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SHEAR MOMENT INTERACTION FOR DESIGN OF STEEL BEAM-TO-COLUMN CONNECTIONS

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SUMMARY

The moment capacity of structural beams is influenced by the presence of shear in the beam. In general, the presence of shear reduces the moment carrying capacity of the beam. Thus, the design of beam-to-column connections using capacity design concept should consider the presence of shear in the beam. Current design code specifications for design of beam-to-column connections do not consider the effect of reduction in maximum developable moment capacity in the beam, due to the presence of shear. This may result in the heavier connections. In this paper, the interaction between shear and moment, for various AISC beam sections is investigated. Shear-moment curves are analytically developed for a spectrum of AICS sections. It is observed that the shear-moment interaction for the AISC sections may be represented using straight lines. This interaction is used to estimate the maximum shear and moment that is likely to be developed in the beam during extreme earthquake shaking. These shear and moment estimates can be used to design the connections for the beam-column interface. Beam length-to-depth ratios for which the shear-moment interaction becomes significant are calculated. It is shown that for deep beams, neglecting the presence of shear can overestimate the design moment demands, and underestimate the design shear demands for the beam-to-column connections.

INTRODUCTION

The existing procedures for the design of beam-to-column connections using the capacity design concepts, calculate the maximum capacity (moment and the corresponding equilibrium shear force) of the beam, and use these to estimate the sizes of the connections elements. For beam-to-column connections in moment resisting frame (MRF) buildings, the connections are subjected to moments and shear forces simultaneously. As long as the behavior is in the linear elastic range, the effect of moment and shear are independent, and they do not influence each other. However, in the inelastic range, the effect of moment and shear force acting together is different from the effect of moment and shear acting independently. Thus, for beam-to-column connections designed to resist earthquake loads, where the behavior is expected to go into the inelastic range, the interaction between moment and shear force should be considered.

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PAST STUDIES ON SHEAR-MOMENT INTERACTION OF BEAMS

In the past, a few research attempts were made to include shear-moment interaction in beam design. In the shear-moment interaction for I-sections based on the maximum shear strength criterion for yielding (Hodge and Brooklyn, 1957), the yield strength F_y was assumed to be the limiting strength, and strain-hardening of steel was not considered. The shear-moment interaction was not found to be significant in shallow beams having depth-to-span ratios less than about 0.1.

Approximate shear-moment (*V-M*) interaction curves for deep plate girders based on the web tension-field action (Basler, 1962) and without strain-hardening of material, shows no interaction between shear *V* and moment *M* so long as the flanges does not yield. But, the web shear capacity drops quickly once yielding of the beam flanges initiates.

In deep girders with small *l/d* ratios, the *V-M* interaction can be significant. The AISC-LRFD code, based the findings for deep girders (Cooper et al., 1978), prescribes the following tri-linear interaction between shear and bending moment (Figure 1):

$$\begin{aligned} V_u &= \phi V_n && \text{for } M_u < 0.75\phi M_n \\ \frac{M_u}{\phi M_n} + 0.625\frac{V_u}{\phi V_n} &\le 1.375 && \text{for } 0.75\phi M_n < M_u < \phi M_n\,, \\ M_u &= \phi M_n && \text{for } V_u < 0.6\phi V_n \end{aligned} \tag{1}$$

where M_u and V_u are the flexural and shear strengths considering interaction, respectively, M_n and V_n are the nominal flexural and shear capacities of the section without considering any interaction, and the resistance factor $\phi = 0.9$.

SHEAR-MOMENT INTERACTION

In this study, a fiber model is employed to obtain the *V-M* interaction curves of thirteen AISC W-sections (namely W36×300, W33×240, W27×177, W21×142, W24×160, W18×114, W16×96, W14×426, W14×84, W12×190, W12×58, W10×112, and W8×67) (AISC, 1989). The cross-section is discretized into fibers of 1 mm thickness that are parallel to the major axis of bending (Figure 2). The dimensions of the beam section are rounded-off to the nearest millimeter.

The curvature of the cross-section is increased in steps, from zero to a maximum value corresponding to the rupture strain ε_r at the extreme fiber of the cross-section. At each level of curvature, the normal strain ε_{xx} in each fiber is calculated. The corresponding normal stress σ_{xx} in each fiber is estimated using an explicit and smooth form of stress-strain curve with strain-hardening (Murty and Hall, 1994) (Figure 2(b)). The shear stress τ_{xy} is estimated using the von Mises yield criterion for steel:

$$\sigma_{xx}^2 + 3\tau_{xz}^2 = Y^2\,, \tag{2}$$

where *Y* is the ultimate stress F_u. From these fiber stresses, the shear capacity *V* and moment *M* capacities of the section are estimated, using

$$V = \int_{-d_b/2}^{d_b/2} \tau_{xy}\, dy \text{, and} \tag{3}$$

$$M = \int_{-d_b/2}^{d_b/2} \sigma_{xx}\, \bar{y}\, dy\,, \tag{4}$$

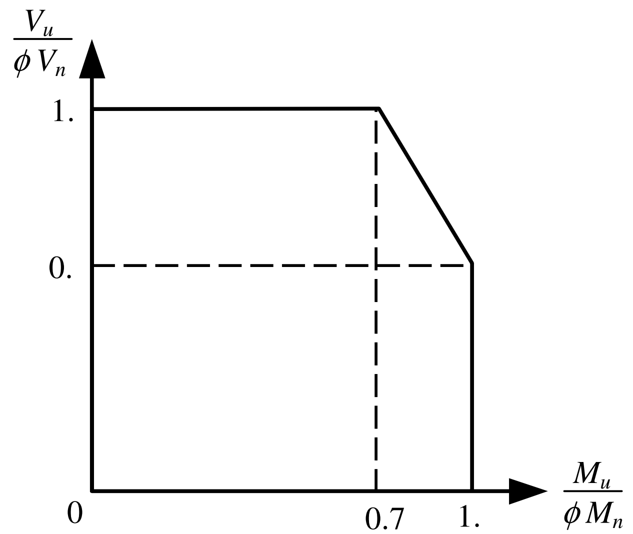

Figure 1: AISC-LRFD shear-moment interaction. Shear-moment interaction is prescribed only for I-shaped plate girders with slender webs.

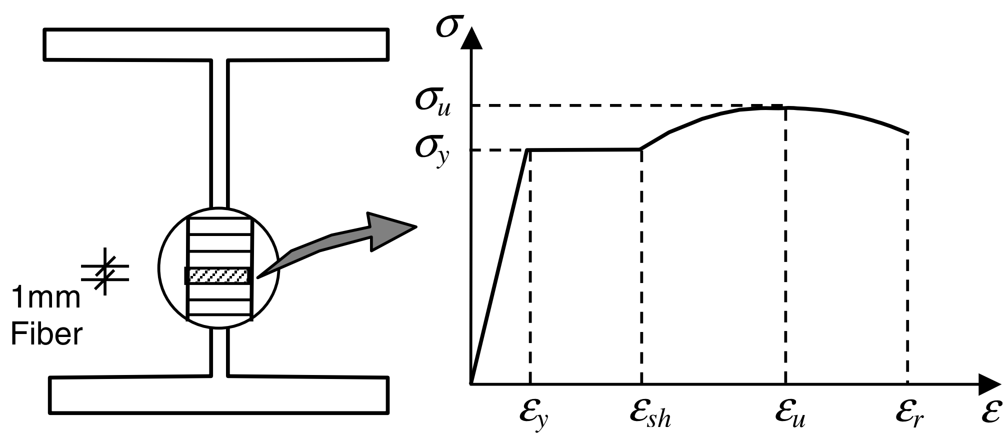

Figure 2: Fiber model of W-sections: (a) Discretisation of the beam section across the cross-section, and (b) Explicit form of stress-strain curve of steel (Murty and Hall, 1994) used in this study.

where $\bar{y}$ is the distance of the fiber from the neutral axis. The uniaxial stress-strain curve of steel (Figure 2(b)) has an increase in the stress up to the ultimate strain ε_u. When σ_{xx} is equal to F_u, the shear stress τ_{xy} is zero from Eq.(2). For strains larger than ε_u, the normal stress σ_{xx} drops below F_u. At this stage, Eq.(2) suggests that the shear stress τ_{xy} in fibers is non-zero. However, in this study, fibers strained beyond ε_u are assumed to have no shear capacity. Further, flanges and web of the beam are assumed to develop their ultimate strength without undergoing local buckling. The nominal shear strength V_p ($=\tau_y t_{bw} d_b$) and the nominal bending moment capacity M_p ($=F_y Z_p$) of the section are used to normalize the actual shear and moment capacities, respectively; here τ_y is the yield shear stress corresponding to a state of pure shear, and given by $\tau_y = F_y \big/ \sqrt{3}$.

Normalized *V-M* interaction curves for thirteen AISC W-sections are shown in Figure 3, for $R_y = 1.0$ and $F_u/F_y = 1.5$ (for A36 steel), and for $R_y = 1.0$ and $F_u/F_y = 1.3$ (for A572 Grade 50 steel). These F_u/F_y values are based on the coupon tests of specimen tested during some of the AISC and SAC tests (Engelhardt and Sabol, 1998; Malley and Frank, 2000). The value of R_y (the ratio of actual yield strength F_y^* to the minimum specified yield strength F_y) is taken as 1.0 while developing the *V-M* curves. But, the actual value of R_y needs to be applied while calculating the final probable moment capacity of the beam. Thus, the *V-M* curves are made independent of the uncertainty in the estimation of yield strength of steel.

The normalized *V-M* interaction curves for the thirteen cross-sections considered (Figure 3) are nearly identical. Furthermore, the interaction between shear and bending moment is weak for moment smaller than yield moment M_y (SF_y) and for shear smaller than yield shear V_y ($\tau_y t_{bw} d_b$); this reiterates the observations reported in literature (Hodge, 1962; Cooper, et al., 1978). The idealized upper bounds of the normalized *V-M* interaction curves are also shown in Figure 3 by three linear segments, given by

$$\begin{aligned} \frac{V}{V_p} &= \beta && \text{for} \quad 0 \le \frac{M}{M_p} \le \frac{M_y}{M_p} \\ \frac{V}{V_p} + \alpha\frac{M - M_y}{M_p} &= \beta && \text{for} \quad \frac{M_y}{M_p} < \frac{M}{M_p} \le \beta, \text{and } \frac{V_y}{V_p} < \frac{V}{V_p} \le \beta,, \\ \frac{M}{M_p} &= \beta && \text{for} \quad 0 \le \frac{V}{V_p} \le \frac{V_y}{V_p} \end{aligned} \tag{5}$$

where β = $R_y(F_u/F_y)$ is the beam overstrength factor and $\alpha = \left(1 - \dfrac{V_y}{\beta V_p}\right) \Big/ \left(1 - \dfrac{M_y}{\beta M_p}\right)$ the shear-moment interaction factor. The point on the upper bound curve where the shear interaction becomes significant corresponds to $V = V_y$.

In the idealization of *V-M* interaction, the presence of shear does not affect the moment capacity of the section as long as the shear is less than or equal to V_y. The straight-line idealization of the *V-M* curves has two distinct points, namely point A (F_u/F_y, V_y/V_p) and point B (M_y/M_p, F_u/F_y). For a beam shear link of length L_o corresponding to the overstrength plastic hinges shown in Figure 4(b), the moment M and shear V are related by

$$\frac{L_o}{2} = \frac{M}{V}. \tag{6}$$

Thus, for point A,

$$\frac{L_{oA}}{d} = \frac{3}{d}\left(\frac{F_u}{F_y}\right)\left(\frac{M_p}{V_p}\right), \tag{7}$$

and for point B,

$$\frac{L_{oB}}{d} = \frac{2}{d}\left(\frac{F_y}{F_u}\right)\left(\frac{S}{Z}\right)\left(\frac{M_p}{V_p}\right), \tag{8}$$

where L_{oA} and L_{oB} are the shear link lengths corresponding to points A and B, respectively (Figure 3); S and Z are the elastic and plastic section modulii, respectively.

Table 1 shows the *L/d* values corresponding to the critical point A and B for the 13 AISC W-sections considered in this study. The *L/d* values are obtained for both A36 steel and A572 Gr 50 steel. For A36 steel, the *L/d* values above which the effect of shear on the moment capacity of the beam can be ignored (flexural beams), ranges from about 7 to 12; and for A572 Gr 50 steel, these limiting *L/d* values are slightly smaller than those for A36 grade steel. Thus, for e.g., for a beam of W21×142, the length of the shear link below which the shear reduces the moment is 4.9m, which is equivalent to beam span (column center-to-center) of about 6-7m. Having spans of 6-7m and smaller may be a common occurrence, particularly in

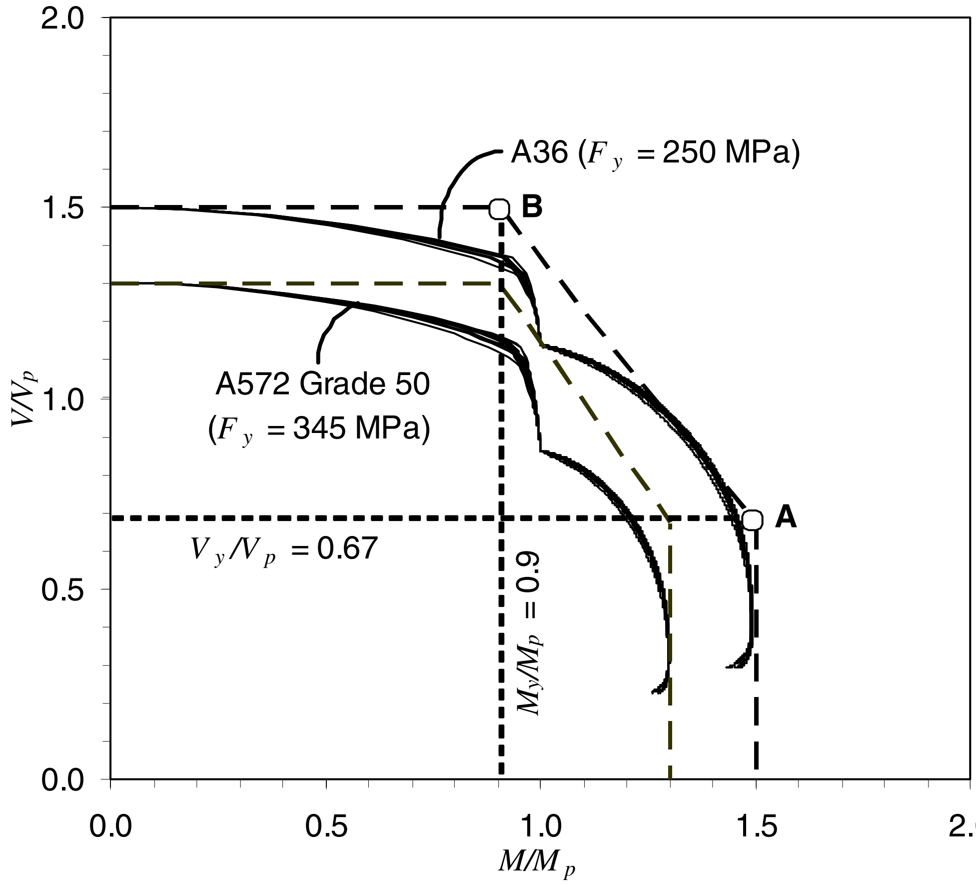

Figure 3: Normalized *V-M* interaction surfaces for AISC W-sections with idealized upper bound for minimum specified yield strength and F_u/F_y = 1.5 (F_y = 250 MPa) and F_u/F_y = 1.3 (F_y = 345 MPa), and R_y =1.0.

industrial buildings. Further, the *L/d* value below which the effect of shear on moment capacity of the beam becomes significant (shear beam) is a function of the F_u/F_y ratio. Thus, beams with larger F_u/F_y ratios need larger *L/d* values to have their flexural behavior unaffected by shear.

DESIGN OF BEAM-TO-COLUMN CONNECTIONS

The design of beam-to-column connections as per capacity design concepts requires that the connections be designed for the maximum moment and shear that are expected to be developed in the beam. In the existing method for design of beams, a section with M_p larger than the maximum bending moment demand *M* is selected. It is then ensured that V_p of the section is larger than the maximum shear demand *V*.

On the idealized *V-M* plane (Figure 5), point C corresponds to the upper bound of the code design procedure for beams, i.e., the design point can be anywhere below and to the left of point C; this is indicated by the shaded region in Figure 5. As seen from Figure 5, beams designed by the code procedure incorporate large overstrength corresponding to the area under the idealized limiting *V-M* curve and beyond the shaded region.

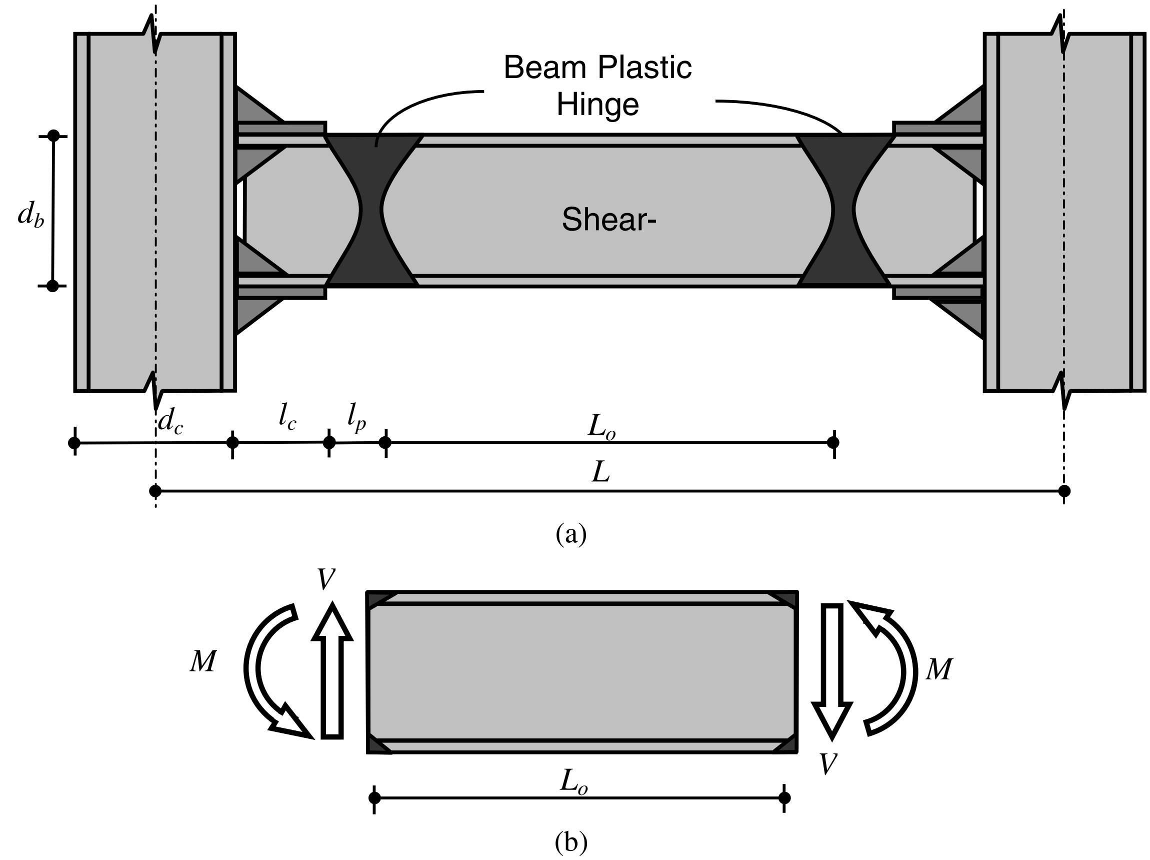

Figure 4: Overstrength plastic hinge-based shear link: (a) Frame subassemblage showing shear-link and locations of plastic hinges, and (b) free body diagram of the shear-link.

Table 1: Limiting *L/d* values for the AISC W-sections used to develop the *V-M* interaction curves in this study.

Sr. No.	W-*Section*	d_b mm	t_w mm	S_b $\times 10^{-4}$m^3	Z_b $\times 10^{-4}$m^3	$\frac{M_p}{V_p}$ m	A36 (F_u/F_y =1.5) $\frac{L_{oA}}{d_b}$	A36 (F_u/F_y =1.5) $\frac{L_{oB}}{d_b}$	A572 Gr 50 (F_u/F_y =1.3) $\frac{L_{oA}}{d_b}$	A572 Gr 50 (F_u/F_y =1.3) $\frac{L_{oB}}{d_b}$
1	W36×300	933	24	181.1	205.7	1.59	7.67	2.00	6.65	2.31
2	W33×240	851	21	132.9	150.5	1.46	7.71	2.02	6.68	2.33
3	W27×177	694	18	80.8	91.3	1.27	8.20	2.15	7.11	2.48
4	W21×142	545	17	52.0	58.5	1.09	9.03	2.38	7.83	2.74
5	W24×160	628	17	67.8	76.0	1.23	8.83	2.33	7.66	2.69
6	W18×114	469	15	36.1	40.6	1.00	9.60	2.52	8.32	2.91
7	W16×96	415	14	27.2	30.5	0.91	9.85	2.61	8.54	3.01
8	W14×426	475	48	115.9	142.5	1.08	10.25	2.47	8.89	2.85
9	W14×84	360	11	21.5	23.8	1.04	13.03	3.47	11.29	4.01
10	W12×190	365	27	43.1	51.0	0.90	11.06	2.77	9.59	3.20
11	W12×58	310	9	12.8	14.2	0.88	12.77	3.42	11.07	3.94
12	W10×112	289	19	20.7	24.2	0.76	11.87	3.01	10.29	3.48
13	W8×67	229	15	9.9	11.5	0.58	11.38	2.91	9.86	3.35

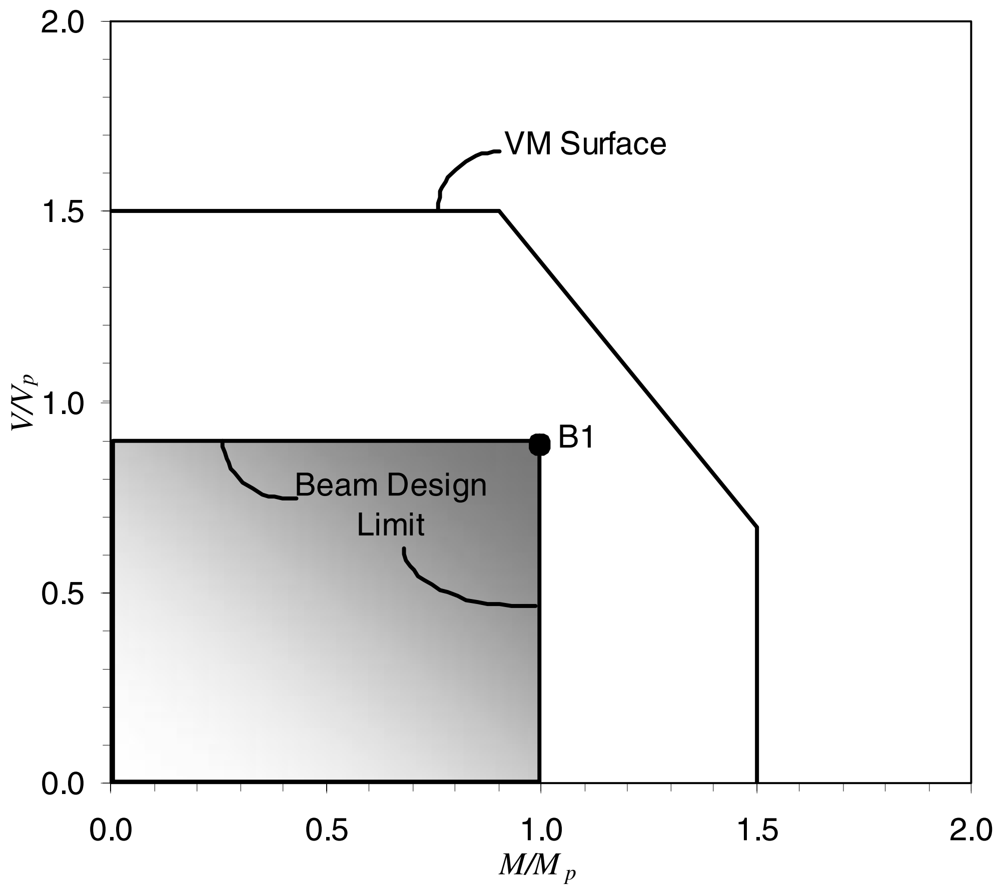

Figure 5: Idealized limiting *V-M* interaction curve for design of beam-to-column connections with minimum specified yield strength and *Fu/Fy* =1.5 (for A36 steel) compared with the design limits employed in the existing beam design procedures.

In industrial buildings it is possible to have the beam *L/d* ratios which result in strong *V-M* interaction, i.e., due to the interaction between shear and moment, the connection design moment can be less than that corresponding to point A (Figure 3). In such cases designing the beam-to-column connections for moment corresponding to point A will result in an unnecessarily strong connection for moment. However, this may also result in a connection that is inadequate to transfer the resulting shear. A detailed procedure for the design of beam-to-column connections while including the effects of *V-M* interaction is given elsewhere (Arlekar and Murty, 2004).

SUMMARY AND CONCLUSIONS

The current design of beam-to-column connections for steel MRFs does not consider the *V-M* interaction; the connections are designed for maximum probable moment expected to be developed in the beam, and the corresponding equilibrium compatible shear. This paper, examines the shear-moment interaction for AISC W-sections to evaluate its effect on the design of beam-to-column connections. For beams with small *L/d* ratios, the current design procedures for beam-to-column connections will result in an over design for moment resistance and an under design for shear resistance. This may be unsafe, particularly in cases where the beam shear is high, i.e., in short span beams. Design procedure to incorporate the shear-moment interaction in the estimation of the connection design forces needs to be developed. An attempt to develop such a procedure has been made by the authors is listed in the references.

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