



## Study On Rock Microtremors Spectra Ratio

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### SUMMARY

Microtremors spectral ratio (H/V) is one of site effect measure method. It is assumed that the spectra ratio of the horizontal to vertical microtremors spectra on the rock surface equal unity. There is not satisfying theoretical explanation about it. The assumption is discussed in this paper. Microtremors is separated into body wave microtremors and surface wave microtremors. Body wave microtremors are supposed by superposing different incidence angle inclined P-wave, S-wave, P-SV waves. Surface wave microtremors is assumed as Rayleigh wave on elastic half space. The body wave microtremors spectra ratio expression of the horizontal to vertical spectra on the elastic half rock surface is deduced in the paper. The characteristics of microtremors spectral ratio of the horizontal to vertical spectra on the Poisson rock surface are discussed. It is shown that the H/V is close to unity for body microtremors and 0.68 for surface waves microtremors on the Poisson rock surface approximately. The spectral ratio of the horizontal to vertical spectra for surface microtremors is 0.54-0.79, when the Poisson ratio of rock changes from 0.5-0.

### INTRODUCTION

Site effect evaluation is important work for microzonation or site evaluation of important facilities. Present approaches to site effect evaluation are either theoretical or empirical. The theoretical approach is expensive for requiring detail geotechnical information. The empirical approaches rely on weak-to-strong motion or microtremors data recorded on the site. Microtremors measurements have become a little popular from 1989. Since 1950', Microtremors have been used basically in four different ways in relation with site response estimation: absolute spectra, spectra ratios with respect to reference site, Nakamura method (the ratio of horizontal and vertical spectra of one station or H/V) and velocity structure inversion through array records. The first three methods are used to estimate the site response in earthquake engineering. Though existed some questions for each method, Nakamura method seems have more quarrels than other methods about their theoretical foundation. Many engineering scientists considered that Nakamura method has not enough theoretical grounding, for numerous assumptions when the method is established. So far, there is not satisfying theoretical explanation about it. Microtremors spectral ratio method assumed that the spectra ratio of the horizontal to vertical microtremors spectra on the rock surface equal unity. Is it right? There is not theoretical explanation.

Numerical simulation is used investigate Nakamura method by Lachert and Bard<sup>[1]</sup>. They concluded that (1)

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the H/V is independent of the source excitation function; (2) H/V for obliquely incident SV waves show several peaks whose position is independent of the incidence angle, and whose fundamental position corresponds to the H/V peak for noise simulation; (3) The shape of H/V is largely controlled by the polarization curve of fundamental Rayleigh waves; The result show that the H/V method of microtremors gives reliable indication of the resonance frequency of a horizontally layered structure, but it seems still premature for deriving the amplification at resonance frequency from a theoretical point of view. Most of other papers about Nakamura method usually testify the method using the earthquake observation and microtremors measures [2]-[5].

Objectively, microtremors may be composed by P-SV waves or/and surface waves. The source of microtremors may be on surface of site or/and under site. Lachet and Bard discussed the noise excited by the surface source. The other part microtremors constituted by the underground source is discussed in the paper. In this paper, we will discuss one assumption used in H/V method, which is the spectral ratio of horizontal to vertical component is unity on the rock surface. The theoretical expressions of spectral ratio between horizontal and vertical component for inclined SV-wave and P-SV waves are deduced by elastic half space model. The spectral ratio characteristics of horizontal to vertical component for P-SV waves are studied. It is shown that the spectral ratio of microtremors horizontal to vertical component on Poisson rock surface is equal to unity for body wave microtremors.

## 2 Methodology

What kind of main wave components are included in microtremors? It is constituted by P-wave, SV-wave, P-SV wave and/or surface waves. Microtremors is assumed as P-wave, SV-wave, P-SV wave and/or surface waves. The former study or applications on microtremors usually assume the microtremors is S-waves or Rayleigh waves. These assumptions did not account for the stochastic of microtremors. In fact, it seems reasonable to consider microtremors as superposing different incidence angle inclined P-SV waves or/and Rayleigh wave. Underlying will deduce the H/V expression formula of microtremors on the rock surface.

2.1 the H/V expression of microtremors by SV-wave incidence to a free surface

Suppose a upward plane SV-wave incidence to a free surface at incidence angle  $\theta$  (see Fig. 1). The total wave field are comprised by reflected P-wave potential  $\phi$ , incidence and reflected SV-wave scalar potential  $\psi$ . The total displacement ratio of horizontal and vertical component can be expressed [6]:

$$R_s = \frac{U_0}{W_0} = \left[ -\sin \theta \frac{F_p}{E_s} + \left(1 - \frac{F_s}{E_s}\right) \cos \theta \right] \left/ \left[ A \left(-\frac{F_p}{E_s}\right) + \left(1 + \frac{F_s}{E_s}\right) \sin \theta \right] \right. \quad (1)$$

$$\begin{aligned} A &= \sqrt{\left(\frac{v_s}{v_p}\right)^2 - \sin^2 \theta} & \sin \theta \leq \frac{v_s}{v_p} \\ &= -i \sqrt{\sin^2 \theta - \left(\frac{v_s}{v_p}\right)^2} & \sin \theta > \frac{v_s}{v_p} \end{aligned} \quad (2)$$

Where  $U_0$  and  $W_0$  is the displacement spectrum of horizontal and vertical component on the rock surface spectrum, respectively.  $R_s$  is the spectral ratio between horizontal and vertical component for inclined SV-wave.  $E_s, F_p, F_s$  is the amplitudes of incidence SV-wave, reflect P-wave and reflect SV-wave, respectively.  $v_s, v_p$  is the wave speed of SV and P-waves.  $\theta$  is incidence angle.

Considering the sources of microtremors are very complicated, it is most probably that the microtremors is consisted of different incidence angle SV-wave. If we superpose the spectrum of different incidence angle ( $0^\circ$ – $90^\circ$ ) wave, we get the average spectrum ratio expression

$$\bar{R}_s = \frac{\sum_{\theta=0}^{90} U_0}{\sum_{\theta=0}^{90} W_0} = \int_0^{90} \left[ -\sin\theta \frac{F_p}{E_s} + \left(1 - \frac{F_s}{E_s}\right) \cos\theta \right] d\theta \bigg/ \int_0^{90} \left[ A \left(-\frac{F_p}{E_s}\right) + \left(1 + \frac{F_s}{E_s}\right) \sin\theta \right] d\theta \quad (3)$$

$\bar{R}_s$  is the superposed spectrum ratio of different incidence angle.

From equation (2) and equation (3), we know that the H/V of SV wave have nothing to do with the frequency of wave.

2.2 the H/V expression of microtremors by P-wave incidence to a free surface

Follow 2.1 we have the H/V expression of microtremors by P-wave incidence to a free surface<sup>°</sup>

$$\bar{R}_p = \frac{\sum_{\theta=0}^{90} U_0}{\sum_{\theta=0}^{90} W_0} = \int_0^{90} \left[ \sin\theta \left(1 + \frac{F_p}{E_p}\right) + \sqrt{\frac{v_p^2}{v_s^2} - \sin^2 \theta} \frac{F_s}{E_p} \right] d\theta \bigg/ \int_0^{90} \left[ \cos\theta \left(1 - \frac{F_p}{E_p}\right) + \sin\theta \frac{F_s}{E_p} \right] d\theta \quad (6)$$

$\bar{R}_p$  is the superposed spectrum ratio of different incidence angle. Other symbols are same as in equation

(1) and equation (2).

2.3 the H/V expression of microtremors by P-SV waves incidence to a free surface

For P-SV we have<sup>°</sup>

$$\bar{R}_{ps} = \frac{\sum_{\theta=0}^{90} U_{ps}}{\sum_{\theta=0}^{90} W_{ps}} = \int_0^{90} \left\{ \left[ \sin\theta \left(1 + \frac{F_p}{E_p}\right) + \sqrt{\frac{v_p^2}{v_s^2} - \sin^2 \theta} \frac{F_s}{E_p} \right] E_p + \left[ -\sin\theta \frac{F_p}{E_s} + \left(1 - \frac{F_s}{E_s}\right) \cos\theta \right] E_s \right\} d\theta \bigg/ \int_0^{90} \left\{ \left[ \cos\theta \left(1 - \frac{F_p}{E_p}\right) + \sin\theta \frac{F_s}{E_p} \right] E_p + \left[ A \left(-\frac{F_p}{E_s}\right) + \left(1 + \frac{F_s}{E_s}\right) \sin\theta \right] E_s \right\} d\theta$$

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$\bar{R}_{ps}$  is the superposed spectrum ratio of P-SV waves at different incidence angle. A is same as in equation (4), and other symbols are same as in equation (1) and equation (2).

2.4 The H/V of surface wave microtremors on the Poisson rock surface

The ratio of horizontal composition to the vertical composition of the surface wave on free Poisson rock field is discussed. From elastic dynamics, the horizontal and vertical displacement of Rayleigh waves are expressed<sup>[8]</sup>

$$\left. \begin{aligned} u_0 &= 0.42C \cos\left(\omega t - \frac{\omega}{V_R} x\right) \\ w_0 &= 0.62C \sin\left(\omega t - \frac{\omega}{V_R} x\right) \end{aligned} \right\} \quad (8)$$

Where  $u_0$  and  $w_0$  is the horizontal and vertical displacement of Rayleigh waves respectively.  $\omega$  and  $V_R$  are the round frequency and velocity,  $x$  is the direction of waves propagation, C is a constant, then we can constitute the ratio of the horizontal and vertical displacement of Rayleigh waves

$$u_0/w_0 = 0.68 \cos(\omega t - \omega/V_R \cdot x) / \sin(\omega t - \omega/V_R \cdot x) \quad (9)$$

From (9) and its fourier transfer, we have  $U_0/W_0 = 0.68$ . Where  $U_0, W_0$  represents the surface spectra of horizontal and vertical on free Poisson rock, respectively. In practice, the most of Poisson ratio of rock are 0.25 approxmaely.

### 3 The H/V characteristics of microtrmeors on the Poisson rock surface

H/V expressions of different kinds of microtremors are deduced in part 2 of the paper. The volumes of microtremors H/V are discussed in this section. Microtremors are assumed by P-wave, SV-wave, P-SV waves or/and Rayleigh wave. Their spectra ratio characteristics are discussed. It is shown that the spectral ratio, The spectral ratio of H/V on the surface rock is nearly equal to 1 for inclined incidence P-SV wave at any incidence angle ( $0^\circ$ -  $90^\circ$ ) on the rock surface (Fig. 2).

For the surface wave microtremors composed by Rayleigh wave, when the Poisson ratio of half elastic space changes from 0 to 0.50, the ratio varies from 0.54 to 0.79. When the Poisson ratio of half elastic space is 0.25, the ratio is 0.68. This is the surface microtremors spectrum ratio of horizontal to vertical component on free Poisson rock.

### 4 EXAMPLE

Figure 3 and figure 4 are the H/V of microtremors observed in Lijiang Basin of China by Seismology Bureau of Yunnan and Kyoto University. Figure 3 is observed at 15:05:05, on August 5, 1997. The record time is 15 minutes. Figure 4 is observed at 18:00:00, on August 5, 1997. The record time is 15 minutes also. The observe site is at the same place on the rock. We know that most of rock can be assumed Poisson rock<sup>[10]</sup>. The spectral ratio of horizontal to vertical (H/V) of microtremors is between 0.9-1.1 in figure 3. It shows that the main part of microtremors is body waves. In figure 4, the H/V of observed microtremors is between 0.6-0.8. It shows that the main component of microtremors belongs to Rayleigh waves.

### 5 CONCLUSION

1. The H/V is close to unity for body wave microtremors for superposed P-wave, SV-wave and /or P-SV waves at different incidence angle.
2. The H/V is changes from 0.54 to 0.79 for the surface wave, when the Poisson ratio of half elastic space from 0 to 0.50. For the Poisson half elastic space, the ratio is 0.68.

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