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JOINT HAZARD OF EARTHQUAKE SHAKING AT MULTIPLE LOCATIONS

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SUMMARY

A number of applications require estimates of the levels of earthquake shaking that are likely to occur concurrently at multiple locations. Examples include the evaluation of extended lifelines networks and the estimation of damage costs in a region. Uniform hazard maps are not appropriate for these purposes because the earthquake sources that are the principal contributors to the hazard generally vary between locations. Unless the hazard at all locations in a region is dominated by a single fault, the mapped values at different locations are likely to be produced by different earthquakes. Scenario maps are often used to assess the levels of motion that are likely to occur together (i.e. in the same earthquake) across a lifelines network or region. The usual implementation of the scenario approach is that the motions in a given earthquake are uniformly high or low at all locations with respect to the median motion for each location e.g. at the 50-percentile or 84-percentile level everywhere. This corresponds to an implicit assumption that the random variability in earthquake motions is contributed totally by variations between different earthquakes of the same magnitude and location (between-earthquake variation), with none of the random variability occurring between different locations within a single earthquake (within-earthquake variation). Modern attenuation relations indicate that most of the variation in fact consists of within-earthquake variation. This paper discusses the methodology for assessing the joint probabilities of exceedance of given ground motions at multiple locations. It presents examples of the sometimes large differences between the levels of motion occurring jointly at multiple locations compared to those indicated by scenario approaches, and their different implications for redundant and non-redundant networks. It also indicates how joint probabilities affect estimated damage costs in a region.

INTRODUCTION

In seismic hazard analysis, the probability of earthquake shaking is typically estimated individually at one or many sites. However, a number of applications require estimates of the levels of earthquake shaking

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that are likely to occur concurrently at multiple locations. Examples include the evaluation of extended lifelines networks and the estimation of damage costs in a region.

Lifeline systems may have vulnerable elements at two or more points of a network, but with redundancy in the system so that failure of the network as a whole occurs only when there is simultaneous failure of certain combinations of elements. Then there is interest in knowing whether the vulnerable elements are likely to be simultaneously affected by shaking of sufficient strength to disable them. Conversely, there may be several critical points, with failure at any one of them causing failure of the system. In that case, there is interest in knowing how often the strongest shaking at any one of the critical points is sufficient to cause failure.

When facilities at different sites are considered together, the probability of failure at all sites simultaneously can be no greater than the smallest of the individual site probabilities of failure, while the probability of failure for at least one of the sites can be no less than the largest of the individual failure probabilities. These cases are relevant for redundant and non-redundant systems respectively. Just how close the actual probabilities are to these bounds depends largely on how much within-earthquake variability there is in the levels of motion that could be experienced at equivalent sites in a given earthquake.

Similar considerations come into play when assessing earthquake damage costs for a portfolio of properties or on a regional basis. The total damage depends on the strength of shaking that occurs at multiple locations in a single event, and may be very different from that estimated by summing the damage costs from considering the properties either jointly in a scenario approach in which the ground motions are totally correlated within a given earthquake, or individually, assuming that the ground motions are uncorrelated at different sites in the same earthquake.

A rigorous methodology for assessing the joint hazard at a number of sites has been presented by Rhoades & McVerry [1, 2], taking into account the different components of uncertainty in an attenuation relation. Here we review that methodology and apply it to examples relevant to systems with redundancy or multiple critical points, and use it to demonstrate the different damage cost estimates that result from different assumptions about the within- and between-earthquake variability of the motions.

It needs to be pointed out that the situation we are discussing is joint hazard for sites that are sufficiently distant from each other that an analysis of ground motion coherency (e.g. Abrahamson et al. [3]) is not applicable. Coherency analyses are generally applicable for separations up to a few hundred metres, while the joint-hazard methodology presented here is for scales of kilometres to tens of kilometres, corresponding to the separations between sites that may be shaken strongly by the same earthquake. The correlation between motions at different sites in a joint-hazard analysis results from a particular earthquake producing motions that are generally higher or lower than the median motions expected for that earthquake. Under the assumptions of the partitioning of the variance in the attenuation model, the correlation coefficient is independent of the separation between the sites. Conversely, coherency, which is related to the passage of the same waves past closely-spaced sites, drops rapidly as the separation increases.

METHODOLOGY

The strength of shaking at a particular location from a given earthquake is estimated by attenuation relations fitted to strong-motion data from past earthquakes. The strength of shaking may depend on many different features of the earthquake source, the state of the ground at the remote site and the state of the intervening crust in the path of the seismic waves that cause the shaking. Attenuation relations typically include only a few of the most obvious explanatory variables: earthquake magnitude, the distance from source to site, site conditions and perhaps one or two others. The effects of all other variables, measured or unmeasured, become part of the error term which is used to estimate the uncertainty in future events.

In the "random-effects" model (Brillinger & Preisler [4]; Abrahamson & Youngs [5]) which has been used to develop many modern attenuation relations, the error is partitioned into "between-earthquake" and "within-earthquake" components of uncertainty. Both components are potentially reducible by improved understanding of the process, through better modelling of the source effects and site effects respectively. But at a given state of knowledge, both types of uncertainty are real, and are viewed as the cause of random variability in future events. The "random-effects" model is the starting point of our analysis.

Rhoades & McVerry [1] have presented the methodology where the attenuation model is expressed in terms of a general function $g(a_i)$ of an earthquake ground motion measure a_i such as the peak ground acceleration (pga) or 5% damped response spectrum acceleration and also includes both epistemic variability and the effects of errors in the values of the independent variables such as magnitude. Our presentation here will be for $\log a_i$, which is the standard form for most acceleration-based attenuation models. Also, as we are primarily interested in aspects of joint hazard analysis rather than the effects of epistemic uncertainty and errors in the variables, we will neglect those in the presentation of the methodology.

Single earthquake source analysis

In the "random-effects" attenuation model, the peak ground acceleration or response spectrum acceleration a_i at n locations i in an earthquake of known source properties is represented by

$$\log a_i = h(e, s_i, \hat{\beta}) + \delta + \varepsilon_i \qquad i = 1, \cdots, n \tag{1}$$

where e denotes variables that are properties of the earthquake source (e.g. the earthquake magnitude, tectonic setting and focal mechanism), s_i denotes variables that are properties of site location i (e.g. its distance from the source, the stiffness of the ground at the site, and path characteristics), $\hat{\beta}$ is a vector of parameter estimates, and h is a suitable function. The error random variables δ and ε_i represent the between-earthquake and within-earthquake components of variability and are assumed to be independent and normally distributed with variances σ_δ^2 and σ_ε^2, respectively. Both δ and ε are considered to be components of "aleatory" uncertainty, i.e. they represent variability of the data from the fitted model. As shown by Rhoades and McVerry [1, 2], it is also possible to include the effects of epistemic uncertainty in the parameter estimates $\hat{\beta}$, but the presentation here is restricted to the aleatory uncertainty for more direct comparison with the standard procedure for single-site hazard estimates.

The value of δ is common to all sites, it being a random variable determined by the earthquake. On the other hand, the value of ε_i in a particular earthquake depends on the site. The total aleatory component of error is

$$\eta_i = \delta + \varepsilon_i \tag{2}$$

The total aleatory variance σ^2 is given by

$$\sigma^2 = \sigma_\delta^2 + \sigma_\varepsilon^2 . \tag{3}$$

From Equations (1) and (2), the joint probability that any given threshold levels $a_1,\cdots, a_n$ are exceeded at $i=1,\ldots n$ respectively, given the source characteristics e, satisfies

$$P(A_i \geq a_i, i=1,\cdots,n \mid e) = P[\eta_i \geq \log(a_i) - h(e,s_i), i=1,\cdots,n]. \tag{4}$$

This can be rewritten as

$$P(A_i \geq a_i, i=1,\cdots,n \mid e) = P[\varepsilon_i \geq \log(a_i) - h(e,s_i) - \delta, i=1,\cdots,n]. \tag{5}$$

Noting that ($\varepsilon_i, i=1,\cdots,n$) and δ are independent normal random variables, the joint probability of the given levels of acceleration being exceeded at the n sites is given by

$$P(A_i \geq a_i, i=1,\cdots,n \mid e) = \frac{1}{\sigma_\delta}\int_{-\infty}^{\infty}\prod_{i=1}^{n}\left\{1-\Phi\left[\frac{\log a_i - h(e,s_i) - \delta}{\sigma_\varepsilon}\right]\right\}\phi\left(\frac{\delta}{\sigma_\delta}\right)d\delta \tag{6}$$

where ϕ and Φ are the standard normal probability density function and cumulative distribution function, respectively, i.e.,

$$\phi(u) = (2\pi)^{-1/2}\exp(-u^2/2) \tag{7}$$

$$\Phi(z) = \int_{-\infty}^{z}\phi(u)du \tag{8}$$

For the case of two locations, the joint probability of exceedance of acceleration a_1 at location 1 and a_2 at location 2 can be written in terms of a standardized bivariate normal distribution, a result that has also been recognised by Wesson & Perkins [6] and Leonard & Steinberg [7].

$$P(A_1 \geq a_1, A_2 \geq a_2|e) = \left[2\pi sqrt\left(1-\rho^2\right)\right]^{-1}\int_{z_1}^{\infty}\int_{z_2}^{\infty}\left[-\left(x_1^2 - 2\rho x_1 x_2 + x_2^2\right)/\left(2\left(1-\rho^2\right)\right)\right]dx_1 dx_2 \tag{9a}$$

$$= L(z_1, z_2; \rho) \tag{9b}$$

where

$$z_i = (\log a_i - h(e, s_i))/\sigma \qquad i=1, 2 \tag{10}$$

and

$$\rho = (\sigma_\delta / \sigma)^2 \tag{11}$$

$L(z_1, z_2; \rho)$ is a standard tabulated function (e.g. Abramowitz & Stegun [8], clause 26.3.3). ρ is referred to as the correlation coefficient.

Returning to the general situation of multiple sites, two extreme cases are where all the variability is either within earthquakes $(\sigma_{\delta} = 0)$ or between earthquakes $(\sigma_{\varepsilon} = 0)$.

When all the variability occurs within earthquakes, the product $(1/\sigma_{\delta})\phi(\delta/\sigma_{\delta})$ becomes the delta function, and the integral in (6) reduces to the product terms with δ=0. This expression is just the product of the individual probabilities of exceedance, an intuitive result when the strength of motion at one location in a given earthquake is not a function of the strengths of motion at other locations.

In the second extreme case, where all the variability occurs between earthquakes, each of the $\Phi(\cdot)$ terms in the product expression is a unit step function, with 1 - $\Phi(\cdot)$ stepping down from 1 to 0 when $log(a_i) > h(e,s_i) + \delta$. The product is nonzero only when this inequality holds for all *i*. The integral of the $\phi(\delta/\sigma_{\delta})$ term then leads to the conditional probability being the minimum conditional probability of the individual cases. Again, this is an intuitive result when the strengths of motions at different sites in a given event are deterministic functions of the strength of motion at one site, as in the case of no within-earthquake component of variability.

If the individual probabilities are small, the ratio between these extremes is large. The difference between the product and the smaller of the individual probabilities can become very large as the number of sites increases, or as the individual probabilities become small for extreme hazard levels.

This methodology requires the variability associated with the attenuation to be partitioned into within-earthquake and between-earthquake components. In models that include this separation, it has generally been made as a requirement of the regression methodology used in developing the attenuation expressions; the partitioning has been mainly of academic interest as far as application to hazard estimation is concerned, and several models that make use of the partitioning in their regression methodology unfortunately do not report the individual components of the variance. Here the partitioning of the variability is an integral part of the joint probability hazard analysis. If the partitioning is not available, we must either make some assumptions about it, based for example on ρ values of other models developed from similar datasets, or obtain upper and lower bound estimates of the joint hazard by assuming that all of the variability is between-earthquakes and within-earthquakes respectively. An important feature of peak ground acceleration and response spectrum attenuation models is that most of the variance lies in the within-event term i.e. the correlation coefficient is small (e.g. $\rho=0.125$ for the Boore-Joyner-Fumal [9] pga attenuation model, and from 0.16 for magnitudes 5 or less up to 0.36 for magnitudes of 7 and greater for pga in the New Zealand attenuation model of McVerry et al. [10]).

Multiple earthquake source analysis

Let us now consider the joint hazard rate $\lambda(a_1,\cdots, a_n)$, i.e. the average rate of occurrence of events in which the level of shaking exceeds $a_1,\cdots, a_n$ simultaneously at *n* points when there are multiple earthquake sources. The contribution $\lambda_e(a_1,\cdots, a_n)$ that an individual earthquake source *e* makes to this rate is given by

$$\lambda_e(a_1,\cdots,a_n) = \frac{1}{T_e} P(A_i \geq a_i, i = 1,\cdots,n \mid e) \tag{12}$$

where T_e is the average recurrence interval for earthquake event *e,* and $P(A_i \geq a_i, i = 1,\cdots,n \mid e)$ is given by equation (6). If all possible earthquake events and their average recurrence times are known, the joint hazard rate can be computed as

$$\lambda(a_1,\cdots,a_n) = \sum_e \lambda_e(a_1,\cdots,a_n) \tag{13}$$

The probability that at least one instance of simultaneous exceedance will occur in a time period of length T is given by

$$P_T(A_i \geq a_i, i = 1, \cdots, n) = 1 - \exp[-\lambda(a_1, \cdots, a_n)T] \; . \qquad (14)$$

The ratio of the joint exceedance rate to the individual exceedance rates of given levels of motion at two sites depends on two factors. The first factor is the extent to which both sites are affected by the same mix of earthquakes. As sites become further apart, they are less likely to be affected simultaneously in the same event, and their joint exceedance rates can be very low even when the individual exceedance rates are quite high. The second factor is the degree of variability in motions within a given earthquake. If the within-event variability is small, the joint probability of exceedance for any event approaches the smaller of the individual probabilities of exceedance, while when this variability is high the joint probability approaches the product of the individual exceedances.

The scenario approach that is used commonly to tackle joint hazard situations, such as for reliability analysis of extended lifelines systems or for loss estimates for insurance purposes, requires that the sites are affected by a similar mix of earthquakes, with the selected scenario dominating. Most scenario approaches choose a particular percentile level of motion for the individual sites, corresponding to an implicit assumption that the joint probability of exceedance is the same as the individual probabilities of exceedance. This assumption would be correct if there were no within-event variability, but in practice can considerably over-estimate joint probabilities of exceedance when there is significant within-event variability.

EXAMPLES

A number of examples are presented to demonstrate the concepts and applications of joint hazard probabilities, and to point out the differences from standard single-site analyses. The first example is a simple single-source joint-hazard analysis for two sites, as may be appropriate for a region where the seismic hazard is dominated by a single active fault. The second example demonstrates the joint hazard for three sites with a low-activity fault source at a moderate distance from the sites within a region of distributed seismicity that can be represented by a series of point sources. In both of these examples, the joint-hazard results are used to demonstrate the different hazards implied for networks with redundancy compared to those for which the failure of any one of multiple critical nodes leads to network failure. These hazard levels vary in different ways with the correlation coefficient of the single-event earthquake motions. Finally, the concepts are extended to loss estimates from all events for three sites in a region where there is a dominant fault.

JOINT HAZARD FOR A SINGLE SOURCE

This example illustrates a situation where a scenario approach is usually assumed to be appropriate, namely where the hazard at the sites of interest is dominated by a single source.

Consider sites in Upper Hutt and Wellington, both at distances of less than 3 km from the Wellington-Hutt Valley segment of the Wellington fault but separated by about 30 km along the fault. The scenario uses the Wellington fault parameters and attenuation model from a recently published New Zealand National Seismic Hazard Model (NZNSHM) (Stirling et al. [11]). In the NZNSHM, this segment of the Wellington fault is modelled as producing only characteristic magnitude 7.3 earthquakes, at an average recurrence interval of 600 years. The 50-, 84- and 90-percentile peak ground accelerations estimated for these sites in the Wellington fault event are about 0.6g, 0.95g and 1.1g for Wellington, and 0.65g, 1.0g and 1.15g for Upper Hutt. The hazard at both of these sites is dominated by this segment of the Wellington Fault for return periods of about 500 years and greater, especially beyond about 1000 years. The hazard curves for the two sites are very similar, for both the Wellington-Hutt Valley fault event alone, and when all events are taken into account. The Wellington-Hutt Valley fault 50-, 84- and 90-percentile acceleration values correspond to individual site return periods of about 700 years, 2500 years and 4000 to 5000 years respectively when the contributions of all events are taken into account. The example presented here is for the Wellington fault scenario alone.

A complete presentation of the distribution of joint hazard at two locations requires a surface of probability of exceedance as a function of the pairs of accelerations at the two sites, as shown by Wesson & Perkins [6}. For simplicity, the results here are presented in the form of hazard curves for the situation where the acceleration levels are the same at the two locations, corresponding to a particular cross-section through the general hazard surface. Other cross-sections that may of interest are those where the acceleration is held constant at one site and varied at the other, as used by Leonard & Steinberg [7], with a series of curves produced for different values of the selected hazard at the reference location (e.g. the hazard curves at Upper Hutt for accelerations of 0.1g, 0.2g through to 0.6g at Wellington).

The joint hazard curves for Wellington and Upper Hutt presented in Figure 1 are in normalized form. The normalisation is in terms of the single-site exceedance rates at Wellington, which are essentially the same as those for Upper Hutt, so they represent the joint exceedance as a function of acceleration as a fraction of the single-site exceedance rate. The normalised joint hazard curves are shown for a selection of values of ρ, the ratio of the between-event to total aleatory variance, covering the entire possible range from 0 to 1. In the NZNSHM, ρ is magnitude-dependent, ranging from 0.16 for magnitudes of 5 and less to 0.36 for magnitudes of 7 and greater. These values are similar to those found in other attenuation models. The curve for $\rho=0.36$ appropriate for the Wellington-Hutt Valley fault segment is included in the figure.

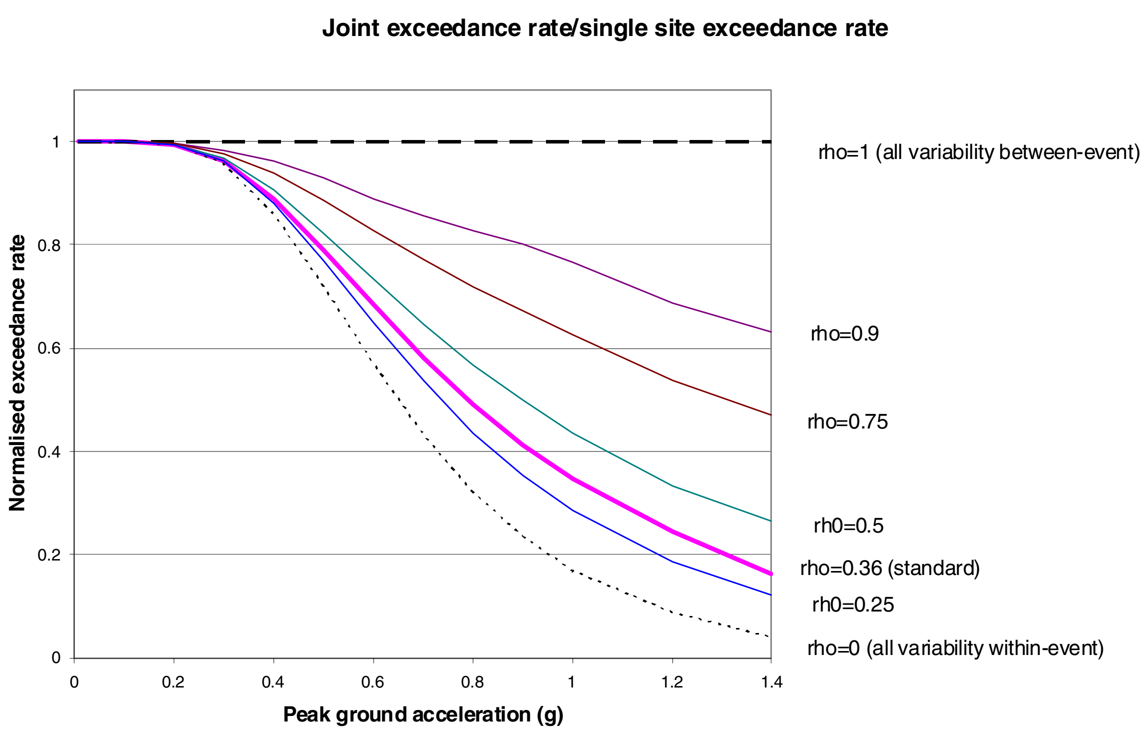

Figure 1. Joint exceedance rate normalised by Wellington single-site exceedance rate for PGA in Wellington-Hutt Valley fault event. Each curve is for a different value of ρ (ratio of between-event variance to total aleatory variance) shown at the right hand side of the plot.

For no within-event variability *(ρ=1)*, the normalised joint exceedance rate is 1.0, the usual assumption in scenario approaches. When all the variability is within events *(ρ=0)*, the normalised joint exceedance rate falls well below 1.0, with the ratio decreasing as the acceleration level increases. As the joint probabilities of exceedance for this case are the product of the individual probabilities, the normalised joint exceedance rate is the same as the individual exceedance probability for a given acceleration level, i.e. 0.5 at the 50-percentile level, 0.16 at the 84-percentile level and 0.1 at the 90-percentile level. For the actual ρ-value of the attenuation model, the normalised exceedance rates are 0.62, 0.35 and 0.30 at the 50-, 84- and 90-percentile levels, respectively. These are closer to the $\rho=0$ values than to the $\rho=1$ values that correspond to scenario analyses. The results depart rapidly from the $\rho=1$ case as ρ reduces from 1.

These results have important implications for the standard scenario approach. For critical lifelines facilities, 84-percentile or occasionally 90-percentile scenarios may be considered. The 84-percentile acceleration value resulting from the Wellington-Hutt Valley scenario corresponds to individual site return periods of about 2500 years according to the NZNSHM estimates, a return period at about the top end of the range that is considered for most lifelines systems. However, for the ρ-value associated with the attenuation model, this lengthens to about 7500 years for the joint return period, or about 15,000 years if the $\rho=0$ case applied. The 90-percentile levels show an even more exaggerated effect. These return periods are well beyond what is appreciated in most uses of the scenario approach.

For a network system with redundancy, requiring failure at both locations for the network to fail, within-event variability reduces the joint probability of failure compared to the case where the variability is all between events. However, for a non-redundant network where failure at either location causes failure of the network, the converse is true: the network is less reliable when there is variability within events than when all the variability is between events. This result arises because, for two sites, the probability of exceedance of the critical accelerations at one or both of the sites in a single event is $P_1 + P_2 - P_{12}$, where P_1 and P_2 are the individual probabilities of exceedance and P_{12} is the joint probability. Obviously, this sum increases as the joint probability decreases. When all variability occurs between events, the probability of exceedance at one or both of locations becomes the larger of the individual exceedance probabilities, while for all within-event variability, it approaches the sum of the two individual probabilities if the probabilities are small, because P_{12} is the product of the two small probabilities.

Comparisons of the probabilities of exceedance of the critical accelerations in a Wellington Fault earthquake are shown in Figure 2 for redundant versus non-redundant systems with critical sites at Wellington and Upper Hutt for values of ρ of 0.36 (the standard case), 0 (all within-event variability) and 1 (all between-event variability).

For the redundant case (corresponding to the joint probabilities that have been discussed to date), the probabilities vary from the product of the individual probabilities *(ρ=0)* to the smallest individual probability, corresponding to that at Wellington *(ρ=1)*. For the non-redundant case, the probabilities of exceedance range upwards from the larger of the individual probabilities for $\rho=1$ to a value that becomes close to the sum of the individual site probabilities as the acceleration increases. Redundancy obviously has much greater benefit in the $\rho=0$ case (all within-event variability) than in the $\rho=1$ case. In fact, if the individual site probabilities were exactly equal, the probabilities of exceedance of the critical accelerations would be the same for the individual sites, the redundant case and the non-redundant case, and there would be no benefit in redundancy for sites with equal hazard in a given event if the critical accelerations are well-defined, with no variability in the damage versus acceleration function. This is clearly an unrealistic result in practical situations, but is what comes out of a standard scenario analysis. For both systems, the case for the actual value of ρ in the attenuation model *(ρ=0.36)* is closer to the $\rho=0$ case than to the value for $\rho=1$ that is inherent in scenario analyses.

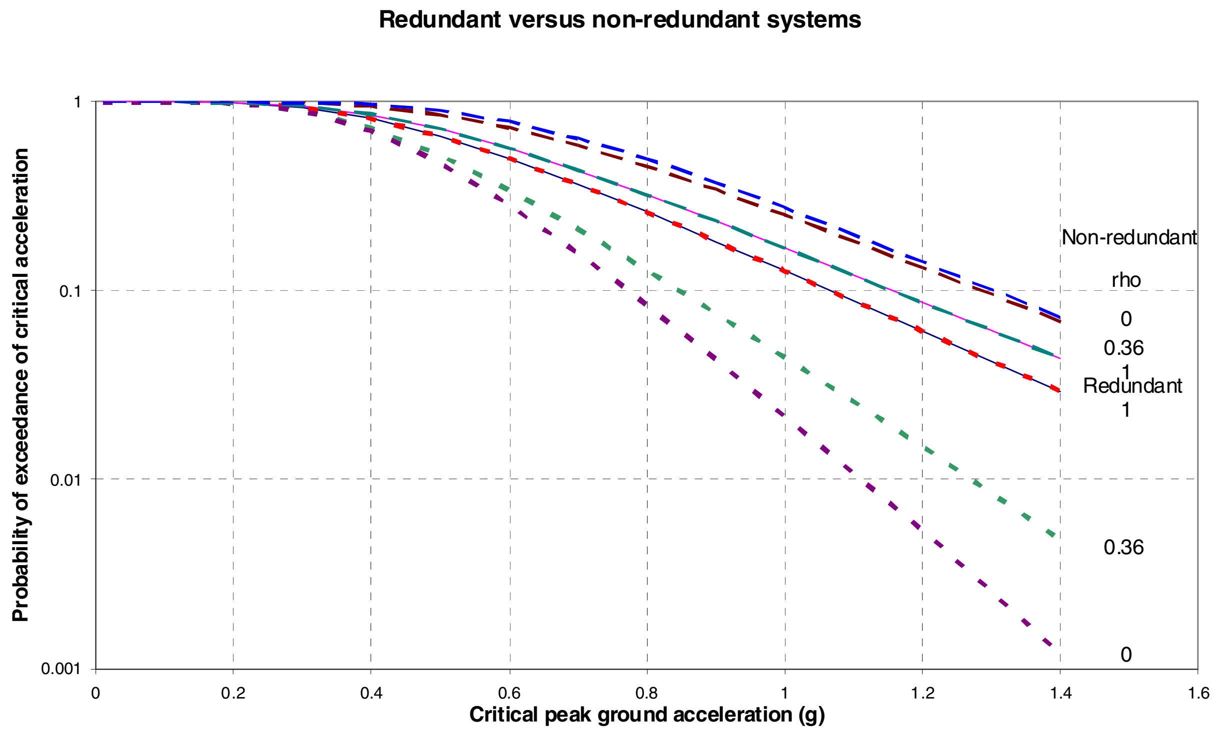

Figure 2: Comparison of probabilities of exceedance of critical peak accelerations for redundant and non-redundant systems with critical sites at Wellington and Lower Hutt for a Wellington Fault earthquake. The individual site curves are shown as thin lines under the two ρ=1 cases.

The variation of joint hazard with ρ is more pronounced for single-source scenarios than when multiple sources are considered. For example, theoretical expressions that we have derived and examples carried out including all sources in the NZNSHM both show that the curves for a range of ρ values become more compressed when multiple events are considered. For multiple events, some events will affect some sites more than others, so that even in the $\rho=0$ case the results show benefits from redundancy.

Let us consider now an example involving multiple sources.

JOINT HAZARD AT THREE SITES IN A REGION OF MULTIPLE POINT SOURCES

Figure 3 shows three sites in the Auckland area, a relatively low seismicity region of New Zealand. They are exposed to earthquakes from the extension of the northern segment of the Kerepehi fault into the Hauraki Gulf, and from two seismicity regions, A and B, in which earthquakes are treated as point sources. The Kerepehi fault is the largest known surface fault in the Auckland area, modeled in this study as producing magnitude 6.9 earthquakes with an average recurrence interval of 5000 years.

The eastern region B has historically had a higher level of seismicity than the western region A, which includes the Auckland urban area. The seismicity parameters assumed for each region are given in Table 1, where a_4 is the expected number of earthquakes of magnitude 4 or greater per 1000 km^2 per year, b is the Gutenberg-Richter b-value, and $M_{\max}$ is the maximum magnitude. The rate of occurrence of earthquakes exceeding magnitude M is given in events/1000km^2/yr by

$$\lambda(M) = a_4[10^{-b(M-4)} - 10^{-b(M_{\max}-4)}]. \quad (15)$$

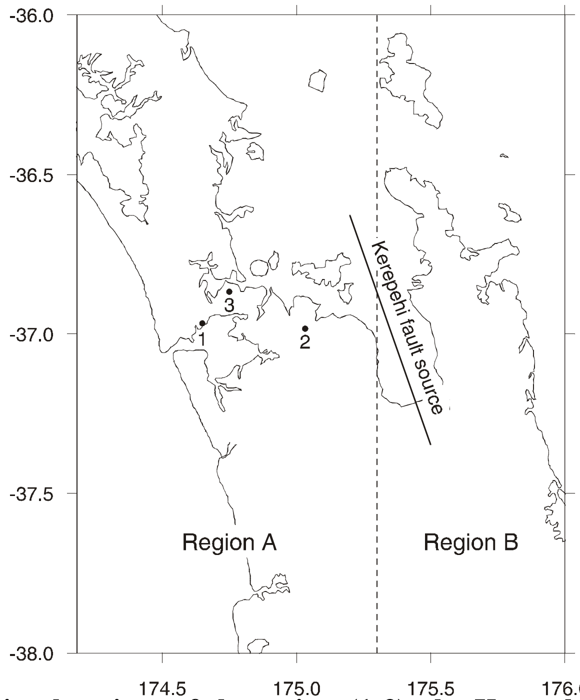

Figure 3. Map showing locations of three sites (1-3), the Kerepehi fault source and the seismicity regions A and B.

Table 1. Seismicity parameters for Regions A and B

Region	a_4	b	M_{max}
A	0.01	0.89	7.0
B	0.103	1.27	7.0

The three sites could represent, for example, three critical nodes of a lifelines network. We seek to evaluate the joint probability of earthquake shaking exceeding a given peak horizontal ground acceleration (pga) at any one of the sites or simultaneously at all three sites over a 1000 year period using the method of Monte Carlo simulations. This involves generating many random catalogues composed of both characteristic earthquakes on the Kerepehi fault (M6.9, mean recurrence interval 5000 years) and seismicity distributed in regions A and B (parameters in Table 1).

Here we adopt a simple attenuation model given by

$$\log_{10}(a_i) = -1.24 - \log_{10} R_i + 0.28M - 0.0022R_i + \delta + \varepsilon_i \quad (11)$$

$\sigma_\delta = 0.08$

$\sigma_\varepsilon = 0.23$

where M is earthquake magnitude, a_i is the peak horizontal acceleration at site i, and

$$R_i = (D_i^2 + h^2)^{1/2} \quad (12)$$

D_i is the horizontal distance in kilometres from the source to the site and $h=6.57$ is a depth parameter common to all earthquakes. Note that σ_ε is about three times larger than σ_δ, showing that most of the variability occurs in the within-earthquake term in this case, with $\rho=0.11$.

Monte Carlo techniques were used to generate 200 simulated 1000-year catalogues of earthquakes with $M \geq 5.0$. For each earthquake a value of δ was simulated and for each site and earthquake a value of ε_i. From each earthquake the acceleration at each of the three sites was noted and also the maximum and minimum acceleration across the two sites. The probability of any given level of shaking being exceeded was estimated from the resulting statistics, together with the probabilities that a given threshold of acceleration is exceeded simultaneously at all three sites and for at least one site. The results are shown in Figure 4.

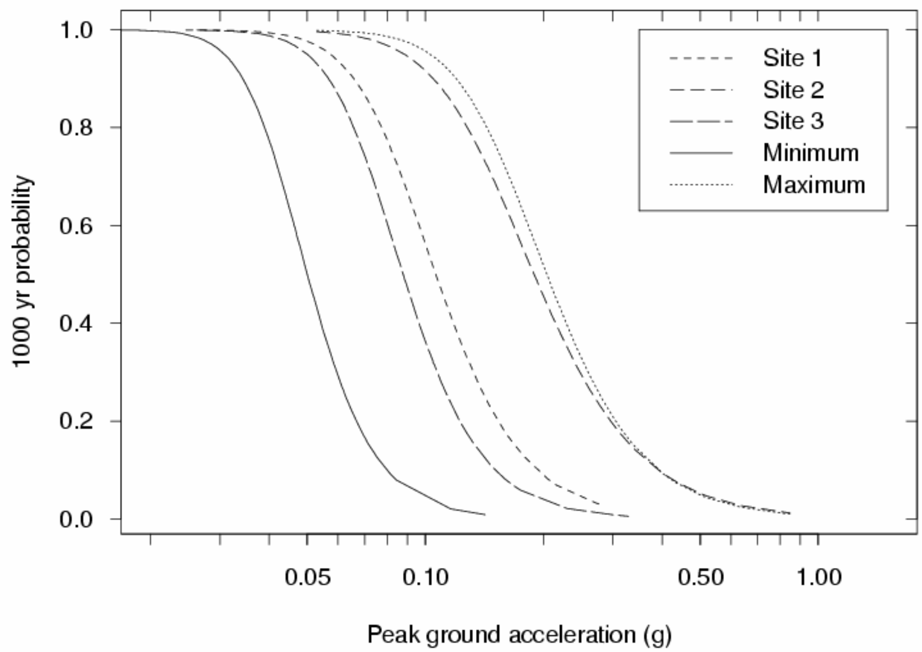

Figure 4. 1000-year probabilities of exceedance for the peak ground accelerations at the three Auckland region sites (Fig. 3) and for the maximum and minimum pgas across the three sites, corresponding respectively to the probabilities of exceedance of critical accelerations for non-redundant and redundant systems with critical facilities at the three locations.

The probability that a given threshold of acceleration is exceeded simultaneously at all three sites is the probability that the minimum of the accelerations at the three sites exceeds the threshold. Assuming that the critical accelerations are the same at all three locations, as they may be for some standard equipment for which the design is not varied to reflect different hazard levels at different locations in a region, this is the probability of exceeding the critical acceleration for network failure with redundancy. Similarly, the probability that a given threshold is exceeded for at least one site is the probability that the maximum of the three sites exceeds the threshold, and corresponds to network failure in the case of multiple critical points, failure of any one of which leads to network failure. In this situation there is a much reduced probability of failure at all points compared to that at just one point, presumably because there are few individual earthquakes that cause strong motions at all three locations.

For a region of distributed seismicity where there is no dominant source, this example demonstrates that the return periods for the joint exceedance may be several times the return periods for the individual sites, with the ratio increasing as the level of shaking increases. Also, although not demonstrated by the results shown here, the return periods of joint exceedance of critical levels of shaking are much less dependent on the relative sizes of the components of variability.

COMPARISON OF JOINT HAZARD WITH SCENARIO APPROACH

As already noted, a common approach in studies of lifeline systems in earthquakes is to develop a scenario earthquake. In the scenario approach, one seeks to identify a single earthquake source and return period that epitomize the earthquake hazard across a region. This is easy to do only when the earthquake hazard is dominated by a single source, as in the Wellington region. In the Auckland region, it is difficult to achieve because the distributed seismicity, although low compared with other New Zealand areas, dominates compared with the hazard contributed by the most obvious candidate for the "largest earthquake" scenario, the Kerepehi fault. Generating appropriate point-source scenarios was difficult. Four different scenarios designed to produce pgas of about 0.2g across much of the urban area each produced up to 0.45g for parts of the region. The problem was overcome by selecting a scenario earthquake centred offshore to the east of site 3 and north of site 2, that produced shaking intensities within the main urban area (in the isthmus around site 3) close to those estimated from a uniform hazard model, with the unrealistic higher accelerations near the selected source lying offshore or on largely uninhabited islands.

Even where a single event contributes most of the hazard, the assumption implicit in these approaches is that the return period for the joint hazard levels is the same as the return periods for the individual sites. The single earthquake source example for the Wellington Fault showed that the joint probability of the critical motions given that the earthquake occurs may be considerably less than the individual probabilities, particularly as the acceleration increases. Conversely, within-earthquake variability increases the possibility of obtaining an extreme motion at one of multiple locations compared to the single-site probabilities. The ratios of the accelerations resulting from joint-hazard analyses to those given by scenario analyses with no within-event variability depart further from unity as the individual probabilities decrease, i.e., as the events become more extreme. It is usually extreme events that are most significant for analysis of lifeline systems. Even when a single scenario can be identified, the standard scenario approach tends to be conservative for joint probabilities relevant to redundant systems, but under-estimates critical accelerations for non-redundant systems with multiple critical points.

The probability of ground shaking exceeding given thresholds at various locations in a lifeline system is of course only part of the problem for lifeline seismic risk analysis. What we would like to know is the probability of partial or complete loss of function of the various components of the system, and the consequent effects on the ability of the system to provide service to its customers after an earthquake. This depends not only on the uncertainty in the ground motions, but also on the uncertainty in the seismic performance and functionality of each component when subjected to a given level of ground shaking. The uncertainty in estimating component performance is by no means trivial.

In summary, there are clear disadvantages associated with the scenario approach:

- It may not be possible to find a single scenario that adequately represents the hazard.
- The scenario PGAs cannot be relied on for all locations in the region.
- Usually there is no inclusion of within-event variability, which greatly affects the joint-hazard at multiple locations.
- The scenario is often only designed around a single return period.

LOSS ESTIMATES

The within- and between-earthquake components of the variance in earthquake ground-shaking attenuation models are now being used in New Zealand in earthquake loss modeling (Smith [12]) to account for the correlation of earthquake ground shaking at multiple locations within the same earthquake. The joint hazard across multiple locations is handled through Monte Carlo techniques.

As an example of joint hazard on loss estimates, consider an enterprise that operates at three locations in the Wellington region, where the hazard is dominated by the Wellington Fault. Two sites are within 2 km of the Wellington fault, so have very similar hazard levels, while the third is about 8 km distant from it, so has slightly reduced hazard. The total replacement values of the properties at the three locations are $62M, with about 40% of the exposure at the lower hazard site.

Loss estimates were performed through a damage ratio model based on Modified Mercalli intensities (Cousins [13]). In the damage ratio approach to loss estimation, the loss is defined as the product of the damage ratio D_r and the replacement value, where D_r is defined as

$$D_r = \frac{\text{cost of material damage to property at risk}}{\text{replacement value of property at risk}} \tag{13}$$

The mean damage ratio is a function of the intensity of shaking and is given by a model of the form

$$\overline{D}_r = A \times 10^{\left(\frac{B}{MMI - C}\right)} \tag{14}$$

where $\overline{D}_r$ is the mean damage ratio, *MMI* the shaking intensity, and *A*, *B* and **C** are constants. A log-normal distribution is used for the damage ratio distribution for a given intensity, with the standard deviation of D_r assigned the same value as the mean damage ratio.

Intensities were modeled by the Smith [14] implementation of the Dowrick and Rhoades [15] attenuation model. For a Wellington Fault earthquake, the region encompassing the fault has median intensities of MM9 or greater. The intensity attenuation model has a total standard deviation $_{l}\sigma_{MMtotal}$ of 0.43. Assuming that pga increases by a factor of 1.5-2 for a unit increase in intensity, $\sigma_{MMtotal}$ of 0.43 corresponds to a total standard deviation of *ln(pga)* of 0.17-0.30, far less than typical values of 0.45 to 0.70 in pga attenuation models. The values are greater when individual site intensities rather than isoseismal intensities are considered. With such a low variance, the effects from variability in intensity are not great, and the effects in variation of ρ are masked by the variability in the damage ratio model. To investigate better the effect of ρ on damage costs for a ground motion model with variability more typical of published attenuation models, the variance of the intensity model was increased to make it consistent with that of the pga attenuation model used in the New Zealand National Seismic Hazard Model as used previously in the Wellington Fault example. The total standard deviation was increased to 1, consistent with that of 0.45 for *ln(pga)* if pga increases by about about a factor of 1.6 with unit change in intensity. The correlation coefficient ρ was taken as 0.36, as in the pga attenuation model for magnitudes greater than 7. The losses were estimated with this increased total variance for ρ values of 0.36, 0 and 1, as shown in Figure 5.

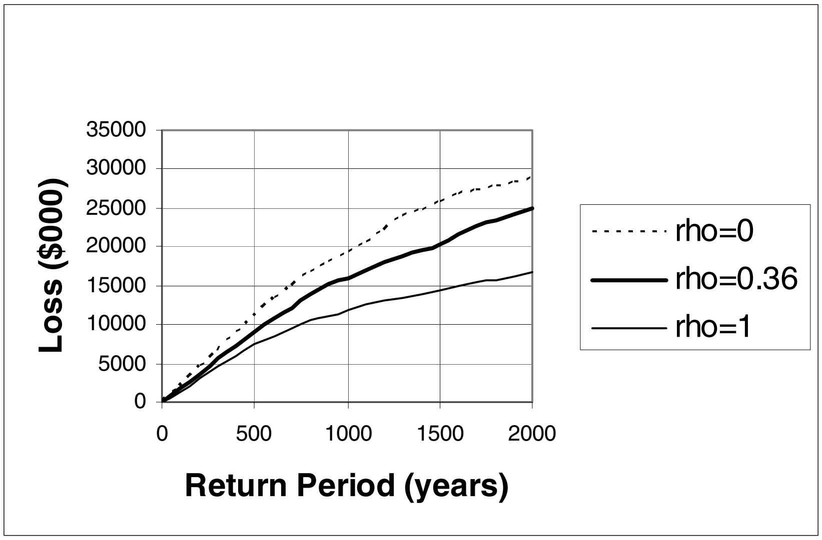

Figure 5: Loss estimates for a portfolio spread across three locations in the Wellington region for three values of ρ, with ρ=0.36 being consistent with a New Zealand pga attenuation model.

The losses show a strong dependence on ρ, with the $\rho=1$ case showing the lowest loss and $\rho=0$ the greatest. As in the other examples, the estimates for the actual ρ value of the attenuation model lie closer to the $\rho=0$ than to the $\rho=1$ case. These results appear at odds with those of Wesson and Perkins [6], who stated that "the risk of a large loss to a portfolio is significantly greater if all the variability is interevent than if all the variability is intraevent". Their statement is true for the largest losses, but may require very long return periods before it takes effect. There may be a broad range of return periods for which intraevent (i.e within-earthquake) variability gives greater loss estimates, depending on the combination of the exposure and hazard at different locations. The loss curves for this example have been continued out to 20,000 years return period, and show similar behaviour with ρ. For a small portfolio as in this example, extreme losses at one location may have greater probability than the same total loss made up of more moderate losses at all localities, especially if the damage ratios change rapidly with intensity. The maximum motion, which is greatest for the $\rho=0$ case, then governs rather than the joint hazard, which is greatest for the $\rho=1$ case.

CONCLUSIONS

A general method has been presented for estimating the joint hazard of given thresholds of strong shaking being exceeded at two or more sites in the same earthquake. The joint hazard rather than individual site hazards is useful where the continuation of a system, activity or lifeline depends on at least one of several critical facilities at different sites remaining operational. The maximum acceleration exceeded at one of multiple sites is also important for lifelines studies, for non-redundant systems with multiple critical nodes. The distribution of the maximum acceleration hazard depends on a combination of hazards at the individual sites and their joint hazards. Joint and maximum hazard also affect estimation of earthquake losses over multiple sites. The examples presented here are simple, but the method can be applied with no great difficulty to situations of much greater complexity, e.g. complex networks involving any number of sites, with variable thresholds of shaking at different sites, and attenuation models that account for more site and source effects. Epistemic variability can be included as well as the aleatory variability included in the examples presented here

The joint hazard method has distinct advantages over the scenario approach commonly applied in lifelines studies. Unlike the scenario approach, it can adequately represent the hazard at all locations in the region, and for all return periods, and can formally account for uncertainties. Quite apart from the difficulty of determining appropriate scenario events when the hazard in an area is not dominated by a single earthquake source, even for single events the scenario approach can give misleading results depending on how the variability in motions is treated. Applying the variability uniformly across the scenario region, i.e. taking 50-, 84- or 90-percentile values everywhere, over-estimates joint probabilities of motions at multiple sites but under-estimates the maximum motion at one of many sites. This means that scenarios are conservative for redundant systems, but non-conservative for systems with multiple critical locations without redundancy.

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