

VIBRATION ANALYSIS OF REINFORCED CONCRETE BEAM MEMBERS INCLUDING BOND-SLIP OF REINFORCEMENT

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SUMMARY

In this paper, a technique of analyzing the vibration problem of reinforced concrete beam members including bond-slip of the reinforcements is proposed. A displacement field is introduced to formulate the vibration problem of reinforced concrete beam members including bond-slip of reinforcement, consisting of 2+m independent variables corresponding to the longitudinal and vertical displacements u and w, as well as, the bond-slip displacements S_1, S_2, \ldots, S_m for the "m" steel layers.

The paper derives a finite element formulation of the problem and concludes with numerical examples with selected bond moduli and damping resulting from bond-slip, to examine the effects of bond-slip on the bending vibration of reinforced concrete beams.

INTRODUCTION

Reinforced concrete (RC) members require good bond between the concrete and reinforcement in order to function as hybrid elements. In general, the assumption of perfectly bonded behavior is reasonable only for the elastic response of RC members. The bond-slip phenomenon becomes important for RC members after cracks form, or for RC members with unbonded tendons.

Recently, there has been increased interest in structural framing systems which make use of concrete members post-tensioned with unbonded tendons, for example PRESSS [1]. One of the authors has conducted research on the analytical methods for concrete beams [2][5][7][8] and concrete slabs [3][4][6][9] taking into account the bond-slip of reinforcement including material non-linearity and geometrical nonlinearities. This research has been based on only statics. To date, there has been little research considering the vibration analysis of RC members with bond-slip of reinforcement.

This paper deals with the free-vibration analysis and sinusoidal dynamic analysis of reinforced concrete beam members considering the bond-slip of reinforcement.

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FORMULATION FOR RC BEAM MEMBER INCLUDING BOND-SLIP OF REINFORCEMENT

Fundamental assumptions for analysis

The following fundamental assumptions were made for the present analysis:

- (1) The proposed analytical method is based on small displacement theory, with the additional assumption that bending and axial forces dominate the deformation of the beam members. Here, transverse shear deformations are not taken account.
- (2) Concrete is assumed to behave elastically with cross sections remaining plane when deformed.
- (3) Reinforcement is assumed to be elastic bar members carrying only axial load. The reinforcement is arranged in m-layers in the concrete cross section along the span direction.
- (4) Bond-slip occurs along the curve of the reinforcement, between the concrete and each reinforcement layer. The relationship between bond stress and slip displacement is assumed linear.

Displacement fields

Displacement fields we must establish are axis displacement u in the axial direction, bending displacement w in the vertical direction perpendicular to the axis, and bond-slip displacements S_i (i=1, ..., m), for a total of 2+m independent variables.



Fig. 1 Coordinate System and Symbols of Element

Displacement, acceleration, strain, and stress in concrete and reinforcement

Concrete

With reference to the coordinate system in Fig. 1, the displacement u_c and acceleration \ddot{u}_c in concrete are given by,

$$u_c = u - zw', \qquad \ddot{u}_c = \ddot{u} - z\ddot{w}' \tag{1}$$

Strain and stress at the point are given by,

$$\varepsilon_c = u'_c = u' - zw'' \qquad \sigma_c = E_c u'_c = E_c \left(u' - zw''\right) \tag{2}$$

where z is height from center of RC beam section, and E_c is the Young's modulus of concrete.

Reinforcement

The displacement u_{Si} and acceleration \ddot{u}_{Si} of an arbitrary *i* th-layer of reinforcement are given by,

$$u_{si} = u - z_{si}w' + S_i, \quad \ddot{u}_{si} = \ddot{u} - z_{si}\ddot{w}' + \ddot{S}_i$$
 (3)

Strain and stress at the point are

$$\varepsilon_{si} = u'_{si} = u' - z_{si}w'' + S'_i \qquad \sigma_{si} = E_{si}u'_{si} = E_{si}(u' - z_{si}w'' + S'_i)$$
(4)

where z_{Si} is height of *i*-layer of reinforcement from center of RC beam section, and E_{Si} is the Young's modulus of the *i* th-layer of reinforcement.

Sectional force

The axial force, N, and bending moment, M, at the section level of a RC beam, taking the bond slip of reinforcement into account are given by,

$$N = \left(A_0 E_c + \sum_{1}^{m} A_{si}^* E_{si}\right) u' - \left(\sum_{1}^{m} A_{si} E_{si} z_{si}\right) w'' + \sum_{1}^{m} A_{si} E_{si} S_i'$$
(5)

$$M = \left(\sum_{1}^{m} A_{si}^{*} E_{si} z_{si}\right) u' - \left(E_{c} I_{0} + \sum_{1}^{m} A_{si}^{*} E_{si} z_{si}^{2}\right) w'' + \sum_{1}^{m} A_{si} E_{si} z_{si} S_{i}'$$
(6)

The shear force Q is then given by differentiating the bending moment M with respect to x

$$Q = \left(\sum_{1}^{m} A_{si}^{*} E_{si} z_{si}\right) u'' - \left(E_{c} I_{0} + \sum_{1}^{m} A_{si}^{*} E_{si} z_{si}^{2}\right) w''' + \sum_{1}^{m} A_{si} E_{si} z_{si} S''_{i}$$
(7)

where A_0 is the sectional area of RC beam, A_{Si} is the sectional area of the *i* th-layer of reinforcement, and A_{Si}^* is the converted sectional area taking into account the difference in the Young's modulus of concrete and reinforcement, and I_0 is the geometric moment of inertia for plain concrete.

Principal of virtual work

The principal of virtual work can be written

$$\delta U - \delta T - (\delta W - \delta D) = 0 \tag{8}$$

where δU is the variation in strain energy, δT is the variation in kinetic energy, δD is the variation in damping energy and δW is the virtual work of the external force.

Kinetic energy

The kinetic energy of the model beam (with span L) is

$$T = T_{cx} + T_{stx} + T_{cz} + T_{stz} \tag{9}$$

where T_{cx} and T_{stx} are kinetic energies in the axis direction of the RC beam and T_{cz} and T_{stz} are kinetic energies in the vertical direction.

The variationals of each kinetic energy ∂T_{cx} , ∂T_{stx} , ∂T_{cz} , ∂T_{stz} are given by

$$\delta T_{cx} = -\int_0^L \left(\int_{-t/2}^{t/2} \rho_c B_0 \ddot{u}_c \delta u_c dz - \sum_1^m \rho_c A_{si} \ddot{u}_c \delta u_c \right) dx \tag{10}$$

$$\delta T_{stx} = -\int_0^L \sum_{1}^m \rho_{si} A_{si} \ddot{u}_{si} \delta u_{si} \, dx \tag{11}$$

$$\delta T_{cz} = -\int_0^L \left(\int_{-t/2}^{t/2} \rho_c B_0 \ddot{w} \delta w dz - \sum_1^m \rho_c A_{si} \ddot{w} \delta w \right) dx \tag{12}$$

$$\delta T_{stz} = -\int_0^L \sum_{1}^m \rho_{si} A_{si} \ddot{w} \delta w \, dx$$
(13)

Substituting (1) and (4) into (10), (11), (12), (13) and in turn into (9) gives

$$\delta T = -\int_{0}^{L} \left[\delta u \Big(\rho_{c} A_{0} \ddot{u} + \sum_{1}^{m} \rho_{si} \bar{A}_{si} \ddot{u} - \sum_{1}^{m} \rho_{si} \bar{A}_{si} z_{si} \ddot{w}' + \sum_{1}^{m} \rho_{si} A_{si} \ddot{S}_{i} \Big) \right] \\ + \delta w' \Big(\rho_{c} I_{0} \ddot{w}' - \sum_{1}^{m} \rho_{si} \bar{A}_{si} z_{si} \ddot{u} + \sum_{1}^{m} \rho_{si} \bar{A}_{si} z_{si}^{2} \ddot{w}' - \sum_{1}^{m} \rho_{si} A_{si} z_{si} \ddot{S}_{i} \Big) \\ + \delta w \Big(\rho_{c} A_{0} + \sum_{1}^{m} \rho_{si} \bar{A}_{si} \Big) \ddot{w} + \sum_{1}^{m} \delta S_{i} \rho_{si} A_{si} (\ddot{u} - z_{si} \ddot{w}' + \ddot{S}_{i}) \Big] dx$$
(14)

Strain energy

The strain energy of the beam model (with span L) is

$$U = U_c + U_{st} + U_{bs} \tag{15}$$

where U_c and U_{st} are the strain energies in the concrete and reinforcement, respectively, and U_{bs} is the energy of the bond-slip in the concrete and reinforcement.

The variationals of each kinetic energy U_c , U_{st} and U_{bs} are given by

$$U_{c} = \frac{1}{2} \int_{0}^{L} \int_{-t/2}^{t/2} B_{0} E_{c} \varepsilon_{c}^{2} dz dx - \frac{1}{2} \int_{0}^{L} \sum_{1}^{m} A_{si} E_{c} \varepsilon_{c}^{2} dx$$
(16)

$$U_{st} = \frac{1}{2} \int_0^L \sum_{1}^m A_{si} E_{si} \varepsilon_{si}^2 dx$$
(17)

$$U_{bs} = \frac{1}{2} \int_0^L \sum_1^m A_{bsi} \tau_{bi} S_i dx = \frac{1}{2} \int_0^L \sum_1^m A_{bsi} K_{bi} S_i^2 dx$$
(18)

where K_{bi} is the bond modulus of *i* th-layer of reinforcement, A_{bsi} is the bond surface area per unit length of the *i* th-layer of reinforcement and τ_{bi} is the bond stress of the *i* th-layer of reinforcement given by $\tau_{bi} = K_{bi}S_i$.

Substituting (1), (2), (3)and (4) into (16), (17), (18) and in turn into (15) gives

$$\delta U = \int_{0}^{L} \left[(A_{0}E_{c} + \sum_{1}^{m} A_{si}^{*}E_{si})u'\delta u' - (\sum_{1}^{m} A_{si}^{*}E_{si}z_{si})(u'\delta w'' + \delta u'w'') + (I_{0}E_{c} + \sum_{1}^{m} A_{si}^{*}E_{si}z_{si}^{2})w''\delta w'' + \sum_{1}^{m} A_{si}E_{si}(S_{i}'\delta u' + \delta S_{i}'u') - \sum_{1}^{m} A_{si}E_{si}z_{si}(\delta S_{i}'w'' + S_{i}'\delta w'') + \sum_{1}^{m} A_{si}E_{si}S_{i}'\delta S_{i}' + \sum_{1}^{m} A_{bsi}K_{bsi}S_{i}\delta S_{i} \right] dx$$
(19)

Damping

In the response analysis presented here, we consider both modal damping, δD_{mod} , and damping produced by the bond-slip of the reinforcement, δD_{mod} . The modal damping matrix can be obtained from the eigenvalues, eigenvectors and damping ratios for each of the modes considered. The damping energy of bond-slip δD_b is given by

$$\delta D_b = \int_0^L \left(\sum_{i=1}^m C_{bsi} A_{bsi} \dot{S}_i \delta S_i \right) dx \tag{20}$$

where C_{bsi} is the coefficient of viscous damping associated with the bond-slip of the *i*th-layer of reinforcement.

Therefore, δD in the Eq. (8) is given by

$$\delta D = \delta D_{mod} + \delta D_b \tag{21}$$

Virtural work of external load

The virtual work of external loading δW is different for each load condition. For the case where the external loads consist of axial load, P_u , vertical uniformly distributed load, q_z , and tension load in the *i* th-layer of reinforcement, P_{si} , the virtual work of the external loads is given by

$$W = \int_{0}^{L} (P_{u}(t)u'_{0} + q_{z}(t)w + \sum_{1}^{m} P_{si}(t)S'_{i})dx$$

$$\delta W = \int_{0}^{L} (P_{v}(t)\delta u'_{0} + q_{z}(t)\delta w + \sum_{1}^{m} P_{si}(t)\delta S'_{i})dx$$
(22)

$$W = \int_{0}^{m} (P_{u}(t)\delta u_{0} + q_{z}(t)\delta w + \sum_{1}^{m} P_{si}(t)\delta S_{i})dx$$

= $\left[P_{u}(t)\delta u_{0} + \sum_{1}^{m} P_{si}(t)\delta S_{i}\right]_{0}^{L} + \int_{0}^{L} q_{z}(t)\delta wdx$ (23)

EQUATIONS FOR FINITE ELEMENT METHOD

Displacement functions and their derivatives

The reinforced concrete beam elements proposed in this paper are shown in Fig. 2. The displacement functions associated with each generalized displacement are

$$u = \begin{bmatrix} 1 & x \end{bmatrix} \{ \alpha_{u0} & \alpha_{u1} \}^T = \begin{bmatrix} f_1 \end{bmatrix} \{ \alpha_u \}$$
(24)

$$w = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \{ \alpha_{w0} & \alpha_{w1} & \alpha_{w2} & \alpha_{w3} \}^T = \begin{bmatrix} f_3 \end{bmatrix} \{ \alpha_w \}$$
(25)

$$S_i = \begin{bmatrix} 1 & x \end{bmatrix} \{ \alpha_{si0} & \alpha_{si1} \}^T = \begin{bmatrix} f_1 \end{bmatrix} \{ \alpha_{si} \}$$
(26)



Fig. 2 Concrete Beam Element

where $\{\alpha_u\}$, $\{\alpha_w\}$ and $\{\alpha_{Si}\}$ are the generalized displacements of Δu , Δw and ΔS_i (*i*=1, ..., *m*.), respectively.

Thus, the derivative functions of these displacements needed in the subsequent formulation are

 $u' = [f'_1]\{\alpha_u\} = [0 \ 1]\{\alpha_u\} = [f'_u][C_u^{-1}]\{\delta u\}$ (27)

$$w' = [f'_3]\{\alpha_w\} = \begin{bmatrix} 0 & 1 & 2x & 3x^2 \end{bmatrix}\{\alpha_w\} = \begin{bmatrix} f'_w \end{bmatrix} \begin{bmatrix} C_w^{-1} \end{bmatrix}\{\delta w\}$$
(28)

$$w'' = [f_3'']\{\alpha_w\} = [0 \ 0 \ 2 \ 6x]\{\alpha_w\} = [f_w''][C_w^{-1}]\{\delta w\}$$
(29)

$$S'_{i} = [f'_{1}]\{\alpha_{si}\} = [0 \ 1]\{\alpha_{si}\} = [f'_{si}][C^{-1}_{si}]\{\delta S_{i}\}$$
(30)

where $\{\Delta u\}, \{\Delta w\}$ and $\{\Delta S_i\}$ are the nodal displacement vectors for $\Delta u, \Delta w$ and ΔS_i , respectively; $[C_u^{-1}], [C_w^{-1}]$ and $[C_{Si}^{-1}]$ are the inverse matrices of the modal damping matrix [C] for $\Delta u, \Delta w$ and ΔS_i (*i*=1,...,*m*.), respectively.

The nodal acceleration vectors $\{\Delta \ddot{u}\}$, $\{\Delta \ddot{w}\}$ and $\{\Delta \ddot{S}_i\}$, and the nodal velocity vectors $\{\Delta \dot{u}\}$, $\{\Delta \dot{w}\}$ and $\{\Delta \dot{S}_i\}$ are obtained in the same manner.

Finite element equation for vibration analysis

Substituting Eqs. (24)-(30) into Eq. (8), we get the following the dynamic equilibrium equation formulated in terms of the generalized finite element coordinate system for vibration analysis of reinforced concrete beams:

$$\begin{bmatrix} [M_{uu}] & [M_{us}] & [M_{uS}] \\ [M_{ww}] & [M_{wS}] \\ sym. & [M_{SS}] \end{bmatrix} \begin{cases} \Delta \ddot{u} \\ \Delta \ddot{w} \\ \Delta \ddot{S} \end{cases} + \begin{bmatrix} [C_{uu}] & [C_{us}] & [C_{uS}] \\ [C_{ww}] & [C_{wS}] \\ sym. & [C_{SS}] \end{bmatrix} \begin{cases} \Delta \dot{u} \\ \Delta \dot{w} \\ \Delta \dot{S} \end{cases} + \begin{bmatrix} [K_{uu}] & [K_{uw}] & [K_{uS}] \\ [K_{ww}] & [K_{wS}] \\ sym. & [K_{SS}] \end{bmatrix} \begin{cases} \Delta u \\ \Delta w \\ \Delta S \end{cases} = \begin{cases} P_u(t) \\ P_w(t) \\ P_S(t) \end{cases}$$
(31)

where $\{\Delta S\}$ is the slip displacement vector of the nodes with respect to all *m* reinforcement layers; and $\{P_u(t)\}, \{P_w(t)\}\$ and $\{P_s(t)\}\$ are the load vectors corresponding to the displacement vectors $\{\Delta u\}$, $\{\Delta w\}\$ and $\{\Delta S\}$, respectively.

EXAMPLE OF NUMERICAL CALCULATION

Analytical model

The example presented in this section considers the elastic free vibration and sinusoidal dynamic analysis of the model shown in Fig. 3. For sinusoidal dynamic analysis, nodes are established in the cross-section as shown in Fig.4 in order to reduce computational effort, For free vibration analysis, however, the concrete element is divided into halves. This model is a pre-cast RC beam connected by tendons to column stubs through joints made of epoxy resin adhesives. In the analyses, we assume tendons are fixed at the ends of the model (Nodes 1 and 11) and that the concrete in the column stubs is rigid. Table 1 shows the material properties assumed for the concrete, tendon and adhesive.



Table 1 Material Properties of Analytical Model

	Concrete	Tendon	Adhesive
Density (kg/m ³)	$2.3*10^{3}$	$7.85*10^3$	$1.25*10^{3}$
Young's Modulus (N/m ²)	$2.1*10^{10}$	$2.1*10^{11}$	$1.0*10^{9}$

Free vibration analysis

The eigenvalue analysis of the model is performed using Jacobi's method. As is standard practice, damping is neglected in the eigenvalue analyses. The parameter of interest in the analysis is the bond modulus K_b of the tendons. We analyze the case of unbonded tendons and the case corresponded to grouted tendons, for which the bond moduli are assumed to be $1.0*10^6$ N/m³ and $1.0*10^{11}$ N/m³, respectively. Figs. 5 and 6 show the axial deformation mode, the vertical deformation mode and the bond-slip modes for the first six natural modes of vibration. In these figures, the mode shapes have been normalized to a maximum value of 1.0.

Fig. 5 shows the results of eigenvalue analysis for the model with unbonded tendons. In the 1st, 2nd and 4th modes, vertical deformation predominates and it is seen that bond-slips occur in all cases. In the 3rd mode, axial deformation predominates and bond-slips occur almost equally in directions opposite to that of the axial deformation. In the 5th mode, bond-slip at the top and bottom occurs in the opposite direction, whereas, in the 6th mode, bond-slip at the top and bottom occur in the same direction. The natural frequencies of 5th mode and 6th mode are nearly identical.

Fig. 6 shows the results of eigenvalue analysis of the model with grouted tendons. In this case, vertical deformation predominates in the 1st, 2nd, 4th and 5th modes, and axial deformation predominates in the 3rd and 6th modes. Although bond-slip occurs at the edges of beam, it is very small in all of the modes considered.

If the natural frequencies of the two cases are compared for the first four modes (modes with similar shapes), the natural frequencies of the model with grouted tendons are about 4%-6% higher than those of the model with unbonded tendons, depending on the different values of bond modulus.



Fig. 6 Natural vibration mode ($K_b = 1.0 \times 10^{11} \text{ N/m}^3$)

Sinusoidal dynamic response analysis

In the sinusoidal dynamic response analysis, a concentrated sinusoidal load is applied at the center of the model. The amplitude of the load is 10KN with input periods of 0.025s, 0.05s, 0.1s and 0.2s. For the analysis presented here, only the first 10 cycles of loading are considered. Fig.7 shows the input load in for the case where the input period is 0.025s.

The parameters of interest in the analysis are the bond moduli and whether or not viscous damping corresponding to bond slip of tendons is included. The bond moduli are assumed to be the same as those in the free vibration analysis $(1.0*10^6 \text{ N/m}^3 \text{ and } 1.0*10^{11} \text{ N/m}^3)$. For all models the modal damping is assumed to be 3%. For the unbonded model case we also include the added viscous damping corresponding to bond slip of tendons. Table 2 shows the model for sinusoidal dynamic response analysis. For Model UD, the coefficient of viscous damping corresponding to bond slip for unit area of bond

surface of tendon is assumed to be $1.5*10^7$ N*s/m/m². This value is reported by Suzuki et al. [10] who conducted experiments on unbonded tendons which used an asphalt polymer as the tendon covering material.

In the sinusoidal dynamic response analysis, we adopt the Runge - Kutta method to solve the system of coupled differential equations given by Eq. (31).



Fig.7 Period of Input Load is 0.025s

Table 2 Analytical Models for Sinusoidal Dyn	namic Response Analys	sis
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	Tendon Bond	Damping	
	Condition		
Model UN	Unbonded	Modal damping	
Model UD	Unbonded	Modal damping + Bond-slip damping	
Model GM	Grouted	Modal damping	

Fig.8 shows the vertical displacement at Node 6 (center of model) for the input periods: (i) 0.025s, (ii) 0.05s, (iii) 0.1s and (iv) 0.2s. In case (i), the sinusoidal dynamic responses are different depending on the model. The response of Model GM tends to be larger than that of Models UN and UD, with the maximum absolute values for the three models, UN, UD and GM, being 0.144mm, 0.142mm and 0.150mm, respectively. This seems to depend on the natural frequency of the 1st mode, which for Model GM is closer to the input period than for the other two models. In case (ii), the response of Model GM is smaller than that of the other models, with the maximum absolute values for the three models, with the maximum absolute values for the three models, UN, UD and GM, being 0.525mm, 0.455mm and 0.365mm, respectively. In case (iii), the absolute maximum values for the three models, UN, UD and GM are 0.174mm, 0.167mm and 0.150mm, respectively, and in case (iv), 0.120mm, 0.115mm and 0.103mm, respectively. Comparing Model UN and Model UD, the maximum response decreases 1.4%, 13%, 4.0% and 4.2% when viscous damping of bond-slip is included, for cases (i), (ii), (iii) and (iv) respectively. From this it is clear that the damping effect of bond-slip decreases the near the response is to resonance, but in the cases when the input period is shorter or longer than the natural period, the damping effect of bond-slip is small.



Fig. 8 Vertical displacement at the center of analytical model

Fig. 9 shows the sinusoidal dynamic responses of the bond-slip of the bottom tendon at Nodes 2,3,4 and 5. Plots (i), (ii), (iii) and (iv) correspond to input periods 0.025s, 0.05s, 0.1s and 0.2s, respectively. It can be seen that the bond-slip response of Model UN is the largest at Node 5 (the center of the shear span of the beam), and that it is larger than those of the other models for each period of input. The bond-slip responses of Model UN at Nodes 2 and 3 (the nodes in stub) are very small except for case (i). The bond-slip of Model UD for each period of input is almost the same at Nodes 2, 3, 4 and 5, but with phase differences. The bond-slip responses of Model GM are smaller than for the other models, and Node 3 (the edge of beam) has the largest amount of slip among all of the nodes. From Fig.9, it is clear that the quantities of bond-slip, the distribution of bond-slip along the axis of the beam, and the phase difference, are different if models have different bond moduli and bond-slip damping.

CONCLUSIONS

This study proposes a model for vibration analysis, based on the finite element method, for concrete beam members including the bond-slip of reinforcement. In this paper, virtual work is used to derive equations based on the finite element method and numerical examples are provided. From the examples presented, it is clear that for free vibration analysis the effect of bond-slip on the characteristics of vibration depends on the different bond moduli assumed, whereas, for sinusoidal dynamic analysis, the effect of bond-slip on the response depends on the different bond moduli assumed, as well as, the bond-slip damping and period of the input.



Fig. 9 Bond-slip of bottom reinforcement at Nodes 2,3,4 and 5

REFERENCES

- Priestley M. J. N., Sritharan S., Conley J.R., and Pampanin S. "Preliminary Results and Conclusions From the PRESSS Five-Story Precast Concrete Test Building", PCI Journal, 11-12/1999, Vol.44, No.6.
- 2. Ueda M. and Dobashi Y. "Analysis of Deformations in Bending of Reinforced Concrete Beams including Bond-Slip of Reinforcement", Proceedings of JSCE, Japan Society of Civil Engineers, 1986.8. (In Japanese).
- 3. Ueda M. and Dobashi Y. "Analysis of Deformations in Bending of Reinforced Concrete Slabs including Bond-Slip of Reinforcement, Part 1: Governing equations in linear region with numerical examples", Journal of Structural and Construction Engineering, Architectural Institute of Japan, 1987.1. (In Japanese).
- 4. Ueda M. "Analysis of Deformations in Bending of Reinforced Concrete Slabs including Bond-Slip of Reinforcement, Part 2: Governing equations in geometrically nonlinear region with numerical examples", Journal of Structural and Construction Engineering, Architectural Institute of Japan, 1988.3. (In Japanese).
- 5. Ueda M. "Governing Equations of Prestressed Concrete Beams with Curved Tendons and Its Numerical Examples by the Finite Element Method", Concrete Research and Technology, Japan Concrete Institute, 1990.1. (In Japanese).
- 6. Ueda M. "Governing Equations of Prestressed Concrete slabs with Curved Tendons and Its Numerical Examples by the Finite Element Method", Concrete Research and Technology, Japan Concrete Institute, 1991.7. (In Japanese).
- 7. Koshikawa T., Ueda M., Uchiyama T. and Wada T., "Elastic Analysis of RC Timoshenko Beams with Various Cross-Sectional Types including Bond-Slip of Reinforcement", Journal of Structural and Construction Engineering, Architectural Institute of Japan, 2002.9. (In Japanese).
- 8. Koshikawa T., Ueda M., Wada T. and Kikuchi M. "Large Deflection Analysis of RC Beams Under Biaxial Bending including Bond-Slip of Reinforcements", Proceedings of the Japan Concrete Institute, 2003. (In Japanese).
- 9. Mizuno M., Ueda M., Kikuchi M. and Uchiyama T., "Material Nonlinear Analysis of Reinforced Concrete Slabs considering the effect of Bond-Slip of Reinforcement", Journal of Structural and Construction Engineering, Architectural Institute of Japan, 2003.7. (In Japanese).
- 10. Suzuki K., et al. "Study on Seismic Damping Effect Using Unbonded Prestressing Tendon Part 1 and Part 2", Summaries of technical papers of Annual Meeting Architectural Institute of Japan, 2001. (In Japanese).
- 11. Washizu K., "Variational methods in elasticity and plasticity", Pergamon Press, 1975.