

# PREDICTORS OF SEISMIC DEMAND OF SMRF BUILDINGS CONSIDERING POST-ELASTIC MODE SHAPE

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# SUMMARY

Predictors of seismic structural demands such as inter-story drift angles that are less time-consuming than nonlinear dynamic analysis can be useful for structural performance assessment and for design. Luco and Cornell proposed a simple predictor extending the idea of modal superposition of the first two modes with the SRSS rule and taking a first-mode inelastic spectral displacement into account. This predictor achieved quite an improvement from simply using the response of an elastic oscillator; however, it cannot capture well the large displacement due to local yielding. The possible improvement of Luco's predictor is discussed in this paper considering a post-elastic mode.

# **INTRODUCTION**

Predictors or estimates of seismic structural demands such as inter-story drift angles that are less timeconsuming than nonlinear dynamic analysis (NDA) can be useful for structural performance assessment and for design. Several predictors have been proposed using the results of a nonlinear static pushover (NSP), which became a practical engineering tool to estimate the inelastic response of a multistory frame in the last decade.

The Limit Strength Calculation method (LSC) was introduced in the Enforcement Order in Japan in 2000 as a seismic design rule for ordinary building structures. Considering the inelastic first-mode response, inter-story drifts are evaluated [1]. However, the higher-order mode responses are neglected, and accordingly, the response at upper stories of a long-period building are not generally estimated accurately [2].

Luco and Cornell [3,4] proposed a ground motion intensity measure that can also be used as a predictor. It uses the first two elastic modes and the square-root-of-sum-of-squares (SRSS) rule of modal combination,

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and takes a first-mode inelastic spectral displacement into account. This predictor achieved quite an improvement from simply using the response of an elastic oscillator. However, it cannot capture well the large displacement caused by local yielding.

Chopra and Goel [5] proposed the Modal Pushover Analysis (MPA) procedure to take the higher-order mode responses into account by carrying out extra NSP's with lateral load patterns proportional to the higher-order mode shapes. Elnashai [6] and Antoniou et al. [7] proposed adaptive load patterns recognizing that invariant load patterns are not compatible with the progressive yielding of a structure during a NSP. Such kinds of procedures could be applied to improve Luco's predictor; however, they would make the predictor more complex and may render it less useful, particularly for preliminary or conceptual design purposes.

This paper first reviews Luco's predictor, and then discusses its possible improvement considering a postelastic mode shape. The accuracy of the predictor is investigated, along with that estimated by the MPA, using "fishbone," or generic frame [8], models of steel moment-resisting frame (SMRF) buildings. The relatively small numbers of degrees of freedom for the fishbone models allow us to consider buildings of several different periods and numerous earthquake ground motions of interest in both Japan and the U.S.

# **BUILDING MODELS**

In order to investigate the accuracy of predictors, two-dimensional fishbone frames of two mid- and one high-rise SMRF buildings are considered; a nine-story building designed according to Japanese practice is denoted as JP9, and nine-story and twenty-story buildings designed according to U.S. practice are denoted as SAC9 and SAC20, respectively. The first and second natural periods,  $T_1$  and  $T_2$ , and Rayleigh damping ratios,  $h_1$  and  $h_2$ , of each building model are listed in Table 1.

The fishbone model of a frame condenses all of the columns in a story into a single column, and all of the beams in a floor into a single rotational beam spring. Accordingly, the number of degrees of freedom can be reduced significantly while keeping almost the same accuracy as NDA of a full-frame model [4,8]. The key assumption is that the rotations at all of the beam-column connections in a floor are identical. The details of this condensation are explained in [8], but a few important characteristics of the fishbone models considered in this paper are listed here:

- (1) The backbone curve of the beam spring for each floor is trilinear, whereas bilinear plastic hinging at the column ends and splices is modeled for all but JP9 (explained below). The ratios of the strain-hardening (or third) slope to the elastic slope for the beams,  $\alpha_B$ , and for the columns,  $\alpha_C$ , of each building model are listed in Table 1. Column P-M interactions due to tributary gravity loads, but not due to varying axial forces caused by overturning, are taken into account.
- (2) Global (but not member)  $P \Delta$  effects are accounted for, with all applicable gravity loads placed on the fishbone column.

Building Model	$T_1$ (sec)	$T_2$ (sec)	<i>h</i> <sub>1</sub> (%)	<i>h</i> <sub>2</sub> (%)	$lpha_{B}$ (%)	α <sub>c</sub> (%)
JP9	1.50	0.56	2.0	2.0	0	0 (base of 1st-story column)
SAC9	2.24	0.84	2.0	1.1	3	3
SAC20	3.82	1.37	2.0	1.2	2	3

Table 1: Structural characteristics of fishbone models

Other details specific to each of the three buildings are provided in the subsections below.

# **JP9 Building**

JP9 is a 9-story SMRF building designed directly as a fishbone model as follows:

The height of each story is 4.0m, and the mass is distributed equally among the floors. The story-shear force distribution coefficient,  $A_i$ , is given by

$$A_i = 1/\sqrt{\alpha_i} \tag{1}$$

in which

$$\alpha_i = \frac{\text{mass above the } i - \text{th story}}{\text{total mass of the building}}$$
(2)

Assuming that the inflection point of each column is located at the mid-height of each story, all the beams yield simultaneously when the normalized base shear,  $C_0$ , is equal to 0.2. The stiffness is designed so that the inter-story drift angle at this point is 1/200 of the height of each story.

When  $C_0 = 0.3$ , the moments at all of the beams and the column base of the first story are equal to their maximum strengths.

The hinging at the first-story column base is modeled as rigid-perfectly-plastic, while the rest of the fishbone column is assumed to be elastic.

The ratio of the elastic stiffness of a beam spring to that of the sum of the adjacent columns is unity.

The moment-rotation characteristics of the beam springs is trilinear with a second stiffness ratio of 1/4 and a third stiffness ratio of 0%.

# **SAC9 Building**

SAC9 is a 9-story perimeter SMRF building designed for Los Angeles conditions by consulting structural engineers as part of Phase II of the SAC Steel Project [9]. Only one of the 5-bay perimeter MRF's is modeled, although gravity loads from half of the building are considered since they contribute to the P- $\Delta$  effect. The interior frames are assumed to resist gravity loads only. It should be noted that, unlike the "M1" model of this building commonly considered by SAC investigators [3,9], the basement is ignored and columns are assumed to be fixed at the ground.

# **SAC20 Building**

SAC20 is a 20-story perimeter SMRF building that, like SAC9, was designed according to pre-Northridge practices for Los Angeles conditions as part of the SAC Steel Project (Phase II). Unlike SAC9, both a 5-bay perimeter MRF and a 3-bay interior gravity frame of SAC20 (as well as columns that are part of the orthogonal perimeter MRF's) are modeled. Separate representations of the perimeter and interior frames (and the additional columns) are linked together at each floor under the assumption of a rigid diaphragm. Furthermore, rather than modeling the shear connections (e.g., in the interior gravity frames) as "pins," they are each attributed stiffness and strength properties reasonably close to those observed in laboratory tests conducted by Liu & Astaneh-Asl [10].

The NSP curves for all of the models using lateral load patterns proportional to the  $A_i$ -distribution given in the Enforcement Order in Japan are illustrated in Figs.1(a)JP9, (b)SAC9, and (c)SAC20. More detailed descriptions of the SAC9 and SAC20 buildings can be found in [4].



# EARTHQUAKE GROUND MOTION RECORDS

In conjunction with the building models described in the previous section, several earthquake record sets are used to evaluate the predictors. The sets include ground motions recorded at both near- and far-field sites in the U.S. and Japan, but only the results using a "nearby-field"set are presented in this paper. The "nearby-field" earthquake records were selected from the PEER Strong Motion Database (http://peer.berkeley.edu/smcat) according to the following criteria: (i) closest distance to the rupture surface less than 16km, (ii) earthquake moment magnitude greater than or equal to 6.0, (iii) recorded on "stiff soil" or "very dense soil and soft rock" (e.g., [11] site classes D or C, respectively), and (iv) high-pass-filter corner frequency less than or equal to 0.25 hertz. Only the strike-normal components are used. Of the resulting 73 nearby-field ground motions, 70 were recorded in California, and the other 3 were recorded in Erzican (Turkey), Tabas (Iran), and Kobe. A detailed list of the earthquakes can be found in [3]. Despite their proximity to the earthquake source, it is important to note that not all of the nearby-field earthquake records are "pulse-like" (i.e., not all exhibit a low frequency, large amplitude pulse in the velocity time history). In fact, less than half of the nearby-field ground motions are recorded in the region where forward rupture-directivity effects are anticipated, and even those are not all pulse-like [3].

Note that a relatively large number of earthquake records are considered here; NDA, carried out here using DRAIN-2DX [12], for this many earthquake records is not overly time-consuming when fishbone models are used.

# **EXISTING PREDICTORS**

#### **First-Mode-Elastic Predictor**

Although more commonly thought of as a ground motion intensity measure, the fundamental-mode spectral displacement,  $S_D(T_1, h_1)$ , can also be considered as a predictor if multiplied by a modal participation factor for inter-story drift angle,  $\theta$ . Such predictors of the peak (over time) inter-story drift angle of the *i*-th story,  $\theta_i$ , and the maximum (over all stories) peak inter-story drift angle,  $\theta_{\max}$ , denoted here as  $\hat{\theta}_i^{1E}$  and  $\hat{\theta}_{\max}^{1E}$ , are expressed in Eqs.(3) and (4), respectively, where *n* is the total number of

stories (or floors) in the given building model. Note that in modal analysis,  $\hat{\theta}^{1E}$  is simply the first-mode elastic estimate of  $\theta$ .

$$\hat{\boldsymbol{\theta}}_{i}^{1E} = \boldsymbol{P}\boldsymbol{F}_{1,i} \cdot \boldsymbol{S}_{D}(\boldsymbol{T}_{1},\boldsymbol{h}_{1}) \tag{3}$$

$$\hat{\boldsymbol{\theta}}_{\max}^{1E} = \max_{i=1}^{n} \left\{ \hat{\boldsymbol{\theta}}_{i}^{1E} \right\}$$
(4)

The participation function of the *j*-th mode,  $PF_{j,i}$ , is defined generally in Eq.(5), where  $\phi_{j,i}$  is the element of the *j*-th modal vector that corresponds to the upper floor of the *i*-th story (i.e., the *i*-th floor, with  $\phi_{j,0} = 0$ ), and  $H_i$  is the height of the *i*-th story (in the same units used for spectral displacement).

$$PF_{j,i} = \Gamma_j \frac{\phi_{j,i} - \phi_{j,i-1}}{H_i}$$
(5)

The participation factor of the *j*-th mode,  $\Gamma_i$ , defined by

$$\Gamma_{j} = \frac{\sum_{i=1:n} \phi_{j,i} \cdot m_{i}}{\sum_{i=1:n} \phi_{j,i}^{2} \cdot m_{i}}$$
(6)

where  $m_i$  is the mass of the *i*-th story. In short, the predictor  $\hat{\theta}^{1E}$  only requires modal vibration properties of the given structure (i.e.,  $T_1$ ,  $h_1$ , and  $PF_{1,i}$ ) and an elastic single-degree-of-freedom (SDOF) time-history analysis to compute  $S_D(T_1, h_1)$ .

# Luco's Predictor

Extending  $\hat{\theta}^{1E}$ , Luco and Cornell proposed a predictor,  $\hat{\theta}^{Luco}$ , that makes use of elastic modal vibration properties, a NSP (nonlinear static pushover) curve, and elastic and inelastic SDOF time-history analyses [4]. As expressed in Eq.(7),  $\hat{\theta}^{Luco}$  is the product of (i) the ratio of a first-mode inelastic spectral displacement,  $S_D^I(T_1, h_1, \delta_y, \alpha)$ , to the first-mode elastic spectral displacement,  $S_D(T_1, h_1, \delta_y, \alpha)$ , to the first-mode elastic spectral displacement,  $S_D(T_1, h_1)$ , and (ii) the elastic estimate of  $\theta$  computed using the first two modes and the square-root-of-sum-of-squares (SRSS) rule of modal superposition. Note that the elastic third mode response can be easily added to  $\hat{\theta}^{Luco}$ .

$$\hat{\theta}_{i}^{\text{Luco}} = \frac{S_{D}^{I}(T_{1}, h_{1}, \delta_{y}, \alpha)}{S_{D}(T_{1}, h_{1}^{0})} \sqrt{[PF_{1,i} \cdot S_{D}(T_{1}, h_{1})]^{2} + [PF_{2,i} \cdot S_{D}(T_{2}, h_{2})]^{2}}$$
(7)

$$= PF_{1,i} \cdot S_D^{I}(T_1, h_1, \delta_y, \alpha) \sqrt{1 + \frac{\{PF_{2,i} \cdot S_D(T_2, h_2)\}^2}{\{PF_{1,i} \cdot S_D(T_1, h_1)\}^2}}$$
(8)

$$\hat{\boldsymbol{\theta}}_{\max}^{\text{Luco}} = \max_{i=1}^{n} \left\{ \hat{\boldsymbol{\theta}}_{i}^{\text{Luco}} \right\}$$
(9)



Figure2: Schematic representation of a NSP curve and the bilinear or trilinear backbone curve fit to it

Recall that the elastic participation functions  $PF_{1,i}$  and  $PF_{2,i}$  were defined above in Eq.(5).

The backbone curve of the inelastic SDOF system is based on the roof drift versus base shear curve from a NSP for a given building model. An elastic-perfectly-plastic backbone curve (i.e.,  $\alpha = 0$ ) is fit to the NSP curve, except when the NSP curve exhibits significant strain hardening and does not degrade; in this latter case, a bilinear backbone curve is fit. The elastic slope of the idealization follows the elastic points of the NSP curve, whereas the perfectly-plastic slope passes through the peak base shear (up to a roof drift angle of 0.10 rad). The intersection of the two slopes provides an estimate of the roof drift angle at yield, denoted ( $\theta_{roof}$ )<sub>y</sub> (see Fig.2(a)), which is translated to  $\delta_y$  according to Eq.(10).

$$\delta_{y} = \frac{(\theta_{roof})_{y} \cdot \sum_{i=1}^{n} H_{i}}{\Gamma_{1} \cdot \phi_{1,n}}$$
(10)

#### **MPA** Approach

In MPA, the inter-story drift angles of a building are evaluated as follows [5]:

- (1) For the *j*-th mode, develop the roof drift versus base-shear pushover curve for the force distribution  $\mathbf{s}_i = \mathbf{m} \boldsymbol{\varphi}_i$ , where **m** is the mass matrix and  $\boldsymbol{\varphi}_j$  is the *j*-th mode vector.
- (2) Idealize the pushover curve as a bilinear curve.
- (3) Determine the backbone curve of an equivalent SDOF system based on the pushover curve obtained in Step (2).  $\delta_{y,j}$  is determined by Eq.(10) for the *j*-th mode.
- (4) Perform NDA to evaluate the maximum drift,  $S_{D,j}^{I}$ , of the equivalent SDOF system.
- (5) Find the roof drift,  $\delta_{i,roof}$ , of the building corresponding to  $S_{D,i}^{I}$  by

$$\delta_{j,roof} = \Gamma_j \cdot \phi_{j,roof} \cdot S_{D,j}^I \tag{11}$$

(6) Extract other desired responses,  $\delta_{j,i}$ , from the pushover database values at roof displacement  $\delta_{j,roof}$ .

- (7) Repeat Steps (1)-(6) for as many modes as required for sufficient accuracy.
- (8) Determine the total response  $\hat{\theta}_{i}^{\text{MPA}}$  by combining the peak modal responses using the SRSS rule.

As a NSP analysis must be carried out for each modal response, the MPA approach requires more computational effort than Luco's predictor. Also, the reasoning behind a NSP with load pattern proportional to a mode shape higher than the 1st mode is unclear. A building model is pushed at some stories while it is pulled at the other stories, but only the roof drift is considered to determine the backbone curve of an equivalent SDOF system. Thus, in the next section, the accuracy of Luco's predictor is investigated, and the possible improvement is considered thereafter.

# ACCURACY OF LUCO'S PREDICTOR

The accuracy of a predictor is expressed by (i) its bias, a, defined by the "median" of  $\theta/\hat{\theta}$ , which is the ratio of the demand computed via NDA of the model structure to the corresponding value of the predictor, and (ii) its "dispersion,"  $\sigma$ , defined by the standard deviation of the natural logarithms of  $\theta/\hat{\theta}$ . The bias, a, and the precision,  $\sigma$ , are equivalently obtained by performing a one-parameter log-log linear least-squares regression of  $\theta$  on  $\hat{\theta}$ . The regression model is expressed by

$$\ln(\theta) = \ln(a) + \ln(\hat{\theta}) + \ln(\varepsilon) \tag{12}$$

in which  $\varepsilon$  is the random error in  $\theta$  given  $\hat{\theta}$  with (by definition) median 1 and dispersion  $\sigma$ . The predictor of  $\theta_{\max}$  (the maximum peak story drift angle over all stories), as well as the predictor of  $\theta_k$  (the peak story drift angle for story k) are compared with the quantities numerically evaluated by NDA in what follows.

Figs.3 (a)-(d) illustrate the regressions of (a)  $\theta_{\text{max}}$  and  $\theta_k$  (k = (b) 1, (c) 5, (d) 9) on the predictor  $\hat{\theta}_{\text{max}}^{\text{Luco}}$ and  $\hat{\theta}_k^{\text{Luco}}$ , respectively, also for the JP9 building. The bias and the dispersion of  $\hat{\theta}_{\text{max}}^{\text{Luco}}$  and  $\hat{\theta}_k^{\text{Luco}}$  are generally small. However, some systematic deviations from the one-to-one line can be observed at the first and ninth stories. At the first story, the predictor tends to underestimate the larger responses; in contrast, it tends to overestimate the responses near yielding ( $\theta_1 \approx 0.005$ ), possibly because of the simple approximation of the backbone curve of the SDOF system as bilinear (see Fig.2(a)). At the ninth story,  $\hat{\theta}_k^{\text{Luco}}$  tends to overestimate the larger responses, while it tends to underestimate the smaller responses, possibly because of the neglect of the modal response higher than the second.

Figs.4 (a) and (b) illustrate, for two different ground motions,  $\theta_k$  and  $\hat{\theta}_k^{\text{Luco}}$  for the JP9 building, plotted story-wise. It can be observed in these figures that the predictor does not capture the large responses at lower stories. In Luco's predictor, only the elastic first- and second-mode shapes are considered, but the large responses at the lower stories after yielding are not predicted well using only the elastic mode shapes. As demonstrated in the next section, it is necessary to consider a post-elastic mode shape.

Since each mode shape changes as plastic hinges are formulated, a post-elastic mode shape must consider the ductility level. One idea is to track the formulation of plastic hinges in the NSP and perform an eigen value analysis for each stage [13]. However, this is complicated. Another possibility is investigated in the next section.



Figure 3: Regressions of  $\theta$  on  $\hat{\theta}^{Luco}$  for the fishbone model of JP9



Figure4:Examples of Luco's predictors

Based on the observations in the previous section, an improved predictor based on Luco's is proposed in this section by considering (i) a post-elastic mode vector dependent upon the ductility level, (ii) a more appropriate backbone curve for the equivalent SDOF system, and (iii) higher-order modal response. Assuming that a post-elastic mode vector,  $\boldsymbol{\phi}_1^I$ , can be approximated by the distribution of story drifts from a NSP, it is determined for each ground motion by taking the following steps (see Fig.5).



Figure 5: Flow for determining the first inelastic mode vector  $\phi_{1,k}^{I}$ 

# PREDICTOR CONSIDERING POST-ELASTIC MODE SHAPE

- (1) Perform a NSP with a lateral load pattern proportional to the  $A_i$ -distribution given in the Enforcement Order in Japan, which takes into account the effects of higher-order modal responses.
- (2) Obtain the story shear force,  $Q_i$ , versus story drift,  $\delta_i$ , curve (see Fig.1), as well as the base shear, Q, versus roof drift,  $\delta_{roof}$ , curve.
- (3) Idealize the  $Q \delta_{roof}$  curve as trilinear with a final strain-hardening ratio  $\alpha = 0$  (see Fig.2(b)).
- (4) Determine the backbone curve of an equivalent SDOF system according to Eq.(10). The second stiffness,  $k_2$ , is determined by

$$k_2 = k_1 \cdot \frac{(K_{roof})_2}{(K_{roof})_1} \tag{13}$$

in which  $k_1$  is the elastic stiffness of the SDOF system, and  $(K_{roof})_1$  and  $(K_{roof})_2$  are the elastic and second stiffnesses of the  $Q - \theta_{roof}$  curve approximated in Step (3).

(5) Perform NDA to evaluate the maximum drift,  $S_D^I$ , of the equivalent SDOF system.

- (6) Taking the reverse of Steps (3) and (4), find the roof drift of the building on the  $Q \theta_{roof}$  curve corresponding to  $S_D^I$ .
- (7) Determine the step number,  $n_p$ , of the NSP at the roof drift angle found in Step (6).
- (8) Find the story drifts at the  $n_p$ -th step of the NSP.
- (9) Use the distribution of story drifts obtained in Step (8) as the post-elastic first-mode vector.

Considering up to the third mode, the proposed predictor of the inter-story drift angle for the i-th story is evaluated by,

$$\hat{\theta}_{i}^{new} = \sqrt{\left\{PF_{1,i}^{I} \cdot S_{D}^{I}\left(T_{1}, h_{1}, \delta_{y}, \alpha\right)\right\}^{2} + \left\{PF_{2,i} \cdot S_{D}(T_{2}, h_{2})\right\}^{2} + \left\{PF_{3,i} \cdot S_{D}(T_{3}, h_{3})\right\}^{2}$$
(14)

where  $PF_{1,i}^{I}$  is evaluated by Eqs.(5) and (6) in which  $\phi_{1,i}$  is replaced with  $\phi_{1,i}^{I}$  determined above.

The proposed predictor can be described as an application of the commonly used modal decomposition and superposition analysis but with the elastic first-mode response replaced with inelastic modal response; i.e., the first-mode elastic spectral displacement,  $S_D(T_1, h_1)$ , is replaced with the first-mode inelastic spectral displacement,  $S_D^I(T_1, h_1, \delta_y, \alpha)$ , and the first mode elastic vector,  $\phi_{1,i}$ , is replaced with the firstmode inelastic vector,  $\phi_{1,i}^I$ . It should be noted that the additional work in the above procedure is minimal as NSP is already carried out to determine an equivalent SDOF system for Luco's predictor.

### NUMERICAL EXAMPLE

Figs.6(a)-(d) illustrate the regressions of (a)  $\theta_{\text{max}}$  and  $\theta_k$  (k = (b) 1, (c) 5, (d) 9) on the predictor  $\hat{\theta}_{\text{max}}^{\text{new}}$  and  $\hat{\theta}_k^{\text{new}}$  for the JP9 building.

The bias of  $\hat{\theta}_{max}^{new}$  becomes closer to unity than that of  $\hat{\theta}_{max}^{Luco}$ , and the dispersion decreases by two-thirds to a half. The underestimation of the large responses at the first story and the overestimation of the responses near  $\theta_1 \approx 0.005$  are improved with respect to Luco's predictor by considering the post-elastic first-mode shape and adopting a trilinear backbone curve for the equivalent SDOF system. The underestimation of the smaller responses at the ninth story is also improved by considering the third-mode response.

Figs.7 (a)-(d) illustrate the regressions of (a)  $\theta_{\max}$  and  $\theta_k$  (k = (b) 1, (c) 5, (d) 9) on the predictor  $\hat{\theta}_{\max}^{MPA}$  and  $\hat{\theta}_k^{MPA}$  for the JP9 building. Although the bias of  $\hat{\theta}_{\max}^{new}$  shown in Fig.6 (a) is a little further from the unity than that of the bias of  $\hat{\theta}_{\max}^{MPA}$ , the dispersion of the former is a little smaller than that of the latter. The accuracy of the inter-story drift angle estimated by the proposed method is basically comparable with that by the MPA; the former is even better than the latter for the first story.

Figs.8(a) and (b) illustrate  $\theta_k$  and  $\hat{\theta}_k^{\text{new}}$ , as well as  $\hat{\theta}_k^{\text{MPA}}$ , plotted story-wise for the same ground motions and building considered in Figs.4 (a) and (b). It can be observed in Fig.8(a) that large inter-story drifts at

the lower stories are well captured by the proposed method as well as by the MPA approach. Although there are still some cases, such as that shown in Fig.8(b), in which the proposed method can provide only a minor improvement from Luco's predictor, the MPA approach provides a similar level of improvement despite of the larger computational effort.





Figure 7: Regressions of  $\theta$  on  $\hat{\theta}^{MPA}$  for the fishbone model of JP9



Figure8: Examples of Proposed predictors

Figs.9(a)-(c) and 10(a)-(c) summarize the bias and the dispersion, respectively, of Luco's predictor, the predictor estimated by the MPA, and that by the proposed method for the (a)JP9, (b)SAC9, and (c)SAC20 buildings. The higher modal responses up to the 5th mode are considered in the proposed method and the MPA approach. The bias of the proposed method is fairly stable and within the range of 0.9 - 1.1 for all of the buildings. The bias of Luco's predictor is comparable with that of the proposed method for all the stories of nine story building except for the ninth story; Luco's approach underestimates the response because of neglecting the modal responses higher than second mode. The error is more notable at the higher stories of the SAC20 building model. The bias of the MPA approach is also within the range of 0.9 - 1.1 for most of the stories for all of the buildings; however a systematic error, slightly overestimating at the lower stories and slightly underestimating at the mid-height stories, can be observed.



It can be observed in Figs.10(a)-(c) that the dispersion of the proposed method is also fairly stable and less than about 0.20 at every story of all of the buildings. The dispersion of Luco's predictor is larger than that of the proposed method for most of the stories of the building models. Again the error is quite notable for the SAC20 building model. The dispersion of the MPA approach is basically comparable with that of the proposed method. However, it is a little larger than that of the proposed method for the JP9 building model, and it is larger than 0.25 at the first story of the SAC9 building model.

For the modal responses higher than the second mode, the proposed method considers only the elastic responses while the MPA considers inelastic responses using higher order pushover analysis. However, as observed in the numerical examples, higher order modal inelastic responses considered in the MPA contribute little to the accuracy of the predictor.

#### DIFFERENCES BETWEEN PROPOSED METHOD AND MPA IN FIRST MODE

For the first mode response, both the proposed method and the MPA consider inelastic response, and their procedures are fairly similar to one another. In both of the procedures, the backbone curve of the equivalent oscillator is determined based on the roof drift from a NSP analysis, the first mode shape is assumed to be proportional to the story drift from the NSP corresponding to the response of the equivalent inelastic oscillator, and the modal responses are combined using the SRSS rule. The notable difference between these two procedures is that the participation factor  $\Gamma_1$  is evaluated using the inelastic first mode

 $\phi_1^I$  in the proposed method, whereas it is evaluated using the elastic first mode vector  $\phi_1$  in MPA. The effect of this difference on the estimation of the first mode inelastic response is investigated in the following. Note that the same bi-linear backbone curve of the equivalent inelastic oscillator is considered for both of the procedures.

The *i*-th story drift of the first mode response found by the proposed method,  $\delta_i^{new}$ , is expressed as,

$$\delta_{1,i}^{new} = \Gamma_1^I \cdot \phi_{1,i}^I \cdot S_D^I \tag{15}$$

in which  $\Gamma_1^I$  is the participation factor evaluated using the inelastic first mode vector,  $\phi_{1,i}^I$ , and  $S_D^I$  is the maximum displacement of the equivalent inelastic oscillator. If  $\phi_{1,i}^I$  is normalized by the largest element of the vector, it can be expressed as,

$$\phi_{\mathbf{l},i}^{I} = \frac{\delta_{\mathbf{l},i}^{\mathrm{MPA}}}{\max_{i=1}^{n} \left\{ \delta_{\mathbf{l},i}^{\mathrm{MPA}} \right\}}$$
(16)

Letting

$$\max_{i=1}^{n} \left\{ \delta_{1,i}^{\text{MPA}} \right\} = \alpha \cdot \delta_{1,roof}^{\text{MPA}}$$
(17)

and substituting Eqs.(11) and (16) into Eq.(15) yields

$$\delta_{1,i}^{new} = \Gamma_1^I \cdot \frac{\delta_{1,i}^{MPA}}{\alpha \cdot \Gamma_1 \cdot \phi_{1,n} \cdot S_D^I} \cdot S_D^I$$
(18)

Thus, the ratio of the first mode response estimated by the MPA to that estimated by the proposed method can be expressed as,

$$\frac{\delta_{1,i}^{\text{MPA}}}{\delta_{1,i}^{new}} = \frac{\alpha \cdot \Gamma_1 \cdot \phi_{1,n}}{\Gamma_1^I}$$
(19)

The ratio of the first-mode responses expressed by Eq.(19) is illustrated in Fig.11 using the JP9 and SAC9 building models. The ratio is unity in the elastic stage. It increases after yielding, but it is less than 0.15 in these examples and not so large. However, it should be noted that the ratio depends on the size of the elastic mode vector  $\phi_1$  as seen in Eq.(19). In this example, the elastic mode vector is normalized by its largest element. In contrast, the size of the inelastic mode vector cancel out in Eqs.(5) and (6) in the proposed method.



Figure 11: Difference of the first mode response between the proposed method and the MPA

### CONCLUSIONS

Predictors of seismic inter-story drift angles for practical use in structural performance assessment and design are investigated in this paper. In addition to the inelastic first-mode spectral displacement considered in Luco's predictor based otherwise on SRSS modal combination, it is proposed to consider (i) a post-elastic first-mode shape approximated by the distribution of story drifts from a NSP at the step corresponding to the maximum drift of the equivalent inelastic SDOF system, (ii) a trilinear backbone curve for the SDOF system, and (iii) the third-mode response for long-period buildings. Numerical examples demonstrate that the proposed predictor is less biased and results in less dispersion than Luco's predictor and is comparable with or sometimes even better than predictions by MPA, which requires more computational effort. Further evaluation of the proposed predictor for additional earthquake records and structures of different fundamental period, heights, and configurations is expected in order to find any limitations.

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