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STRAIN LIMITS FOR PLASTIC HINGE REGIONS OF CONCRETE REINFORCED COLUMNS

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SUMMARY

Establishing reliable acceptance criteria for deformations in structural members is crucial for the successful use of performance-based methods in the design and evaluation of structures. This paper explores extending the applicability of expressions proposed for the ultimate compressive strain in the concrete and for the plastic hinge length of reinforced concrete beams to those for reinforced concrete columns. The limiting concrete strain was evaluated based on a consistent definition for limiting drift of 153 reinforced concrete column tests, and compared with the values obtained using the previously proposed expressions. An optimized equation for limiting compressive strain was developed. This equation indicates a base level of strain in the extreme compression fiber of the concrete of 1% for the column failure criterion used in the evaluation.

INTRODUCTION

Establishing reliable acceptance criteria for deformations in structural members is crucial to the successful execution of performance-based methods for design and evaluation of structures. Limiting values of compressive strain in the concrete within plastic hinge regions are of particular significance to characterize the expected performance of ductile members. W. Gene Corley [3] and A. L. L. Baker [4] proposed expressions for the limiting compressive strain in the concrete and for the plastic hinge length of reinforced concrete members. These expressions were developed on the basis of measured plastic rotations in beams. Because design provisions for beams require lightly reinforced members that fail in a ductile manner, these expressions were derived for and are applicable to members in which the absence of axial compression and the relatively low amount of reinforcement result in a neutral axis depth that is small compared with the overall depth. The focus of this paper is to explore the applicability of these expressions to reinforced concrete columns, where the location of the neutral axis can extend beyond the cross section, and consequently the gradient of flexural strains can be significantly lower.

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The investigation was performed using a database of 153 reinforced concrete columns. All columns included in this analysis were subjected to cyclic lateral displacements, had observable failures, and aspect ratios exceeding 2.5. An original database of 184 specimens was obtained using three main sources: a column database from the University of Washington, research and thesis reports, and publications presented in engineering journals. Details of the column data, including dimensions and material properties, as well as limiting drift ratios, can be found in Brachmann [1]. This database was then reduced to 153 columns due to unreported member properties in some of the tests.

CALCULATION OF LIMITING CONCRETE STRAIN

Limiting Strain in the Concrete

The limiting strain in the extreme compression fiber of each column specimen was calculated for the deformed configuration at the limiting drift. The limiting drift was defined as the lateral drift corresponding to a reduction in shear strength of 20%. The concrete strain in the extreme fiber was determined using the procedure described below.

The total lateral displacement (Δ_t) was defined as the sum of the elastic (Δ_{el}) and plastic displacement (Δ_{pl}) of the member

$$\Delta_t = \Delta_{el} + \Delta_{pl} \tag{1}$$

where

$$\Delta_{el} = \theta_y \left(\frac{2l}{3}\right) = \left(\frac{\phi_y l}{2}\right) \cdot \left(\frac{2l}{3}\right) \tag{2}$$

and

$$\Delta_{pl} = \theta_{pl} \cdot \left(l - \frac{l_p}{2}\right) = \phi_{pl} \cdot l_p \cdot \left(l - \frac{l_p}{2}\right) \tag{3}$$

The rotations θ_y and θ_{pl} are those corresponding to the onset of yielding of the longitudinal reinforcement and the inelastic range of response, respectively (Fig. 1). The distance along the member from the point of maximum moment to the point of contraflexure is defined as l, and the plastic hinge length is represented by l_p. The curvature at yield ϕ_y and the curvature associated with inelastic behavior $\phi_{pl} = \phi_u - \phi_y$, are geometric properties defined using the relationship

$$\phi = \frac{\varepsilon_c}{c} \tag{4}$$

where ε_c is the strain in the extreme compression fiber of the concrete, and c is the neutral axis depth of the member. The limiting strain in the extreme compression fiber of the concrete was determined using Eqs. 1-4.

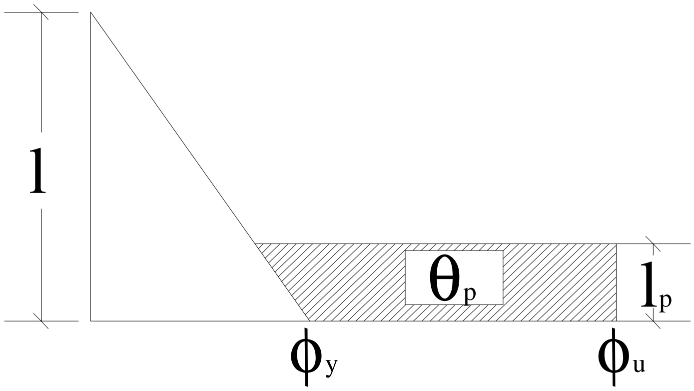

Figure 1. Assumed curvature distribution at the limiting drift

Displacement associated with shear and slip of the reinforcement

The limiting concrete strain was determined using two different definitions for the total drift. For the first case the effects of deformations related to shear and slip of the reinforcement on the limiting drift of columns were neglected. For the second approach, the total limiting drift was defined as the sum of three components

$$\Delta_{\lim} = \Delta_f + \Delta_s + \Delta_b \tag{5}$$

where $\Delta_{\lim}$ is the limiting drift, and Δ_f, Δ_s, and Δ_b are the components related to flexure, shear, and bond slip, respectively. The flexural component of the limiting drift was calculated by subtracting the components related to shear and slip of the reinforcement from the total. The displacement related to shear was assumed to be (Matamoros [5])

$$\Delta_s = \frac{6V}{5bd} * \frac{2(1+\upsilon)}{E} * 10 \tag{6}$$

where V is the maximum measured shear applied to the member, b is the width, d is the effective depth, υ is Poisson's ratio, and E is the modulus of elasticity for the material. The displacement due to slip of the reinforcement was defined as

$$\Delta_b = \theta_b l = \left(\frac{\varepsilon_y f_y d_b}{8\mu(d-c)} \right) * l \tag{7}$$

where θ_b is the rotation of the member due to slip of the reinforcement, f_y is the yield strength of the longitudinal reinforcement, d_b is the diameter of the longitudinal reinforcing bars, and μ is the average bond stress. An average bond stress of $6\sqrt{f'_c}$ was assumed in this analysis. This value was suggested as a lower bound for beams subjected to monotonic loading [A similar value has provided adequate results been used in previous studies to estimate the response of reinforced concrete frames [2]. The flexural displacement, Δ_f, was then determined as the remaining portion of Δ_t as defined in Eq. 5.

Theoretical Equations for Ultimate Concrete Strain

The main goal of the study was to determine whether two equations previously proposed to calculate the limiting compressive strain in the concrete in beams could be used in the case of columns, and to determine how applicable these equations would be for calculating the limiting compressive strain at the limiting drift as defined by Brachmann [1]. The first equation was proposed by Corley [3]

$$\varepsilon_{c\,\mathrm{lim}} = .003 + .02\left(\frac{b}{l}\right) + \left(\frac{\rho_{vs} f_{yh}}{138}\right)^2 \tag{8}$$

where f_{yh} is the hoop steel yield strength in units of megapascals, ρ_{vs} is the volumetric ratio of binding reinforcement, b is the width of the member, and *l* is the distance from the critical section to the point of contraflexure. The second equation was proposed by Baker and Amarakone [4]

$$\varepsilon_{c\,\mathrm{lim}} = 0.0015\left[1 + 150\rho_s + (0.7 - 10\rho_s)\cdot\frac{d}{c}\right] \tag{9}$$

which can expanded to

$$\varepsilon_{c\,\mathrm{lim}} = 0.0015 + 0.225\rho_s + 0.00105\cdot\frac{d}{c} - 0.015\rho_s\cdot\frac{d}{c} \tag{10}$$

where ρ_s is the volumetric transverse reinforcement ratio, c is the neutral axis depth of the member at ultimate moment, and d is the effective depth of the member. The relevance of the parameters used in these expressions was analyzed with the goal of developing an alternative expression to estimate the limiting strain of the selected column tests.

Theoretical Equation for the Plastic Hinge Region

The limiting concrete compressive strain inferred from a known displacement value is dependent on the length of the plastic hinge, l_p. Corley [3] and Baker and Amarakone [4] proposed different expressions for the equivalent plastic hinge length of beams subjected to monotonic loads. According to Corley the length of the plastic hinge is given by

$$l_{p-C} = 0.8\cdot k_1\cdot k_3\cdot\left(\frac{c}{d}\right)\cdot l \tag{11}$$

Baker and Amarakone proposed the use of the following expression:

$$l_{p-B} = 0.5\cdot d + \sqrt{d}\cdot\left(\frac{l}{d}\right) \tag{12}$$

where d is the effective depth of the member (mm), k_1 and k_3 are factors depending on the concrete and steel strengths, c is the neutral axis depth at ultimate moment (mm), and *l* is the distance from the critical section to the point of contraflexure (mm).

Definition of Failure

It should be noted that the definition of limiting drift of the columns analyzed in this paper differs from the definition of limit state adopted by Corley and Baker and Amarakone. In the tests performed by Corley, loads were removed after the maintainable applied load dropped significantly below the maximum load imposed on the beams. The values of limiting strain, curvature, and rotation were recorded at the first visible signs of distress in the compression zone. Due to the constraints of the testing machine, loads could not be maintained at a constant rate as the beams deformed. Baker and Amarakone also used a definition of limiting strain corresponding to the point at which the member began to deform greatly with increasing load, or at the first visible signs of concrete crushing at constant loads.

In the case of the members analyzed in this study, the limiting drift was defined as that corresponding to a 20% reduction of the maximum shear force recorded in the member [1]. Limiting strains were calculated based on the limiting drifts, according to Eq. 1 to 4.

ANALYSIS OF THE DATA

Normalization of Expressions

Equations 1 to 4 show that the limiting compressive strain in the concrete is dependent upon the limiting drift and the plastic hinge length, l_p that is adopted in the calculation. While Corley and Baker and Amarakone used the expressions shown in Eq. 11 and 12, respectively, a simple estimate of l_p equal to d/2 was used in the analysis presented in this paper. A review of the data showed that there was no improvement in the accuracy of the estimated limiting drift obtained using Eq. 11 or 12 in lieu of this simple assumption.

To facilitate an objective comparison between the different equations presented in this paper, the limiting compressive strain expressions proposed by Corley and Baker and Amarakone were normalized to account for the different plastic hinge lengths associated with those expressions derived. Limiting strains for an equivalent plastic hinge length l_p equal to d/2 were calculated by equating the estimated rotation for each of the expressions derived using Eqs. 11 and 12 over the respective plastic hinge length to that in a plastic hinge with a length of d/2 (Eq. 13)

$$\theta_{p-CB} = \theta_{p-\text{inferred}} \tag{13}$$

which leads to

$$\phi_{p-CB} \cdot l_{p-CB} = \phi_{p-\text{inferred}} \cdot \left(\frac{d}{2}\right) \tag{14}$$

and consequently

$$\frac{\varepsilon_{c\,\lim-CB} \cdot l_{p-CB}}{c} = \frac{\varepsilon_{c\,\lim-\text{inferred}} \cdot (d/2)}{c}. \tag{15}$$

Equation 16 was then used to normalize the expressions

$$\varepsilon_{c\,\lim-norm} = \varepsilon_{c\,\lim-CB} \cdot \left(\frac{l_{p-\text{inferred}}}{d/2}\right) = \varepsilon_{c\,\lim-\text{inferred}} \tag{16}$$

Normalized values of the limiting compressive strain estimated using the expressions proposed by Corley and Baker and Amarakone were used for the comparisons presented in this paper.

Figures 2 and 3 show the ratio of the limiting compressive strain in the concrete obtained from the Corley and Baker and Amarakone expressions to the values inferred from the limiting drift as define by Brachmann [1] (including for shear and slip deformations). Figures 4 and 5 show these ratios in the case where the effect of slip and shear deformations are neglected and it is assumed that the entire rotation is attributed to flexural deformations only. The solid line in the figures represents the trend line for the mean ratio, while the broken lines correspond to one standard deviation above and below the mean. The data are plotted separately for axial load ratios ($P/A_g f'_c$, where P is the axial load, A_g the cross sectional area, and f'_c the compressive strength of concrete) less than and exceeding 0.1, to indicate differing trends for beam and column specimens. The overall mean and standard deviation for the beam and column specimens are indicated on the figures.

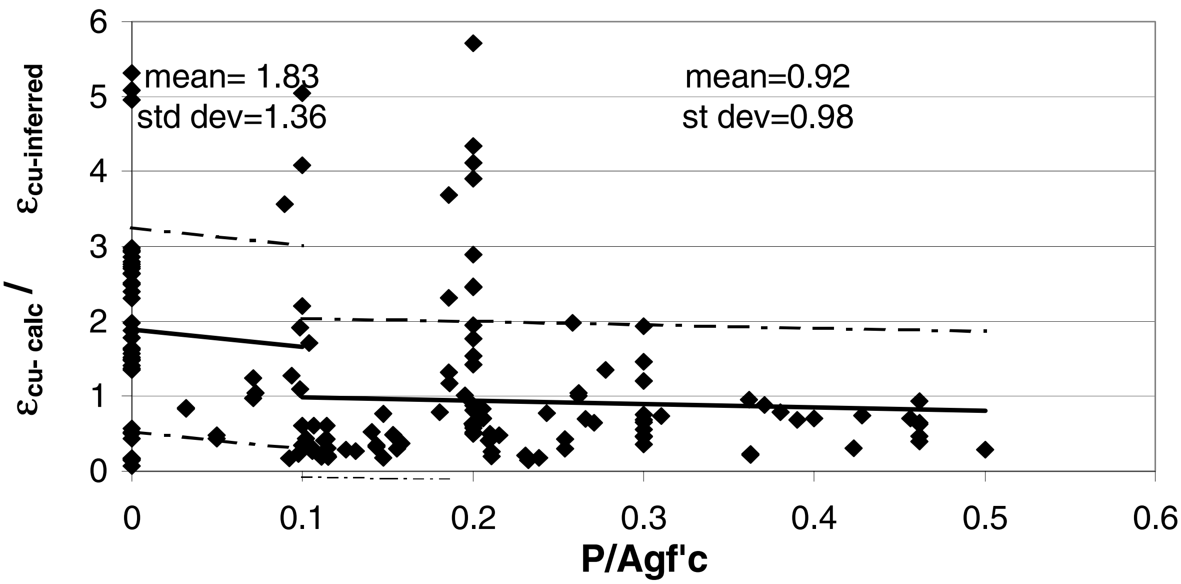

Fig. 2. Ratio of measured to inferred limiting compressive strain in the concrete according to the expression by Corley based on the flexural deformation, without the effects of deformations due to shear and slip of the reinforcement.

Evaluation of Figs. 2-5 indicates that the expression proposed by Corley typically overestimated the values of strain in the beam specimens for total and flexural displacements values. For columns, the expression overestimated much less for flexural displacements, and appears to work quite well when total displacement is utilized. The expression proposed by Baker and Amarakone underestimates the strain values at nearly all points. It is also interesting to note that for both proposed expressions, the column specimen data shows greater scatter than the beam data when flexural displacements are considered, but that this observation is reversed when total displacements are used and the column data has a lower standard deviation than the beam data.

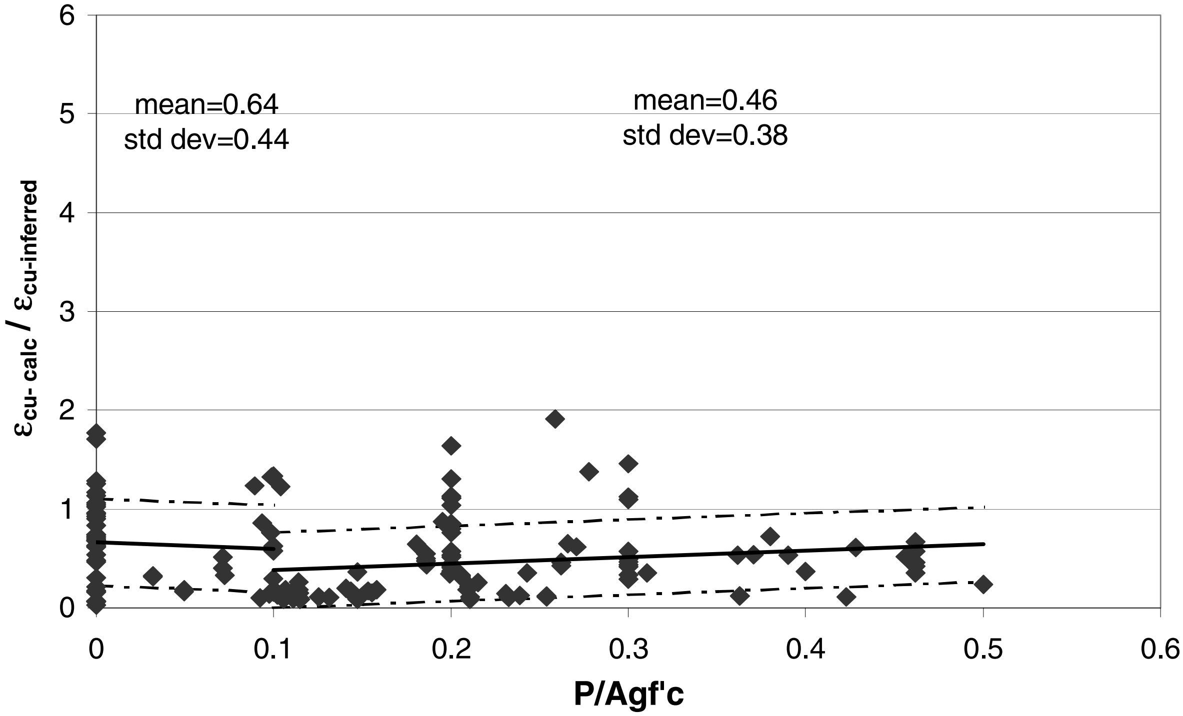

Fig. 3. Ratio of calculated to inferred limiting compressive strain according to the expression by Baker and Amarakone based on the flexural deformation, without the effect of deformations related to shear and slip of the reinforcement.

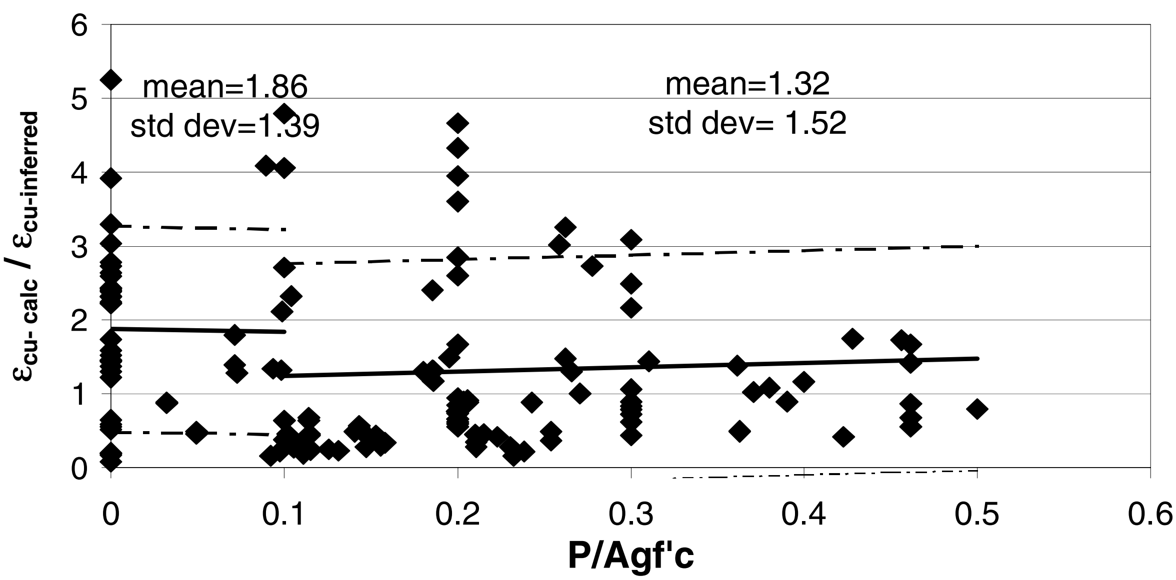

Fig. 4 Ratio of calculated to inferred limiting compressive strain in the concrete according to the equation proposed by Corley based on the total deformations, neglecting the effect of deformations associated with shear and slip of the reinforcement.

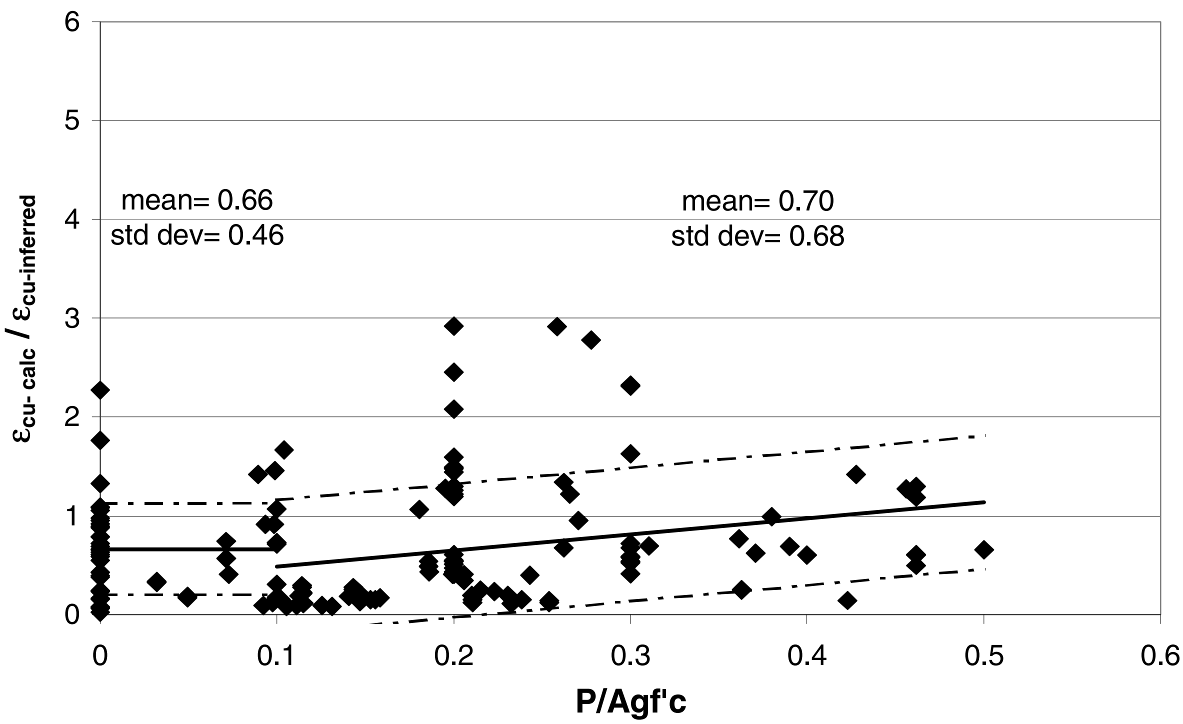

Fig. 5. Ratio of calculated to inferred limiting compressive strain in the concrete calculated using the expression by Baker and Amarakone based on total deformations, neglecting the effect of deformations related to shear and slip.

Linear Regression

A linear regression analysis was performed using the limiting compressive strains inferred from the selected column data and the parameters used in the equations proposed by Corley and Baker and Amarakone. The primary objective of this analysis was to determine the parameters with the lowest standard error, and therefore the best statistical fit to the data. The standard error calculated for each coefficient is shown in Table 1.

The regression analysis indicated a significant level of scatter in the data, as indicated by coefficients of variation on the order of 1. The expression proposed by Corley resulted in values of limiting strain that were closer to those inferred from the data based on the definition of limiting drift used by Brachmann[1]. The equation proposed by Baker and Amarakone resulted in more conservative estimates of the limiting strain, on the order of 65% of those obtained with the definition of limiting drift used by Brachmann.

From the regression analysis, it was noted that the parameter $(\rho_s f_{yh})^2$ from the equation proposed by Corley and the parameter d/c from the expression proposed by Baker and Amarakone had the lowest standard errors. These results indicate that the strain gradient and the amount of confinement represented by the product of the amount of transverse reinforcement and the yield strength of the reinforcement had the most significance on the limiting compressive strain in the concrete. This is consistent with a considerable body of literature indicating that confinement has the effect of increasing the strain at peak stress and reducing the slope of the descending branch of the stress-strain curve for concrete. It is interesting to note that the volumetric reinforcement ratio (ρ_s), when evaluated as a separate parameter resulted in a much higher standard error than the parameter $(\rho_s f_{yh})^2$.

Because these two parameters provided the best fit, they were maintained for evaluation of later expressions. The remaining parameters (b/*l*, ρ_s, and ρ_sd/c) had relatively large standard error values and were not used to develop an improved equation in later analyses. Although the constant portion of the expressions proposed by Corley and Amarakone had standard error values on the order of 10 to 100 times lower than the parameters eliminated from the study, the error values exceeded the other remaining parameters by a significant factor. Other variations on the exponent n for the parameter $(\rho_s f_{yh})^n$ yielded no significant improvement in statistical fit.

Table 1 Standard Error for Coefficients Determined for Linear Regression Analysis

Parameters	Displacement	Coefficient	Standard Error
Corley	Shear and slip neglected	Constant	0.00641
		b/l	0.02365
		$(\rho_{vs} f_{yh})^2$	0.00003
	Shear and slip considered	Constant	0.00561
		b/l	0.02064
		$(\rho_{vs} f_{yh})^2$	0.00002
Baker	Shear and slip neglected	Const	0.00435
		ρ_s	0.35280
		d/c	0.00102
		ρ_s d/c	0.13440
	Shear and slip considered	Constant	0.00477
		ρ_s	0.34831
		d/c	0.00103
		ρ_s d/c	0.13753

Optimization Analysis

After performing the linear regression to evaluate the significance of the parameters in the expressions by Corley and Baker and Amarakone, an optimization based on the most significant parameters in these equations was performed to attempt to reduce the amount of scatter evident from the first linear regression analysis. The format of the equation used in the optimization included the parameters $(\rho_s f_{yh})^2$ and d/c, as well as a new parameter to investigate the influence of aspect ratio *l*/d. Optimization was achieved as an iterative process by simultaneously changing the values of the coefficients to produce the best statistical fit to the selected data by minimizing the coefficient of variation. The optimization was performed twice. First, the process was executed with the goal of achieving normalized theoretical values of limiting strain that were closest to the inferred values (a mean ratio of calculated to inferred limiting strain equal to unity). A second optimization was performed with no constraint on the mean value of the ratio of these values. The results of this optimization are presented in Table 2, along with the normalized expressions proposed by Corley and Baker and Amarakone.

When the optimization was performed without restricting the mean value of the ratio of theoretical to inferred values of strain (labeled “force opt.” in the table), it can be seen that the coefficients of the parameters for the normalized expression proposed by Corley were similar to the coefficients found through optimization, with the exception of the b/*l* parameter. The optimized equation had a much smaller contribution for the term b/*l* than implied by the Corley expression, and it had an opposite sign (an unlikely relationship to exist in true behavior), which is consistent with the relative large standard error calculated in the previous regression analysis. The statistical significance of the parameters in the expression by Corley, as well as the new set of parameters, did not change significantly when the effect of displacements related to shear and slip of the reinforcement were considered in the optimization of the limiting compressive strain in the concrete. The optimized equation based on the parameters of the equation by Baker and Amarakone showed a similar trend, although the statistical fit was not as good as that of the normalized equation. In addition, statistical analyses based on the parameters by Baker and Amarakone were sensitive to whether the effects of displacements related to shear and slip were considered or not. The analysis neglecting the effects of displacements related to shear and slip of the reinforcement showed much higher constant values and a lower sensitivity to the remaining parameters (smaller coefficients), whereas the analysis that neglected these effects showed a lower constant strain value and was more sensitivity to the parameters (larger coefficients). The change in sign associated with the d/c term in the optimized equations is not the likely to reflect true behavior expected for reinforced concrete members.

Table 2 Results of Optimization of Proposed Equations

Parameters	**Type of disp**	**Analysis**	**Equation**	**Mean**	**St Dev**	**COV**
d/c , l/d, $(\rho_s * f_y)^2$	flexural	force opt	$\varepsilon_{c\ lim} = 0.013-0.0005(d/c) - 0.0001(l/d) + 6.6x10^{-5} (\rho_s * f_y)^2$	1.00	0.72	0.72
	total	force opt	$\varepsilon_{c\ lim} = 0.016-0.0012(d/c)+0.0003(l/d)+0.001(\rho_s * f_y)^2$	1.00	0.66	0.66
Corley b/l , $(\rho_s * f_y)^2$	flexural	opt	$\varepsilon_{c\ lim} = 0.006 - 0.001(b/l)+(f_y * \rho_{vs} /127)^2$	0.67	0.51	0.76
		force opt	$\varepsilon_{c\ lim} = 0.009-0.002(b/l)+(f_y * \rho_{vs} /103)^2$	1.00	0.76	0.76
	total	opt	$\varepsilon_{c\ lim} = 0.006-0.0033(b/l)+(f_y * \rho_{vs} /108)^2$	0.57	0.43	0.74
		force opt	$\varepsilon_{c\ lim} = 0.010-.006(b/l)+(f_y * \rho_{vs} /81)^2$	1.00	0.74	0.74
	actual eqtn	normalized	$\varepsilon_{c\ lim} = 0.0047+0.031(b/l)+(f_y * \rho_{vs} /110)^2$	0.68	0.71	1.05
Baker and Amarakone ρ_s, d/c, ρ_s * d/c	flexural	opt	$\varepsilon_{c\ lim} = 0.006+0.317(\rho_s)-0.002(d/c)-0.0019(\rho_s * d/c)$	0.64	0.48	0.75
		force opt	$\varepsilon_{c\ lim} = 0.009+0.49((\rho_s)-0.0003(d/c)-0.03((\rho_s * d/c)$	1.00	0.75	0.75
	total	opt	$\varepsilon_{c\ lim} = 0.0031+0.27((\rho_s)-0.0001(d/c)-0.0344((\rho_s * d/c)$	0.27	0.17	0.64
		force opt	$\varepsilon_{c\ lim} = 0.0113+0.99((\rho_s)-0.0003(d/c)-0.1265((\rho_s * d/c)$	1.00	0.64	0.64
	actual eqtn expanded	normalized	$\varepsilon_{c\ lim} = 0.0018+0.277((\rho_s)+0.0013(d/c)-0.018((\rho_s * d/c)$	0.74	0.92	1.23

When the mean ratio of theoretical to inferred values of the limiting compressive strain in the concrete was forced to a value of unity, it is interesting to note that all optimization results had a constant compressive strain in the concrete to be approximately 1%. This was true even for the optimized equation using only the newly selected parameters. The coefficients of variation improved using the optimized equations in every case, from 1.05 and 1.23 for expressions proposed by Corley and Baker Amarakone, respectively, to 0.66 in the best case using the new parameters. The improvement in statistical fit was similar when identical parameters were used in the comparison, leading to the conclusion that the base limiting compressive strain in the concrete was better represented using a value of 1%.

The expressions shown in the first two rows of Table 2 represent the results of an optimization with the new set of parameters. In some instances, parameters that had a small influence on the data set had signs that are counterintuitive to observed behavior. In these instances it is likely that the effect of the parameters is so small that scatter in the data is considerable more significant. As a result, the following two expressions are suggested for the limiting strain of the concrete. For the case when the limiting strain is applied to calculations that include the effect of displacements related to shear and slip of the reinforcement:

$$\varepsilon_{c\,\mathrm{lim}} = .0013 + \left(\frac{\rho_s f_{yh}}{130}\right)^2 \tag{17}$$

When it is assumed that the total deformation is related to flexure, the limiting strain should be

$$\varepsilon_{c\,\mathrm{lim}} = .0015 + \left(\frac{\rho_s f_{yh}}{30}\right)^2 \tag{18}$$

Ratios of calculated to inferred limiting strains in the concrete based on Eq. 17 and 18 are presented in Fig. 6 and 7. Both figures show that the level of scatter was considerably reduced, although it remains very significant.

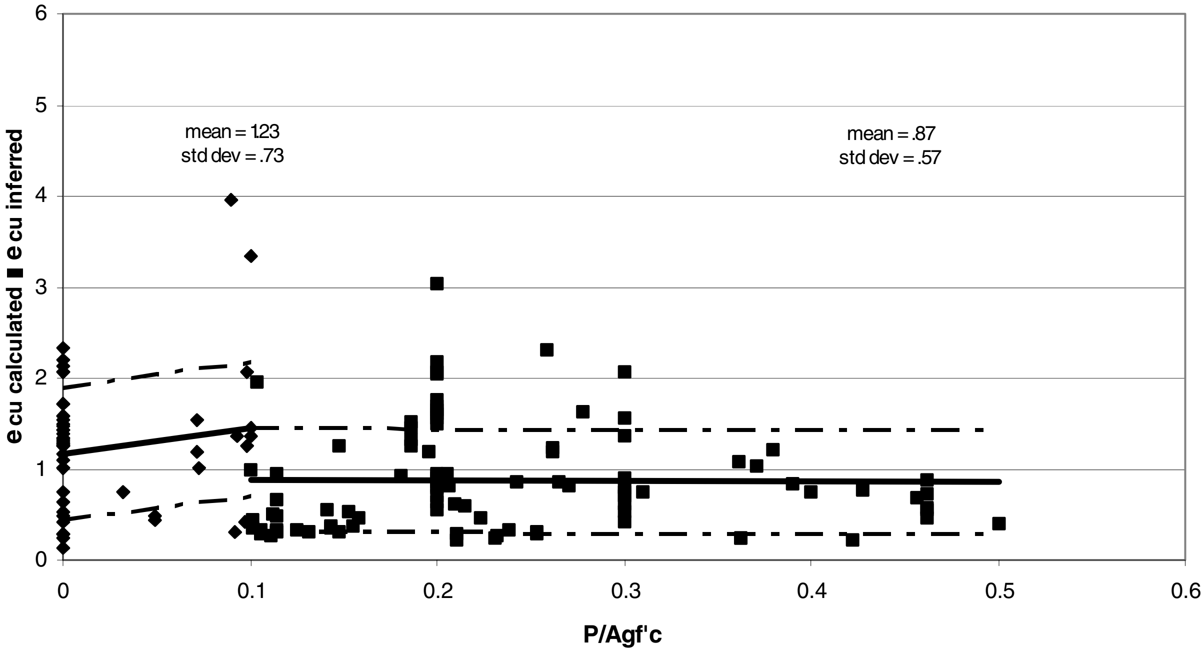

Fig. 6. Ratio of measured to inferred limiting compressive strain in the concrete according to the expression by Corley based on the flexural deformation, without the effects of deformations due to shear and slip of the reinforcement.

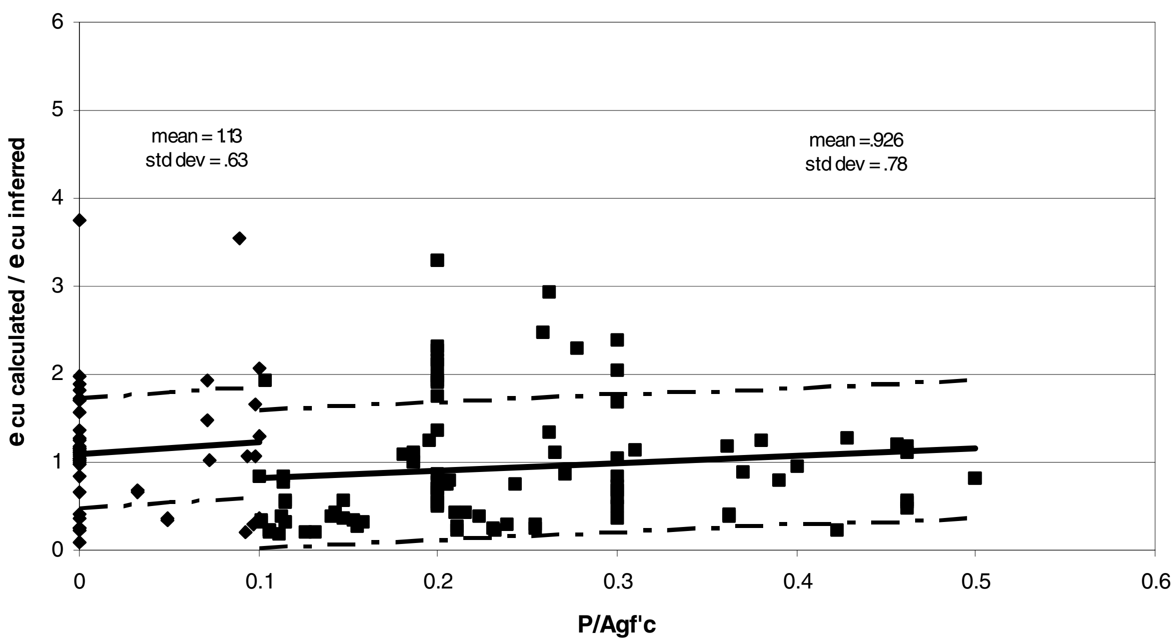

Fig. 7. Ratio of measured to inferred limiting compressive strain in the concrete according to the expression by Corley based on the flexural deformation, without the effects of deformations due to shear and slip of the reinforcement.

CONCLUSIONS

The evaluation of an expanded database of reinforced concrete members tested under lateral loads allowed for a new statistical analysis of the relationship between the limiting compressive strain in the concrete and relevant material and member parameters. The accuracy of the relationship between limiting compressive strain and limiting drift was not improved by using more detailed expressions that accounted for the effect of the spread of plasticity at the end of the member in lieu of the simple relation d/2. Independent of the parameters used to estimate the limiting compressive strain in concrete, the base strain needed to achieve a drift corresponding to a reduction in shear strength of 20% was on the order of 1%.

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