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SUGGESTION AND THEORETICAL EVALUATIONS ON THE PERFORMANCE OF A NEW METHOD TO EXTRACT PHASE VELOCITIES OF RAYLEIGH WAVES FROM MICROTREMOR SEISMOGRAMS OBTAINED WITH A CIRCLULAR ARRAY

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SUMMARY

We present a formulation of a new method to determine phase velocities of Rayleigh waves from microtremor waveforms obtained with a circular array of seismometers, as well as practical considerations on the technical details of the method. In our method the seismograms are averaged along the circumference at each time step with an appropriate set of weights. The spectral ratio of two different kinds of such time histories, obtained by averaging with different sets of weights, depends solely on the product rk (k: wavenumber, r: array radius); we can derive information on k since r is known. According to the results of theoretical calculations based on a deductive approach and conducted for the case where the wavenumber is a single-valued function of frequency, an array of three sensors is expected to be sufficiently useful in realistic situations where microseisms arrive from many directions. The theoretical calculations also revealed that the analysis method is valid for wavelengths upward of $4r$; that the presence of incoherent noise makes analysis in long wavelength ranges difficult; and that the upper limit of the analyzable wavelength is roughly equal to $40r$ and $10r$ for SN ratios of 100 and 10, respectively.

INTRODUCTION

Microseisms can be utilized for relatively cost-effective reconnaissance surveys of the underground structure; the dispersion characteristics of the surface waves, which can be obtained by analyzing microseismic records from an array of seismic sensors, serve as a constraint for inferring the velocity structure beneath the observation site.

Aki [1], in his general theory of the analysis of microseisms, presented a method to determine the phase velocities from the vertical component of microseismic records. Aki's method requires a set of sensors placed along a circumference plus another at its center. The whole information on the wavefield in question, which in the general case is composed of multiple plane waves arriving from different directions

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with different intensities and in different modes, is integrated into a single quantity, namely the azimuthal average of the correlation coefficients. This quantity, according to his theory, contains information on the phase velocities of the waves alone: the effects of the arrival directions and amplitudes are canceled out in the above process.

Henstridge [2] recast Aki's method in a simpler form using a spectral representation based on the theory of stationary random processes. He also developed an original formulation, independent of Aki's, which is general enough to include Aki's method as a special case but is based on a stronger assumption that the phase velocity is a single-valued function of the frequency. His formulation not only helps to see Aki's method in a different light but is also more useful from a practical point of view; for example, only through Henstridge's formulation can one theoretically examine the effects the use of a finite number of sensors and their arrangement have on the estimates of the phase velocities.

In the present study, we have developed a new method of microseismic exploration on the extension of Henstridge's [2] theory, lifting the assumption of single-valuedness described above and introducing a new technique to allow for arrays of sensors placed at uneven intervals along a circumference. Just as Henstridge [2] did, we deal only with the vertical component of microseisms, which is in general cases dominated by Rayleigh waves according to the theory of wave propagation.

METHOD

According to Yaglom [3], any homogeneous random field $z(t, x, y)$, a function of time t and position (x,y), has a spectral representation

$$z(t, x, y) = \iiint \exp(-i\omega t - ik_x x - ik_y y) Z'(d\omega, dk_x, dk_y), \qquad (1)$$

where ω is the angular frequency, (k_x, k_y) is the wavenumber, and Z' is a random spectral measure. This Fourier-Stieltjes integral representation states that every homogeneous random field can be considered as a continuous sum of independent harmonic waves. The random spectral measure Z' is an orthogonal process with regard to frequency and wavenumber, since $z(t,x,y)$ is stationary in time and space. If there exists a frequency-wavenumber spectral density $f'(\omega, k_x, k_y)$, the following relation holds:

$$\begin{aligned} &< Z'(d\omega, dk_x, dk_y) Z'^{*}(d\omega', dk'_x, dk'_y) >= \delta(\omega - \omega')\delta(k_x - k'_x)\delta(k_y - k'_y) \\ &\times f'(\omega, k_x, k_y) d\omega d\omega' dk_x dk'_x dk_y dk'_y , \end{aligned} \qquad (2)$$

where an asterisk denotes the complex conjugate.

It is convenient to shift to the polar form since we are dealing with a circular array:

$$z(t, r, \theta) = \iiint \exp\{-i\omega t - irk\cos(\theta - \phi)\} Z(d\omega, dk, d\phi). \qquad (3)$$

As in the case of the Cartesian coordinate system, $Z(\omega, k, \phi)$ is an orthogonal process with regard to the frequency ω, wavenumber k, and arrival direction ϕ. The following relation holds if there exists a frequency-wavenumber-direction (FWD) spectral density $f(\omega, k, \phi)$:

$$< Z(d\omega, dk, d\phi) Z^*(d\omega', dk', d\phi) >= \delta(\omega-\omega')\delta(k-k')\delta(\phi-\phi') \\ \times f(\omega, k, \phi) d\omega d\omega' kdkdk' d\phi d\phi'. \qquad (4)$$

The FWD spectral density represents the power of the wave component arriving from direction ϕ with frequency ω and wavenumber k.

Let us denote by $z(t, r, \theta)$ the vertical component of the seismogram to be recorded along the circumference of radius r and at the azimuthal location θ, composed of stationary and random plane waves arriving from different directions with different intensities. We postulate that waves come from mutually uncorrelated vibration sources that lie far enough from the seismic array, so that different components of the incident waves are mutually uncorrelated.

We expand the random spectral measure and the FWD spectral density into Fourier series:

$$Z(d\omega, dk, d\phi) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \exp(im\phi) Z_m(d\omega, dk) d\phi, \quad (5)$$

$$Z_m(d\omega, dk) = \int_{-\pi}^{\pi} \exp(-im\phi) Z(d\omega, dk, d\phi), \quad (6)$$

and

$$f(\omega, k, \phi) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \exp(im\phi) f_m(\omega, k), \quad (7)$$

$$f_m(\omega, k) = \int_{-\pi}^{\pi} \exp(-im\phi) f(\omega, k, \phi) d\phi. \quad (8)$$

It follows from equations (4), (6) and (8) that

$$< Z_l(d\omega, dk) Z_m^*(d\omega', dk') >= \delta(\omega-\omega')\delta(k-k') f_{l-m}(\omega, k) d\omega d\omega' kdkdk'. \quad (9)$$

Using equation (5) and the formula

$$\int_{-\pi}^{\pi} \exp[-irk\cos(\theta-\phi) + im\phi] d\phi = 2\pi J_m(rk) \exp[(im(\theta - \pi/2)], \quad (10)$$

we can rewrite (3) as

$$z(t, r, \theta) = \iint \sum_{m=-\infty}^{\infty} J_m(rk) \exp\{-i\omega t + im(\theta - \pi/2)\} Z_m(d\omega, dk). \quad (11)$$

We also expand $z(t, r, \theta)$ into a Fourier series:

$$\alpha_m(t, r) = \int_{-\pi}^{\pi} \exp(-im\theta) z(t, r, \theta) d\theta, \quad (12)$$

$$z(t,r,\theta)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\exp(im\theta)\alpha_m(t,r), \quad (13)$$

Substituting (11) into (12), we get

$$\alpha_m(t,r)=2\pi\exp(-im\pi/2)\iint\exp(-i\omega t)J_m(rk)Z_m(d\omega,dk). \quad (14)$$

Spectral ratio function

By the use of equations (9) and (14), the auto-correlation function for the Fourier coefficient $\alpha_m(t,r)$ is written as

$$<\alpha_m(s,r)\alpha_m^*(s+t,r)>=(2\pi)^2\iint\exp(i\omega t)J_m^2(rk)f_0(\omega,k)d\omega kdk. \quad (15)$$

Wiener-Khintchine's theorem states that the power spectral density $G_m(\omega,r)$, corresponding to the *m*-th coefficient, is given by the Fourier transform of the auto-correlation function, so that

$$G_m(\omega,r)=(2\pi)^2\int J_m^2(rk)f_0(\omega,k)kdk, \quad (16)$$

where we have made use of the fact that f_0 is an even function of ω. Finally, the ratio of power spectral densities of the zeroth and first orders is given by

$$G_0(\omega,r)/G_1(\omega,r)=\int J_0^2(rk)f_0(\omega,k)kdk/\int J_1^2(rk)f_0(\omega,k)kdk. \quad (17)$$

When the wavefield is composed of one or more modes of Rayleigh waves, for example, and the wavenumber is a function of the frequency, either single- or multi-valued, we may use a suffix q to denote quantities pertaining to the $(q-1)$-th mode and model the FWD spectral density as

$$f(\omega,k,\phi)=\sum_{q=1}^{Nq}f^{(q)}(\omega,\phi)\delta(k-k_q(\omega))/k \quad (18)$$

with its Fourier coefficients

$$f_m(\omega,k)=\sum_{q=1}^{Nq}f_m^{(q)}(\omega)\delta(k-k_q(\omega))/k. \quad (19)$$

Substituting (19) into (17), we obtain

$$G_0(\omega,r)/G_1(\omega,r)=\sum_{q=1}^{Nq}J_0^2(rk_q(\omega))a_q(\omega)/\sum_{q=1}^{Nq}J_1^2(rk_q(\omega))a_q(\omega), \quad (20)$$

where

$$a_q(\omega) = f_0^{(q)}(\omega) / \sum_{j=1}^{Nq} f_0^{(j)}(\omega) \quad (21)$$

is the power partition ratio for the *(q-1)*-th mode. The function $\alpha_m(t, r)$, and accordingly the left hand side of equation (20), can be evaluated from records obtained in appropriate field measurements.

Determining the phase velocities
In the special case where the wavenumber is a single-valued function of the frequency, we can determine the phase velocity by

$$c(\omega) = \omega r / M^{-1}[G_0(\omega, r) / G_1(\omega, r)], \qquad (22)$$

where $M^{-1}(\cdot)$ is the inverse of the function $M(\cdot)$, which is defined as

$$M(rk) \equiv J_0^2(rk) / J_1^2(rk). \qquad (23)$$

The reader is referred to the thick and gray curves in Figure 1 for the shape of the function *M(rk)*. Obviously, equation (22) makes sense only in the range where there is a one-on-one correspondence between the function *M(x)* and its argument, namely in the range $rk < 2.4$ or for wavelengths larger than $2.6r$.

Aki [1] suggested a method to determine multiple phase velocities for a wavefield composed of multiple modes. If we measure the wavefield with N_r arrays of different radiuses, we obtain N_r different values of the spectral ratio (20). Together with the compatibility condition

$$\sum_{j=1}^{Nq} a_q(\omega) = 1, \qquad (24)$$

they form a system of N_r+1 equations concerning a set of $2N_q$ frequency-dependent unknowns ($a_1,\ldots,a_{Nq}$; $k_1,\ldots,k_{Nq}$). All we have to do is to find the optimal values of the unknown parameters for each frequency that minimizes, in the least squares sense,

$$S(\omega) = [\sum_{q=1}^{Nr} [G_0(\omega, r) / G_1(\omega, r) - \sum_{q=1}^{Nq} J_0^2(rk_q(\omega)) a_q(\omega) / \sum_{q=1}^{Nq} J_1^2(rk_q(\omega)) a_q(\omega)]^2 \qquad (25)$$

under the constraint of equation (24), where the number N_r of array radiuses should satisfy $N_r \geq 2N_q - 1$. Once the wavenumbers $k_q(\omega)$ $(q = 1, \ldots N_q)$ are obtained, the phase velocities $c_q(\omega)$ are calculated by $c_q(\omega) = \omega / k_q(\omega)$.

PRACTICAL CONSIDERATIONS

In the above formulation, we have postulated that seismograms are available at all points along a circumference of radius *r*. In practice, however, field measurements must be done with a finite number of sensors placed at discrete azimuthal positions. In this section we describe how to estimate α_m in (12) with a circular array of seismic sensors that are not necessarily placed at even spacings because of practical reasons.

When, in equation (13), the terms of up to the K-th order dominate, the sensor output at location (r, θ_n) can be approximated by

$$z(t,r,\theta_n) \approx \frac{1}{2\pi}\sum_{m=-K}^{K}\exp(im\theta_n)\alpha_m(t,r). \qquad (26)$$

Since $z(t,\ r,\ \theta_n)$ takes a real value,

$$\alpha_{-j}(t,r) = \alpha_j^*(t,r), \qquad (27)$$

so that (26) can be rewritten in the matrix form

$$\vec{z} \approx \vec{A}\vec{\alpha}, \quad (28)$$

where

$$\vec{z} \equiv (z(t,r,\theta_1), z(t,r,\theta_2),..., z(t,r,\theta_N))^T. \quad (29)$$

$$\vec{A} \equiv \begin{pmatrix} 1 & 2\cos\theta_1 & -2\sin\theta_1 & \cdots & 2\cos K\theta_1 & -2\sin K\theta_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 2\cos\theta_N & -2\sin\theta_N & \cdots & 2\cos K\theta_N & -2\sin K\theta_N \end{pmatrix}. \quad (30)$$

$$\vec{\alpha} \equiv (\alpha_{r0}(t,r), \alpha_{r1}(t,r), \alpha_{i1}(t,r),..., \alpha_{rK}(t,r), \alpha_{iK}(t,r))^T. \quad (31)$$

where the subscripts r and i denote the real and imaginary parts, respectively, of the Fourier coefficients $\alpha_j(t,r)$. We can estimate $\vec{\alpha}$ by

$$\tilde{\vec{\alpha}} = \vec{D}\vec{z} \quad (32)$$

where $\vec{D} \equiv (d_{ij})$ is a generalized inverse of matrix $\vec{A}$. The relation $N \ge 2K+1$ must hold so that the system of equations (28) may not be underdetermined; for example, the cutoff degree K for the Fourier coefficients cannot be larger than 1 if we use an array of three sensors (N=3) and, on the contrary, we have to use at least five sensors ($N \ge 5$) if we are to set $K \ge 2$ to include the effects of the second and higher order terms in the Fourier expansion (13).

For the simplest case of N=3 and K=1, we have, from equation (32),

$$\begin{pmatrix} \tilde{\alpha}_{r0}(t,r) \\ \tilde{\alpha}_{r1}(t,r) \\ \tilde{\alpha}_{i1}(t,r) \end{pmatrix} = C \begin{pmatrix} 2\sin(\theta_2-\theta_3) & 2\sin(\theta_3-\theta_1) & 2\sin(\theta_1-\theta_2) \\ \sin\theta_3-\sin\theta_2 & \sin\theta_1-\sin\theta_3 & \sin\theta_2-\sin\theta_1 \\ \cos\theta_3-\cos\theta_2 & \cos\theta_1-\cos\theta_3 & \cos\theta_2-\cos\theta_1 \end{pmatrix} \begin{pmatrix} z(t,r,\theta_1) \\ z(t,r,\theta_2) \\ z(t,r,\theta_3) \end{pmatrix}. \quad (33)$$

$$C = \pi / \{\sin(\theta_2-\theta_3) + \sin(\theta_3-\theta_1) + \sin(\theta_1-\theta_2)\}. \quad (34)$$

Directional aliasing

It is obviously not always an appropriate assumption to set K=1 and disregard all terms of higher angular orders, so we would like to examine what effects the use of the approximative formula (33) has on the estimate of the spectral ratio G_0/G_1. Substituting (11) into (33), we have

$$\begin{aligned}&\tilde{\alpha}_0(t,r) = \tilde{\alpha}_{r0}(t,r)\\&= \sum_{j=1}^{3} d_{1j} \iint \sum_{m=-\infty}^{\infty} J_m(rk) \exp\{-i\omega t + im(\theta_j - \pi/2)\} Z_m(d\omega, dk)\end{aligned} \qquad (35)$$

and

$$\begin{aligned}&\tilde{\alpha}_1(t,r) = \tilde{\alpha}_{r1}(t,r) + i\tilde{\alpha}_{i1}(t,r)\\&= \sum_{j=1}^{3} (d_{2j} + id_{3j}) \iint \sum_{m=-\infty}^{\infty} J_m(rk) \exp\{-i\omega t + im(\theta_j - \pi/2)\} Z_m(d\omega, dk),\end{aligned} \qquad (36)$$

so we obtain

$$\begin{aligned}&\tilde{G}_0(\omega,r)/\tilde{G}_1(\omega,r) = F :< \tilde{\alpha}_0(s,r)\tilde{\alpha}_0^*(s+t,r) > / F :< \tilde{\alpha}_1(s,r)\tilde{\alpha}_1^*(s+t,r) >\\&= \sum_{m,n=-\infty}^{\infty} \sum_{j,k=1}^{3} d_{1j}d_{1k} \exp[i(m\theta_j - n\theta_k)] \int J_m(rk)J_n(rk)W_{m-n}(\omega,k)dk\\&/ \sum_{m,n=-\infty}^{\infty} \sum_{j,k=1}^{3} (d_{2j}d_{2k} - id_{2j}d_{3k} + id_{3j}d_{2k} + d_{3j}d_{3k})\\&\times \exp[i(m\theta_j - n\theta_k)] \int J_m(rk)J_n(rk)W_{m-n}(\omega,k)dk\end{aligned} \qquad (37)$$

where

$$W_m(\omega,k) \equiv \exp(-im\pi/2) f_m(\omega,k)k, \qquad (38)$$

and $F:$ denotes the Fourier transform. All terms with suffixes other than m=n=0 for the numerator and $m = n = \pm 1$ for the denominator represent biases coming from the effects of directional aliasing, or errors due to the presence of higher order terms that are not properly taken into account in a discrete numerical scheme because of the finite sampling intervals. The above equation indicates that the effects of directional aliasing are larger in short wavelength ranges (large rk), since those effects are represented by the Bessel functions of higher orders.

When the wavenumber is a function of the frequency, either single- or multivalued, we have

$$\tilde{G}_0(\omega,r)/\tilde{G}_1(\omega,r) = \sum_{m,n=-\infty}^{\infty}\sum_{q=1}^{Nq}\sum_{j,k=1}^{3} d_{1j}d_{1k}\exp[i(m\theta_j - n\theta_k)]J_m(rk_q(\omega))J_n(rk_q(\omega))W_{m-n}^{(q)}(\omega)$$
$$/\sum_{m,n=-\infty}^{\infty}\sum_{q=1}^{Nq}\sum_{j,k=1}^{3}\sum_{j,k=1}^{3}(d_{2j}d_{2k} - id_{2j}d_{3k} + id_{3j}d_{2k} + d_{3j}d_{3k})$$
$$\times\exp[i(m\theta_j - n\theta_k)]J_m(rk_q(\omega))J_n(rk_q(\omega))W_{m-n}^{(q)}(\omega)$$
(39)

where

$$W_m^{(q)}(\omega) \equiv \exp(-im\pi/2) f_m^{(q)}(\omega). \quad (40)$$

The appearance of d_{ij} and $W_m^{(q)}(\omega)$ in equation (39) indicates that the nature of directional aliasing effects depends both on the configuration of sensors and the directionality of the wavefield.

Presence of incoherent noise

In the case where stationary, non-propagating noise $n_j(t)$ contaminates the records of the *j*-th sensor, we can examine, by replacing $z(t,r,\theta_j)$ with $z(t,r,\theta_j)+n_j(t)$ in equation (33), the influence the presence of noise has on the estimate of the Fourier coefficients $\tilde{\tilde{\alpha}}$. If we assume that $n_j(t)$ is uncorrelated with the signal $z(t,r,\theta_j)$ and that $n_i(t)$ and $n_j(t)$ are mutually uncorrelated if $i \neq j$, the necessary modification on the estimated spectral ratio (37) or (39) consists in adding

$$\sum_{j=1}^{3} d_{1j}^2 N_j(\omega) \quad (41)$$

and

$$\sum_{j=1}^{3}(d_{2j}^2 + d_{3j}^2)N_j(\omega) \quad (42)$$

to the numerator and denominator of the right hand side, respectively, where $N_j(\omega)$ represents the power spectral density of the incoherent noise:

$$N_j(\omega) \equiv F :< n_j(s)n_j^*(s+t) >. \quad (43)$$

In the absence of noise, the denominator in equation (37) or (39) approaches 0 as $rk \to 0$, but the above considerations indicate that this does not take place in the presence of noise. As a result, the spectral ratio does not tend to infinity in the long wavelength limit. Since the effects of directional aliasing are small in small *rk* ranges (refer to the thick and gray curves in Figure 1), the level of incoherent noise is expected to be the crucial factor that controls the accuracy of the estimate; its presence leads to the underestimation of the spectral ratio, overestimation of the parameter *rk* and, consequently, to the underestimation of the phase velocity. The presence of incoherent noise can also have certain influence on the estimate of the spectral ratio upward of $rk \approx 2$, but the presence of noise is not the sole factor to influence the estimate in that range of *rk*, because the effects of directional aliasing are far more significant.

Some theoretical calculations

We shall now examine basic properties of directional aliasing by assuming that the wavenumber is a single-valued function of the frequency (*Nq*=1). The FWD spectral density can be written as (44) if waves arrive from a finite range of directions centered on ϕ_0, the power dropping linearly with azimuthal distance from ϕ_d.

$$f^{(1)}(\omega,k,\phi)=\begin{cases} p(\omega)[1/\phi_d-|\phi-\phi_0|/\phi_d^2]\delta(k-k_1(\omega))/k & (\phi_0-\phi_d\le\phi\le\phi_0+\phi_d) \\ 0 & (otherwise) \end{cases} \quad (44)$$

In this case the weights $W_m^{(1)}(\omega)$ defined by (40) are given by

$$W_m^{(1)}(\omega)\equiv p(\omega)\frac{\sin^2(m\phi_d/2)}{(m\phi_d/2)^2}\exp[-im(\phi_0+\pi/2)]. \quad (45)$$

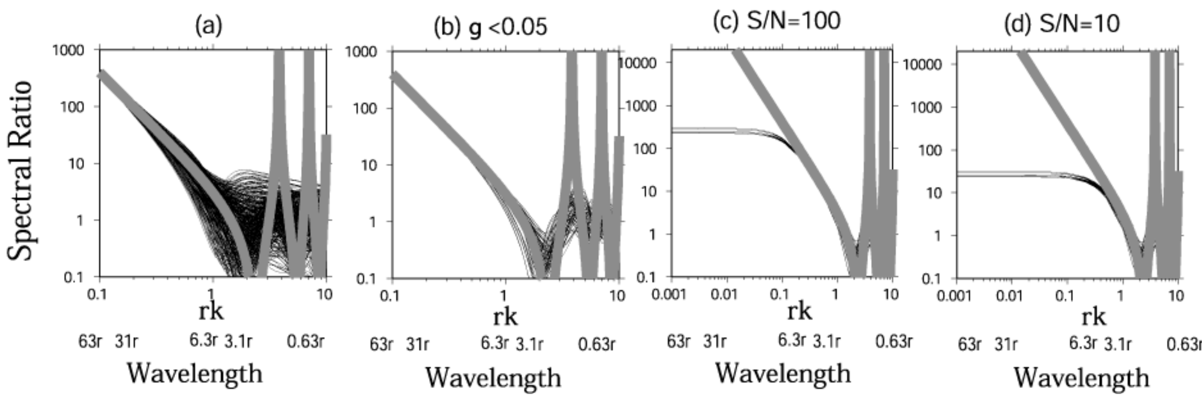

Figure 1 Effects of directional aliasing; three unevenly spaced sensors. Spectral ratios, estimated with three-sensor arrays of different azimuthal configurations, are simultaneously plotted and compared with the true values (thick and gray curve). Waves are assumed to arrive from a direction ϕ_0 of 30 deg with a spreading half-width ϕ_d of 60 deg. (a) all curves; (b) curves for the cases of γ <0.05 alone; (c), (d) same as in (b) except that incoherent noise was incorporated in the data. Note that the abscissa is scaled differently than in (a) and (b).

Figure 1(a) simultaneously plots the spectral ratios estimated with equation (39) for three-sensor arrays of different azimuthal configurations: we chose all possible combinations of three sensor positions out of all 18 points placed at an even interval of 20 deg along a circumference. Equation (45) was used for the shape of W_m; the central direction ϕ_0 of wave incidence was set at 30 deg; the spreading half-width ϕ_d was 60 deg. Figure 1(a) shows that biases coming from directional aliasing, including the effects of the assumption *K*=1 that has been necessitated by the use of just three sensors, are very serious in short wavelength ranges (large *rk*).

We define a parameter γ, an index of unevenness in inter-sensor spacings along the circumference, as a normalized variance of the azimuthal intervals between two adjacent sensors:

$$\gamma \equiv \sum_{i=1}^{3} (\Delta\theta_i - 120)^2 / 86400, \qquad (46)$$

where $\Delta\theta_i$ denotes the azimuthal aperture, measured in degrees, of the *i*-th inter-sensor spacing: $\Delta\theta_1$ + $\Delta\theta_2$ + $\Delta\theta_3$ = 360. The value of γ falls between 0 and 1 depending on the degree of unevenness in inter-sensor spacings; among all cases drawn in Figure 1(a), the maximum value of γ was 0.35.

Figure 1(b) has picked up, out of all curves drawn in Figure 1(a), only those corresponding to values of γ smaller than 0.05, or to cases where the three sensors were placed with minimal unevenness along the circumference. The good agreement between the estimated and true spectral ratio curves indicates that the degree of unevenness in inter-sensor spacings can be the principal factor that limits the efficacy of our method in short wavelength ranges.

The effects of $W_m^{(1)}$ of higher orders decrease with the increasing spreading half-width ϕ_d, and so do the effects directional aliasing. In realistic situations where microseisms arrive from a variety of directions (large ϕ_d), we can therefore presume that an array of three sensors is sufficiently practicable for our method of microseismic exploration, as long as those sensors are not placed too unevenly along the circumference.

Figures 1(c) and (d) show the equivalent of Figure 1(b) in the presence of noise, for frequency-independent signal-to-noise ratios (S/N) of 100 and 10. The spectral ratio saturates for small rk, or in the long wavelength ranges upward of roughly $40r$ and $10r$ for these cases. The deviation of the estimated spectral ratio from the theoretical curve also becomes conspicuous roughly from a wavelength of $4r$ (rk =1.5) downward, irrespective of the value of S/N.

CONCLUSIONS

We have developed a new method to determine phase velocities from the observation of the vertical component of microseisms with a circular array of sensors. According to the results of theoretical calculations conducted for the case where the wavenumber is a single-valued function of frequency, our method is expected to have the following properties: (1) An array of just three sensors is sufficiently practicable in realistic situations; (2) Uneven array configurations are possible, but, as a general argument, it is preferable to place the sensors as evenly as possible along the circumference in order to minimize the effects of directional aliasing; (3) The longest wavelength that is analyzable with our method primarily depends on the signal-to-noise ratio (S/N); (4) The performance in short wavelength ranges is controlled principally by the effects of directional aliasing, and therefore depends on the configuration of the seismic array, although, on the other hand, the requirement that there must be a one-on-one correspondence between our spectral ratio function and its argument sets another limit of analyzability in short wavelength ranges.

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