

EMPIRICAL STUDY ON RELATIONSHIP BETWEEN HORIZONTAL AND VERTICAL GROUD MOTIONS

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SUMMARY

In seismic design of civil engineering structures, vertical component of input ground motion has rarely been taken into account. However, regarding earthquake response of critical structures, the vertical component sometimes plays an important role and needs to be considered in the design. Since the vertical and horizontal ground motions from an earthquake event have some common features such as source and path, it seems natural to think that there should be some close relationship between both components.

In this study, the earthquake ground motion is characterized in terms of Fourier spectra that include amplitude and phase spectra. The amplitude of the earthquake motions observed at a site on the ground surface has often been pointed that many spectral amplitude ratios H/V between horizontal and vertical components resemble each other, like those of the H/V ratios from microtremor observation (Konno and Ohmachi [1]). The H/V ratio is thought to be a site-specific characteristic.

As for the Fourier phase of the horizontal and vertical components, the phase difference or the group delay time is likely to show a simple relationship when it is plotted against frequency. In a higher frequency range, the phase difference between horizontal and vertical components tends to show a hyperbolic relationship. On the other hand, the phase difference becomes constant in terms of the group delay time of horizontal and vertical components in a lower frequency range.

INTRODUCTION

Following the 1995 Hyogoken-nanbu earthquake, the Japan Society of Civil Engineers (JSCE) issued the "Proposal on Earthquake Resistance for Civil Engineering Structures (JSCE[2])". One of the requirements of the proposals is to consider a so-called Level 2 input earthquake motion for seismic design of structures. The Level 2 (L2 for short) motion has been defined as following: The Level 2 Earthquake Motion to be used as an input motion for the seismic design of a specific civil engineering structure is the maximum intensity earthquake motion to be reasonably possible at the site for the structure

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considered. This kind of concept for the input ground motion implies that the motion is a source specific, site specific and structure specific motion. A preferred procedure to determine the motion is thought to consist of selecting a set of scenario earthquakes based on the tectonic setting of the region, to evaluate the respective ground motions at the site taking into account soil characteristics at the site and finally to select the most critical of these inputs based on the response of the structure considered. In accordance with the JSCE proposal, most of the Japanese seismic codes of civil engineering structures have been reviewed and revised with the L2 motion taken into account. However, in most cases, the motions that have been brought into the codes are limited to the horizontal component only of the earthquake motion. When it comes to earthquake response of critical structures such as long bridges and nuclear power plants, the vertical component sometimes plays an important role and is necessary to be considered in the design. Since the vertical and horizontal ground motions from an earthquake event have some common features such as source and path, it seems natural to think that there should be some close relationship between both of the components. Thus, as for the vertical motion to be used as an input motion for the seismic design, it should be reasonably possible and compatible with the horizontal motion prescribed in the codes. This study aims to characterize such vertical motion and to find empirical relationship between horizontal and vertical components of earthquake ground motions.

CHARACTERISTICS OF SPECTRAL AMPLITUDE RATIOS

In this study, the earthquake ground motion is characterized and formulated mainly in terms of Fourier spectra that include amplitude and phase spectra. The characterization and formulation process is explained using a pair of time histories $f_H(t)$ and $f_V(t)$ of strong motion acceleration observed at one of the K-net stations TTR008 during the 2000 Western Tottori earthquake (M_J7.3), Japan, where the subscripts H and V denote horizontal and vertical components, respectively. The 3-components time histories $f_H(t)$ and $f_V(t)$ are shown in Fig.1



TTR008 during the 2000 Tottoriken-western earthquake, Japan

The Fourier spectra $F_H(\omega)$ and $F_V(\omega)$, and the time histories $f_H(t)$ and $f_V(t)$ are related as a Fourier transform pair, and expressed by,

$$\begin{aligned} f_H(t) &\Leftrightarrow F_H(\omega) = \left| F_H(\omega) \right| e^{i\phi_H(\omega)} \\ f_V(t) &\Leftrightarrow F_V(\omega) = \left| F_V(\omega) \right| e^{i\phi_V(\omega)} \end{aligned}$$
 Eq.(1)

Subscript H expresses horizontal components EW or NS and V vertical component UD.

The Fourier amplitude spectra $|F_H(\omega)|$ and $|F_V(\omega)|$ of $F_H(\omega)$ and $F_V(\omega)$ associated with time histories in Fig.1, respectively, are shown in Fig.2.



Fig.2 Horizontal and vertical Fourier amplitude spectra $|F_H(\omega)|$ and $|F_V(\omega)|$.

From Eq.(1), the spectral ratio between the horizontal and vertical components becomes

$$\frac{F_H(\omega)}{F_V(\omega)} = \frac{\left|F_H(\omega)\right|}{\left|F_V(\omega)\right|} e^{i(\phi_H(\omega) - \phi_V(\omega))}$$
Eq.(2)

The amplitude of the earthquake motions observed at a site on the ground surface has often been pointed that many spectral amplitude ratios between horizontal and vertical components (H/V ratios) resemble each other, like those of the H/V ratios from microtremor observation (Lermo and Garcia[3]). Thus, the H/V ratio has been thought to be a site-specific characteristic. The H/V ratio $|R(\omega)|$ of the amplitude spectra $|F_H(\omega)|$ and $|F_V(\omega)|$ shown in Fig.2 is plotted in Fig.3, and is expressed as,

$$\left|R(\omega)\right| = \frac{\left|F_{H}(\omega)\right|}{\left|F_{V}(\omega)\right|}$$
 Eq.(3)



Fig.3 Spectral amplitude ratio $|R(\omega)|$ between horizontal and vertical components

Both of the Fourier spectral ratios in Fig.3 are often stable and similar to each other and, tend to show same peak frequency at 1Hz. This is a fact that the Fourier spectra include the information of the source, path and site-characteristics meanwhile the Fourier spectral ratios are almost independent of the effects of the source and path except for site-characteristics (Yamazaki and Ansary[4]).

THE PHASE DEFFERENCE AND GROUP DELAY TIME

The Fourier phase spectra $\phi_H(\omega)$ and $\phi_V(\omega)$ of $F_H(\omega)$ and $F_V(\omega)$ are plotted in Fig.4.



Fig.4 Fourier phase spectra of horizontal and vertical components plotted between - π and π

The plots in Fig.4 appear much scattered in the range between $-\pi$ and π , especially at around peak frequencies, say at around 1Hz for example. Note in Eq.(2) that on the right hand side, the spectral ratio is related with the phase difference between the horizontal and vertical components. Take the derivative of the phase difference as

$$\frac{d(\phi_H(\omega) - \phi_V(\omega))}{d\omega} = \eta_H(\omega) - \eta_V(\omega)$$
 Eq.(4)

where $\eta_{H}(\omega)$ and $\eta_{V}(\omega)$ are called the group delay times (Papoulis[5]) defined by

$$\eta_{H}(\omega) = \frac{d\phi_{H}(\omega)}{d\omega}$$

$$\eta_{V}(\omega) = \frac{d\phi_{V}(\omega)}{d\omega}$$
Eq.(5)

As is well known, the plots of the group delay times $\eta_H(\omega)$ and $\eta_V(\omega)$ shown in Fig.5 concentrate around the time where large amplitudes are observed in the time histories shown in Fig.1.



Fig.5 Horizontal and vertical group delay times of earthquake record shown in Fig.1

The difference in the group delay time $\eta_H(\omega) - \eta_V(\omega)$ given on the right hand side of Eq.(4) is plotted in Fig.6, which leads to an approximation expressed as,

$$\eta_{H}(\omega) - \eta_{V}(\omega) \approx 0$$

$$\therefore \frac{d\phi_{H}(\omega)}{d\omega} \approx \frac{d\phi_{V}(\omega)}{d\omega}$$

$$\therefore \phi^{H}(\omega) - \phi^{V}(\omega) \approx Const.$$
 Eq.(6)



Fig.6 The difference in the group delay time between horizontal and vertical components

In Fig.6, there seems to be little difference in the group delay time in a lower frequency range from 0.1Hz to 5Hz. In the meantime, the difference in the group delay time appears rather scattered in a higher frequency range. To discuss the phase characteristics in a higher frequency range, that is a shorter period range, the phase difference between horizontal and vertical components is plotted against period as shown in Fig.7. In a short period range up to 0.1 second, hyperbolic curves can be drawn. This is maybe because the vertical and horizontal motion are mainly made up of P-waves and S-waves, and also because the Pwave velocity is higher than the S-wave velocity.



Fig.7 Fourier phase differences between horizontal and vertical components against period.

The phase difference between horizontal and vertical components is defined by

$$\Delta \phi_{H-V}(T) = \phi_H(T) - \phi_V(T)$$
 Eq.(7)

When P and S waves travel the same distance H simultaneously, the phase difference as a function of period T is expressed by

$$\Delta\phi_{H-V}(T) = \frac{2\pi H}{T} \left(\frac{1}{V_s} - \frac{1}{V_p} \right)$$
 Eq.(8)

Where H is the distance, V_P and V_S are the velocities of the elastic body waves.

A approximated curve calculated by Eq.(8) coordinates with the hyperbolic relationship up to 0.1 second in Fig.7. That is to say, this hyperbolic relationship between horizontal and vertical phase in a shorter range is seemed to be due to the difference between the velocities of P and S-waves as expressed in Eg.(8).

CONCLUSIONS

In this paper, the properties of the Fourier amplitude ratios, the Fourier phase and the difference in the group delay time between horizontal and vertical components of earthquake ground motions are discussed. As the results, the Fourier amplitude ratios between horizontal and vertical components are thought to be site-characteristic. As for the difference in the group delay time is likely to be constant in a lower frequency range, meanwhile in a higher frequency range to be random. When the phase difference between horizontal and vertical components is plotted against period, a hyperbolic relationship is seen in the period range shorter than 0.1 second. This is maybe due to a fact that the vertical motion is mainly made up of P-wave and the horizontal motion of S-wave in the surface layers.

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