shaking table are standard design earthquakes designated as " S_1 " and " S_2 " for nuclear facilities in Japan. The maximum input acceleration in S_2 excitation is 4.22 m/s². The excitation cases were named as "4.0S₂", "5.0S₂", indicating that the amplitudes of input acceleration were multiplied by four and five, respectively.

In the early stage of analysis, a whole model was made and analyzed for several excitations. That method was very time consuming. Therefore, the finite element model was simplified and reduced to a half model, as shown in Fig. 23. The model does not include the base slab. It is fixed to the bottom of the cylindrical wall. The height of the additional masses around the wall is adjusted to the top slab thickness. The effect of these simplifications on calculation was investigated by comparing analysis results of both models for several excitations. It was shown to be negligible.

First, eigenvalue analysis is performed. The first mode is dominated by horizontal translation and its natural frequency is 11.4 Hz, which is slightly higher than the observed value of 11.1 Hz. This difference in the natural frequencies can be attributed mainly to the fixed condition at the bottom of the specimen. Generally, actual fixed condition is not perfect as it is in the analysis.

Secondly, static analysis is conducted for self-weight and prestressing loads. Subsequently, time history analyses are performed according to the actual test sequence.



Automatic time increment control is adopted and the basic time increment is set to 0.005 s, as with the analysis of the H-sectioned wall. However, control by the upper limit of the input acceleration increment was inadequate: it caused premature specimen failure, probably because of the delay in the response of heavy top portion. In other words, the response depends to a large extent on the former input acceleration. Another control scheme is introduced to overcome this problem: if acceleration or displacement response of the top slab exceeds the prescribed upper limit, then a control scheme reduces the time increment size and recalculates with a new increment size. In this analysis, allowable maximum responses in acceleration and displacement at the center of the top slab are 500 mm/s² and 0.05 mm, respectively. This cutback scheme engendered a stable solution and did not require control by the limit of input acceleration increment.

In analyses for the design level horizontal excitation, calculated responses for " S_1 " and " S_2 ", in which the specimen responds almost elastically, differed from the measured responses. Detailed examination of the measured responses revealed that the base slab rotated during horizontal excitations. To incorporate the rotational motion in the analysis, average rotational acceleration at the wall bottom is evaluated according to the following procedures. First, the difference in vertical acceleration of two measuring points along the shaking direction is calculated. Then the average rotational acceleration is obtained by dividing the vertical acceleration difference by its distance. That acceleration is applied to the specimen as shown in Fig. 24. Rotational acceleration is applied to each node of the model as translational acceleration according to the angle and the distance from the center of rotation, which is inferred to be the midpoint of two measuring points.

Comparison of horizontal acceleration responses of the top slab by " S_2 " horizontal excitation, in which a few cracks are observed both in the test and the analysis, is shown in Fig. 25a. The acceleration waveform calculated by the analysis represents the measured one with quite good accuracy. Figure 25b compares measured and calculated acceleration responses of the top slab in " $4.0S_2$ " horizontal excitation whose duration time is extended four times longer than the original " S_2 ". Both results agree well except that the calculated acceleration is somewhat larger than the measured one around 7 s and 55 s. Horizontal acceleration caused shear failure of the specimen at 5.7 s. Analysis showed the same type of failure from 5.5 s through 6.0 s.



Final cracking patterns on the outer surface of the cylindrical wall are compared in Fig. 26. The specimen showed shear failure of the cylindrical wall during "5.0S₂" excitation. The failure was initiated near the opening, then progressed in the upper part of the wall. Crack directions and the failure region of the analysis correspond well with the observed ones.

Figure 27 shows a comparison of the transfer functions calculated from the acceleration response of the top slab and the bottom of the cylindrical wall. Although the transfer function obtained from analysis of " $4.0S_2$ " excitation showed a steep peak compared with the test result, a complex shape and several peaks are found both in the test and the analysis. With regard to " $5.0S_2$ " excitation, some differences are seen in the peak frequencies. However, the magnification factor and the change of frequency characteristics obtained from the analysis correspond well with those of the test.





CONCLUSIONS

Constitutive models that feature hysteretic characteristics of reinforced concrete and the method for time increment control and rotational acceleration input are developed and applied for simulation analyses of various types of experiments. The results are summarized as follows:

- 1) The static cyclic analyses reproduced observed load displacement relationships and cyclic degradation characteristics of the specimens with sufficient accuracy.
- 2) In time history analyses, a consistent method; 1% uniform damping for all excitations; can simulate the actual response not only in the elastic range, but also in the nonlinear domain.
- 3) Time increment control by the upper limit of acceleration and displacement response contributes to obtaining reliable solutions in nonlinear dynamic analysis.
- 4) Rotational motion of the shaking table can be considered in the analysis by the proposed method. It leads to precise simulation of the shaking table tests.

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