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SIMPLIFIED ESTIMATION METHOD FOR A SEISMIC RESPONSE CHARACTERISTIC USING STATIONARY RANDOM RESPONSE CHARACTERISTIC

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SUMMARY

Seismic response is nonstationary random process in nature. Mean square value of the response is one of the most important statistical values. Integral of mean square value of the response during seismic excitation is defined as seismic response strength. As seismic excitations, nonstationary artificial time histories are used. An approximate method for obtaining seismic response strength is proposed. Seismic response strength is obtained for various values of the damping ratio and natural period and some types of envelope functions. It is found that seismic response strength obtained form approximate method is equal to that obtained from exact method.

INTRODUCTION

Seismic response is nonstationary random process in nature. Mean square value is one of the most important statistical values of the response of the structure [1]. Mean square value is used to evaluate absorbed energy and cumulative damage of the structure [2]. Theoretical methods for obtaining mean square value of the response of the structure subjected to nonstationary seismic excitation are complicated and time consuming. Thus, approximate methods are proposed [3], [4].

In this paper, integral of mean square response during seismic excitation is defined as seismic response strength. As seismic excitations, nonstationary artificial time histories are used. Nonstationary artificial time histories are generated by multiplying stationary random processes by envelope functions. As an analytical model, a single-degree-of-freedom model is used.

An approximate method for obtaining seismic response strengths of displacement, velocity and acceleration response is proposed. In this method seismic response strength is obtained by multiplying mean square value of stationary response by square of envelope function. Mean square value of stationary response can be easily obtained. As exact method, seismic response strength is obtained from moment

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equations considering nonstationary seismic excitations. Seismic response strength is obtained for various values of the damping ratio and natural period and some types of envelope functions. It is found that seismic response strength obtained form approximate method is equal to that obtained from exact method.

Next, approximate method is applied to seismic response strength of secondary-primary system. When the secondary system is subjected to seismic excitation, the response of the secondary system depends on the dynamic characteristics of the primary system [5]. The response of the secondary system is greatly amplified when the natural period of the secondary system is nearly equal to that of the primary system [6]. For such a case, two-degree-of-freedom system in which the secondary system and the primary system are simulated by single-degree-of-freedom system respectively is used as an analytical model. Seismic response strength is obtained for various values of the damping ratio and the natural period of both systems, mass ratio of the secondary system to the primary system and some types of envelope functions. In this case, seismic response strength obtained form approximate method is also equal to that obtained from exact method.

It is concluded that the proposed approximate method gives exact value of seismic response strength and is simplified and practical method.

ANALYTICAL METHOD FOR SINGLE-DEGREE-OF-FREEDOM SYSTEM

Analytical model and input excitation

As an analytical model, a single-degree-of-freedom system as shown in Fig. 1 is used. The equation of motion with respect to relative displacement of mass to the ground z(x-y) is given as:

$$\ddot{z} + 2\zeta\omega_n\dot{z} + \omega_n{}^2 z = \ddot{y} \tag{1}$$

where $\zeta\left(c/2\sqrt{mk}\right)$ is the damping ratio and $\omega_n\left(\sqrt{k/m}\right)$ is the natural circular frequency. As input excitation $\ddot{y}(t)$, nonstationary white noise given by the following equation is used.

$$\ddot{y}(t) = I(t)s_y(t) \tag{2}$$

where I(t) is envelope function representing amplitude nonstationary characteristics and $s_y(t)$ is stationary white noise. In this study, envelope functions as shown in Fig. 2(a) (type A) and (b) (type B) are used. For Fig.2(a), I(t) is expressed as:

$$I(t) = \frac{e^{-at} - e^{-bt}}{\left|e^{-at} - e^{-bt}\right|_{max}} \tag{3}$$

where a=0.125 and b=0.25. For Fig.2(b), I(t) is expressed as:

$$I(t) = \begin{cases} t^2/16 & ; 0s \le t \le 4s \\ 1.0 & ; 4s \le t \le 15s \\ \exp\{-0.0924(t-15)\} & ; t > 15s \end{cases} \tag{4}$$

Nonstationary response analysis

The mean square response of z is given by the autocorrelation function as follows.

$$R_z(t_1, t_2) = \int_{-\infty}^{\infty} G(\omega, t_1)G^*(\omega, t_2)S_0 d\omega \tag{5}$$

where $G(\omega,t)$ corresponds to the response for nonstationary harmonic excitation and $G^*(\omega,t)$ is complex conjugate function of $G(\omega,t)$. $G(\omega,t)$ is give as;

$$G(\omega,t) = \int_0^t h(t-\xi)I(\xi)e^{i\omega t}d\xi \tag{6}$$

where

$$h(t) = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}\,\omega_n} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t\right) \tag{7}$$

S_0 is power spectral density of stationary white noise, $s_y(t)$ in Eq.(2). $i = \sqrt{-1}$. The autocorrelation function of relative velocity and the cross correlation function of z and $\dot{z}$ are given as:

$$R_{\dot{z}} = \frac{\partial^2 R_z(t_1, t_2)}{\partial t_1 \partial t_2} \tag{8}$$

$$R_{z\dot{z}} = \frac{\partial R_z(t_1, t_2)}{\partial t_2} \tag{9}$$

The mean square response of relative displacement $\sigma_z{}^2$, that of relative velocity $\sigma_{\dot{z}}{}^2$ and the covariance of displacement and velocity $\kappa_{z\dot{z}}$ are given as:

$$\sigma_z{}^2(t) = R_z(t,t) \tag{10}$$

$$\sigma_{\dot{z}}{}^2(t) = R_{\dot{z}}(t,t) \tag{11}$$

$$\kappa_{z\dot{z}}{}^2(t) = R_{z\dot{z}}(t,t) \tag{12}$$

Integral of Eq.(5) is complicated. $\sigma_{z_s}{}^2$, $\sigma_{\dot{z}}{}^2$ and $\kappa_{z\dot{z}}$ can also be obtained by moment equations as:

$$\begin{aligned}
\frac{d\sigma_z{}^2}{dt} &= 2\kappa_{z\dot{z}} \\
\frac{d\sigma_{\dot{z}}{}^2}{dt} &= -4\zeta\omega_n\sigma_{\dot{z}}{}^2 - 2\omega_n{}^2\kappa_{z\dot{z}} + 2\pi S_0\{I(t)\}^2 \\
\frac{d\kappa_{z\dot{z}}}{dt} &= \sigma_{\dot{z}_s}{}^2 - 2\zeta\omega_n\kappa_{z\dot{z}} - \omega_n{}^2\sigma_z{}^2
\end{aligned} \tag{13}$$

Stationary approximation

For simplicity, when I(t) is assumed to be independent of ω, Eq.(5) is expressed as:

$$\sigma_z{}^2(t) = \{I(t)\}^2 S_0 \int_{-\infty}^{\infty} \left|H_d(\omega)\right|^2 d\omega \tag{14}$$

$$\sigma_{\dot{z}}{}^2(t) = \{I(t)\}^2 S_0 \int_{-\infty}^{\infty} \left|H_v(\omega)\right|^2 d\omega \tag{15}$$

Equations (14) and (15) are given as:

$$\sigma_{z_s}{}^2(t) = \{I(t)\}^2 S_0 \frac{\pi}{2\zeta\omega_n{}^3} \tag{16}$$

$$\sigma_{\dot{z}_s}{}^2(t) = \{I(t)\}^2 S_0 \frac{\pi}{2\zeta\omega_n} \tag{17}$$

Equations (16) and (17) are simpler than Eq.(5) or Eq.(13). If approximation of Eqs.(14) and (15) are appropriate, nonstationary random vibration analysis becomes simple. In this paper, when mean square response is obtained by Eqs.(16) and (17), it is referred to as approximate value. When mean square response is obtained by Eq.(13), it is referred to as exact value.

Integral of mean square response with respect to time from 0 to infinity is expressed as:

$$I_z = \int_0^{\infty} \sigma_{z_s}{}^2(t)dt \tag{18}$$

In this paper, I_z is referred to as seismic response strength of relative displacement. Multiplying I_z by k/2 gives total potential energy. For stationary approximation using envelope function type A, I_z is given as:

$$I_z = \frac{(a-b)^2}{2ab(a+b)\left(\left. e^{-at} - e^{-bt} \right|_{max}\right)^2} \frac{\pi}{2\zeta\omega_n{}^3} S_0 \tag{19}$$

Seismic response strength of relative velocity is given as:

$$I_{\dot{z}} = \frac{(a-b)^2}{2ab(a+b)\left(\left. e^{-at} - e^{-bt} \right|_{max}\right)^2} \frac{\pi}{2\zeta\omega_n} S_0 \tag{20}$$

Multiplying $I_{\dot{z}}$ by c gives total dissipated energy by the damper. Mean square response of absolute acceleration of the system is given as:

$$\sigma_{\ddot{x}}{}^2 = 4\xi^2\omega_n{}^2\sigma_{\dot{z}}{}^2 + 4\xi_s\omega_n{}^3\kappa_{z\dot{z}} + \omega_n{}^4\sigma_z{}^2 \tag{21}$$

Then, seismic response strength of absolute acceleration is obtained as:

$$I_{\ddot{x}_s} = \int_0^\infty \sigma_{\ddot{x}_s}{}^2 dt \tag{22}$$

$I_{\ddot{x}}$ is related to total kinetic energy. For stationary approximation using envelope function type A, seismic response strength of absolute acceleration is written as:

$$I_{\ddot{x}} = \frac{(a-b)^2}{2ab(a+b)\left(\left. e^{-at} - e^{-bt} \right|_{max}\right)^2} \left(4\zeta^2\omega_n{}^2\sigma_{\dot{z}} + \omega_n{}^4\sigma_z\right) S_0$$

(23)

For stationary approximation using envelope function type B, seismic response strength of relative displacement is given as:

$$I_z = \frac{17.211\pi}{2\zeta\omega_n{}^3} S_0 \tag{24}$$

Seismic response strength of relative velocity is given as:

$$I_{\dot{z}} = \frac{17.211\pi}{2\zeta\omega_n} S_0 \tag{25}$$

Seismic response strength of absolute acceleration is written as:

$$I_{\ddot{x}} = 17.211\left(4\zeta^2\omega_n{}^2\sigma_{\dot{z}} + \omega_n{}^4\sigma_z\right) S_0 \tag{26}$$

ANALYTICAL METHOD FOR SECONDARY-PRIMARY SYSTEM

Analytical model

In order to examine the response of the secondary system such as pipings, tanks and other mechanical equipment is installed on the primary system such as building, two-degree-of- freedom system as shown in Fig. 3 is used. In this model, the secondary system and the primary system are simulated by single-degree-of-freedom system respectively. The upper system is the secondary system and the lower system is the primary system. The equations of motion with respect to relative displacement of the secondary system to the primary system $z_s(x_s - x_p)$ and relative displacement of the primary system to the ground $z_p(x_p - y)$ are given as:

$$\left.\begin{aligned} &\ddot{z}_s + 2\zeta_s\omega_s(1+\gamma)\dot{z}_s + \omega_s{}^2(1+\gamma)z_s - 2\zeta_p\omega_p\dot{z}_p - \omega_p{}^2 z_p = 0 \\ &\ddot{z}_p + 2\zeta_p\omega_p\dot{z}_p + \omega_p{}^2 z_p - 2\zeta_s\omega_s\gamma\dot{z}_s - \omega_s{}^2\gamma z_s = \ddot{y} \end{aligned}\right\} \tag{27}$$

where $\zeta_s\left(c_s/2\sqrt{m_s k_s}\right)$ and $\zeta_p\left(c_p/2\sqrt{m_p k_p}\right)$ are the damping ratio of the secondary system and that of the primary system, respectively. $\omega_s\left(\sqrt{k_s/m_s}\right)$ and $\omega_p\left(\sqrt{k_p/m_p}\right)$ are the natural circular frequency of the secondary system and that of the primary system, respectively. $\gamma\left(m_s/m_p\right)$ is mass ratio of the secondary system to the primary system. For two-degree-of-freedom system, response energy of the secondary system is focused on.

Nonstationary response analysis

The mean square response of z_s is given by the autocorrelation function as follows.

$$R_{z_s}(t_1,t_2)=\int_{-\infty}^{\infty} G(\omega,t_1)G^*(\omega,t_2)S_0 d\omega \tag{28}$$

where $G(\omega,t)$ is power spectral density function for nonstationary random process and $G^*(\omega,t)$ is complex conjugate function of $G(\omega,t)$. The mean square response of relative displacement of the secondary system $\sigma_{z_s}{}^2$ is given as:

$$\sigma_{z_s}{}^2(t)=R_{z_s}(t,t) \tag{29}$$

Integral of Eq.(28) is complicated. $\sigma_{z_s}{}^2$ can also be obtained by moment equations. Eq.(27) is written as:

$$\dot{\mathbf{z}}=\mathbf{Gz}+\mathbf{f} \tag{30}$$

where

$$\mathbf{z}^T=\{z_s \quad z_p \quad \dot{z}_s \quad \dot{z}_p\} \tag{31}$$

$$\mathbf{G}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_p{}^2 & \omega_s{}^2\gamma & -2\zeta_p\omega_p & 2\zeta_s\omega_s\gamma \\ \omega_p{}^2 & -\omega_s{}^2(1+\gamma) & 2\zeta_p\omega_p & -2\zeta_s\omega_s(1+\gamma) \end{bmatrix} \tag{32}$$

$$\mathbf{f}^T=\{0 \quad 0 \quad -\ddot{y} \quad 0\} \tag{33}$$

The second moments of the response are expressed as [7]:

$$\dot{\mathbf{V}}=\mathbf{GV}^T+\mathbf{VG}^T+\mathbf{D} \tag{34}$$

where

$$\mathbf{V}=\begin{bmatrix} \sigma_{z_p}{}^2 & \kappa_{z_p z_s} & \kappa_{z_p \dot{z}_s} & \kappa_{z_p \dot{z}_s} \\ \kappa_{z_p z_s} & \sigma_{z_s}{}^2 & \kappa_{z_s \dot{z}_p} & \kappa_{z_s \dot{z}_s} \\ \kappa_{z_p \dot{z}_s} & \kappa_{z_s \dot{z}_p} & \sigma_{\dot{z}_p}{}^2 & \kappa_{\dot{z}_p \dot{z}_s} \\ \kappa_{z_p \dot{z}_s} & \kappa_{z_s \dot{z}_s} & \kappa_{\dot{z}_p \dot{z}_s} & \sigma_{\dot{z}_s}{}^2 \end{bmatrix} \tag{35}$$

$$\mathbf{D}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2\pi S_0\{I(t)\}^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{36}$$

κ is covariance.

The second moments of the response are obtained from moment equations as follows.

$$
\begin{aligned}
&\frac{d\sigma_{z_p}{}^2}{dt} = 2\kappa_{z_p\dot{z}_p}\\
&\frac{d\kappa_{z_pz_s}}{dt} = \kappa_{z_s\dot{z}_p} + \kappa_{z_p\dot{z}_s}\\
&\frac{d\kappa_{z_p\dot{z}_p}}{dt} = \sigma_{\dot{z}_p}{}^2 - \omega_p{}^2\sigma_{z_p} + \omega_s{}^2\gamma\kappa_{z_pz_s} - 2\zeta_p\omega_p\kappa_{z_p\dot{z}_p} + 2\zeta_s\omega_s\gamma\kappa_{z_p\dot{z}_s}\\
&\frac{d\kappa_{z_p\dot{z}_s}}{dt} = \kappa_{\dot{z}_p\dot{z}_s} + \omega_p{}^2\sigma_{z_p}{}^2 - \omega_s{}^2(1+\gamma)\kappa_{z_pz_s} + 2\zeta_p\omega_p\kappa_{z_p\dot{z}_p} - 2\zeta_s\omega_s(1+\gamma)\kappa_{z_p\dot{z}_s}\\
&\frac{d\sigma_{z_s}{}^2}{dt} = 2\kappa_{z_s\dot{z}_s}\\
&\frac{d\kappa_{z_s\dot{z}_p}}{dt} = \kappa_{\dot{z}_p\dot{z}_s} - \omega_p{}^2\kappa_{z_pz_s} + \omega_s{}^2\gamma\sigma_{z_s}{}^2 - 2\zeta_p\omega_p\kappa_{z_s\dot{z}_p} + 2\zeta_s\omega_s\gamma\kappa_{z_s\dot{z}_s}\\
&\frac{d\kappa_{z_s\dot{z}_s}}{dt} = \sigma_{\dot{z}_s}{}^2 + \omega_p{}^2\kappa_{z_pz_s} - \omega_s{}^2(1+\gamma)\sigma_{z_s}{}^2 + 2\zeta_p\omega_p\kappa_{z_s\dot{z}_p} - 2\zeta_s\omega_s(1+\gamma)\kappa_{z_s\dot{z}_s}\\
&\frac{d\sigma_{\dot{z}_p}{}^2}{dt} = 2\left(-\omega_p{}^2\kappa_{z_p\dot{z}_p} + \omega_s{}^2\gamma\kappa_{z_s\dot{z}_p} - 2\zeta_p\omega_p\sigma_{\dot{z}_p}{}^2 + 2\zeta_s\omega_s\gamma\kappa_{\dot{z}_p\dot{z}_s}\right) + 2\pi S_0\{I(t)\}^2\\
&\frac{d\kappa_{\dot{z}_p\dot{z}_s}}{dt} = -\omega_p{}^2\kappa_{z_p\dot{z}_s} + \omega_s{}^2\gamma\kappa_{z_s\dot{z}_s} - 2\zeta_p\omega_p\kappa_{\dot{z}_p\dot{z}_s} + 2\zeta_s\omega_s\gamma\sigma_{\dot{z}_s}{}^2 + \omega_p{}^2\kappa_{z_p\dot{z}_p} - \omega_s{}^2(1+\gamma)\kappa_{z_s\dot{z}_p}\\
&\qquad + 2\zeta_p\omega_p\sigma_{\dot{z}_p} - 2\zeta_s\omega_s(1+\gamma)\kappa_{\dot{z}_p\dot{z}_s}\\
&\frac{d\sigma_{\dot{z}_s}{}^2}{dt} = 2\left\{\omega_p{}^2\kappa_{z_p\dot{z}_s} - \omega_s{}^2(1+\gamma)\kappa_{z_s\dot{z}_s} + 2\zeta_p\omega_p\kappa_{\dot{z}_p\dot{z}_s} - 2\zeta_s\omega_s\sigma_{\dot{z}_s}{}^2\right\}
\end{aligned}
\tag{37}
$$

Stationary approximation

For simplicity, when I(t) is assumed to be independent of ω, Eq.(28) is expressed as:

$$\sigma_{z_s}{}^2(t) = \{I(t)\}^2 S_0 \int_{-\infty}^{\infty} \left|H_s(\omega)\right|^2 d\omega \tag{38}$$

where $H_s(\omega)$ is the frequency response function of the secondary system to input excitation. In this case, $H_s(\omega)$ is expressed as:

$$H_s(\omega) = \frac{R_n(\omega)}{R_d(\omega)} \tag{39}$$

$$R_n(\omega) = -\left(2\zeta_p\omega_p\omega i + \omega_p{}^2\right) \tag{40}$$

$$
\begin{aligned}
R_d(\omega) = {}& \omega^4 - \{2\zeta_p\omega_p + 2\zeta_s\omega_s(1+\gamma)\}\omega^3 i - \{\omega_p{}^2 + \omega_s{}^2(1+\gamma) + 4\zeta_p\zeta_s\omega_p\omega_s\}\omega^2\\
& + \left(2\zeta_p\omega_p\omega_s{}^2 + 2\zeta_s\omega_s\omega_p{}^2\right)\omega i + \omega_p{}^2\omega_s{}^2
\end{aligned}
\tag{41}
$$

The integral of Eq.(38) is that for stationary random process. This integral is obtained as:

$$
I_4 = \frac{\pi}{\lambda_4} \frac{\begin{vmatrix} \xi_3 & \xi_2 & \xi_1 & \xi_0 \\ -\lambda_4 & \lambda_2 & -\lambda_0 & 0 \\ 0 & -\lambda_3 & \lambda_1 & 0 \\ 0 & \lambda_4 & -\lambda_2 & \lambda_0 \end{vmatrix}}{\begin{vmatrix} \lambda_3 & -\lambda_1 & 0 & 0 \\ -\lambda_4 & \lambda_2 & -\lambda_0 & 0 \\ 0 & -\lambda_3 & \lambda_1 & 0 \\ 0 & \lambda_4 & -\lambda_2 & \lambda_0 \end{vmatrix}} \tag{42}
$$

where

$\lambda_4 = 1$, $\lambda_3 = 2\varsigma_p\omega_p + 2\varsigma_s\omega_s(1+\gamma)$, $\lambda_2 = \omega_p{}^2 + \omega_s{}^2(1+\gamma) + 4\zeta_p\zeta_s\omega_p\omega_s$,

$\lambda_1 = 2\zeta_p\omega_p\omega_s{}^2 + 2\zeta_s\omega_s\omega_p{}^2$, $\lambda_0 = \omega_p{}^2\omega_s{}^2$, $\xi_3 = 0$, $\xi_2 = 0$, $\xi_1 = (2\varsigma_p\omega_p)^2$, $\xi_0 = \omega_p{}^4$

and

$$
\sigma_{z_s}{}^2(t) = \{I(t)\}^2 I_4 S_0 \tag{43}
$$

The results using Eq.(42) are obtained when left-hand side of Eq.(37) is equal to 0. Thus, Eq.(38) is simpler than Eq.(28) or Eq.(37). If approximation of Eq.(38) is appropriate, nonstationary random vibration analysis becomes simple. In this paper, when mean square response is obtained by Eq.(38), it is referred to as approximate value. When mean square response is obtained by Eq.(37), it is referred to as exact value.

In this case, seismic response strength of z_z is expressed as:

$$
I_{z_s} = \int_0^\infty \sigma_{z_s}{}^2(t)dt \tag{44}
$$

For stationary approximation using envelope function type A, I_{z_s} is given as:

$$
I_{z_s} = \frac{(a-b)^2}{2ab(a+b)\left(\left. e^{-at} - e^{-bt} \right|_{max}\right)^2} I_4 S_0 \tag{45}
$$

Seismic response strength of relative velocity of the secondary system is given as:

$$
I_{\dot{z}_s} = \int_0^\infty \sigma_{\dot{z}_s}{}^2 dt \tag{46}
$$

$I_{\dot{z}_s}$ corresponding to Eq.(45) is obtained by substituting $\xi_3 = 0, \xi_2 = (2\zeta_p\omega_p)^2$ $, \xi_1 = \omega_p{}^4, \xi_0 = 0$ into Eq.(42). Mean square response of absolute acceleration of the secondary system is given as:

$$
\sigma_{\ddot{x}_s}{}^2 = 4\zeta_s{}^2\omega_s{}^2\sigma_{\dot{z}_s}{}^2 + 4\zeta_s\omega_s{}^3\kappa_{z_s\dot{z}_s} + \omega_s{}^4\sigma_{z_s}{}^2 \tag{47}
$$

Thus, seismic response strength of absolute acceleration is obtained as:

$$
I_{\ddot{x}_s} = \int_0^\infty \sigma_{\ddot{x}_s}{}^2 dt \tag{48}
$$

For stationary approximation using envelope function Type A, seismic response strength of absolute acceleration is written as:

$$
I_{\ddot{x}_s} = \frac{(a-b)^2}{2ab(a+b)\left(\left. e^{-at} - e^{-bt} \right|_{max}\right)^2} \left(4\zeta_s{}^2\omega_s{}^2\sigma_{\dot{z}_s} + \omega_s{}^4\sigma_{z_s}\right)S_0 \tag{49}
$$

For stationary approximation using envelope function type B, seismic response strength of relative displacement of the secondary system is given as:

$$I_{z_s} = 17.211 I_4 S_0 \tag{50}$$

Seismic response strength of relative velocity of the secondary system is given as:

$$I_{\dot{z}_s} = \int_0^\infty \sigma_{\dot{z}_s}{}^2 dt \tag{51}$$

Seismic response strength of absolute acceleration of the secondary system is written as:

$$I_{\ddot{x}_s} = 17.211\left(4\zeta_s{}^2\omega_s{}^2\sigma_{\dot{z}_s} + \omega_s{}^4\sigma_{z_s}\right)S_0 \tag{52}$$

NUMERICAL EXAMPLES

Single-degree-of-freedom system

From Table 1 to Table 4, results of integral of mean square response (seismic response strength) of the system are shown. Values of parameters are selected referring to actual systems. S_0=1.0m^2/s^4/rad/s is assumed. Table 1 and Table 2 show seismic response strength for some values of the damping ratio using envelope function type A and type B, respectively. Stationary approximation gives exact values of seismic response strength. Table 3 and Table 4 show seismic response strength for some values of the natural period. In this case, stationary approximation also gives exact values of response energy.

It is concluded that stationary approximation gives exact value of seismic response strength for some values of the damping ratio and the natural period and some types of envelope functions. Seismic response strength of nonstationary response can be obtained easily by stationary approximation.

Secondary-primary system

From Table 5 to Table 12, results of seismic response strength of the secondary system are shown. Table 5 and Table 6 show seismic response strength for some values of the damping ratio of the secondary system using envelope function type A and type B, respectively. Table 7 and Table 8 show seismic response strength for some values of the ratio of mass of the secondary system to the primary system. Table 9 and Table 10 show seismic response strength for some values of the natural period. In these tables, stationary approximation gives exact values of seismic response strength of the secondary system for displacement and acceleration response. For velocity response, stationary application gives smaller value than exact value. Difference is considered to be phase lag angle of velocity response. However, difference between stationary approximation and exact value is small (less than 2%) from practical point of view. From Table 5 to Table 10, results of seismic response strength for the case where the natural period of the secondary system coincides with that of the primary system are obtained. In Table 11 and Table 12, results of seismic response strength for general case where the natural period of the secondary system does not coincide with that of the primary system are shown. In this case, stationary approximation also gives exact values of seismic response strength except for velocity response. However, for velocity response, difference between stationary approximation and exact value is also small.

It is concluded that stationary approximation gives exact value of seismic response strength for some values of the damping ratio, the natural period and mass ratio and some types of envelope functions. Response energy of nonstationary response can be obtained easily by stationary approximation.

CONCLUSIONS

An approximate estimation method for seismic response strength of the system, which is integral of mean square response, is proposed. In this method, statistical properties of stationary response are used. Some numerical results are shown. It is found that the approximate method gives exact values of seismic response strength independent of parameters such as the damping ratio, the natural period and mass ratio and some types of envelope functions.

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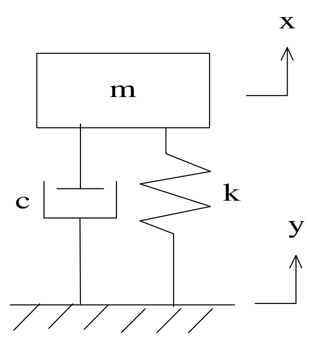
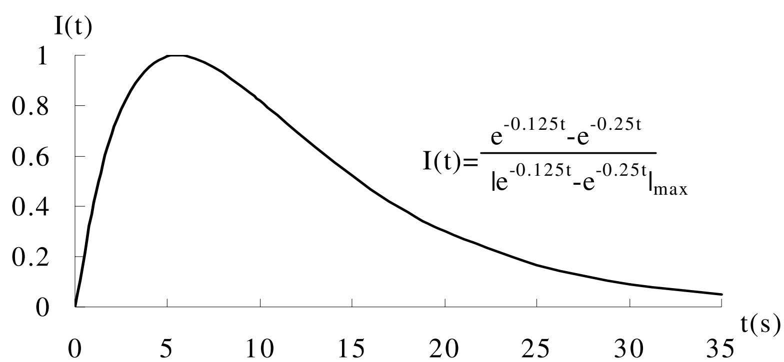
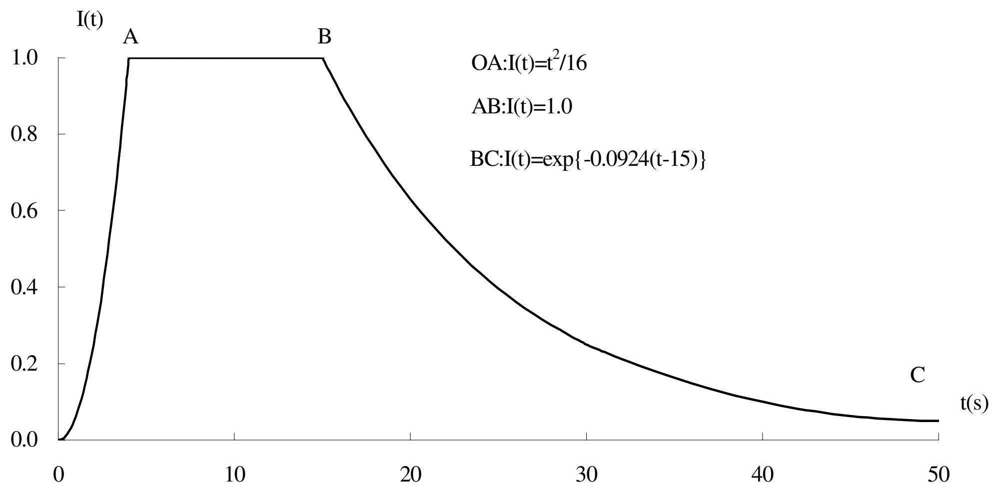


Fig.1 Single-degree-of-freedom system



(a) Type A



(b) Type B

Fig.2 Envelope function

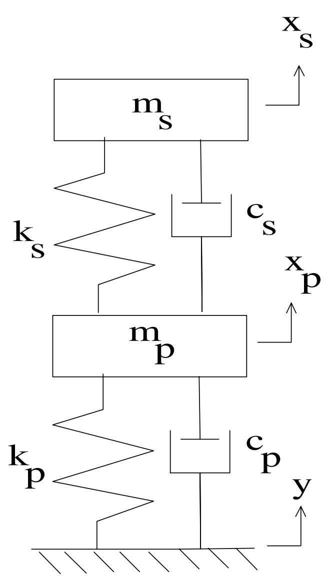


Fig.3 Secondary-primary system

Table 1 Seismic response strength of single-degree-of-freedom system (T_n=1.0s, type A)

ζ_s	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.01	6.76	6.76	2.67×10^2	2.67×10^2	1.05×10^4	1.05×10^4
0.02	3.38	3.38	1.33×10^2	1.33×10^2	5.27×10^3	5.27×10^3
0.05	1.35	1.35	5.33x10	5.33x10	2.13×10^3	2.13×10^3

Table 2 Seismic response strength of single-degree-of-freedom system (T_n=1.0s, type B)

ζ_s	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.01	1.09x10	1.09x10	4.30x10^2	4.30x10^2	1.70x10^4	1.70x10^4
0.02	5.45	5.45	2.15x10^2	2.15x10^2	8.51x10^3	8.51x10^3
0.05	2.18	2.18	8.61x10	8.61x10	3.43x10^3	3.43x10^3

Table 3 Seismic response energy of single-degree-of-freedom system (ζ=0.01, type A)

T_s=T_p(s)	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.2	5.40x10^{-2}	5.40x10^{-2}	5.33x10	5.33x10	5.27x10^4	5.27x10^4
0.5	8.44x10^{-1}	8.44x10^{-1}	1.33x10^2	1.33x10^2	2.11x10^4	2.11x10^4
0.8	3.46	3.46	2.13x10^2	2.13x10^2	1.32x10^4	1.32x10^4
1.0	6.76	6.76	2.67x10^2	2.67x10^2	1.05x10^4	1.05x10^4

Table 4 Seismic response energy of single-degree-of-freedom system (ζ=0.01, type B)

T_s=T_p(s)	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.2	8.71x10^{-2}	8.71x10^{-2}	8.61x10	8.61x10	8.50x10^4	8.50x10^4
0.5	1.36	1.36	2.15x10^2	2.15x10^2	3.40x10^4	3.40x10^4
0.8	5.58	5.58	3.44x10^2	3.44x10^2	2.12x10^4	2.12x10^4
1.0	1.09x10	1.09x10	4.30x10^2	4.30x10^2	1.70x10^4	1.70x10^4

Table 5 Seismic response strength of secondary system (γ=0, ζ_p=0.05, T_s=T_p=1.0s, type A)

ζ_s	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.01	5.70x10^2	5.70x10^2	2.24x10^4	2.20x10^4	8.88x10^5	8.88x10^5
0.02	2.45x10^2	2.45x10^2	9.62x10^3	9.43x10^3	3.82x10^5	3.82x10^5
0.05	6.89x10	6.89x10	2.69x10^3	2.64x10^3	1.08x10^5	1.08x10^5

Table 6 Seismic response strength of secondary system (γ=0, ζ_p=0.05, T_s=T_p=1.0s, type B)

ζ_s	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.01	9.19x10^2	9.19x10^2	3.62x10^4	3.55x10^4	1.43x10^6	1.43x10^6
0.02	3.95x10^2	3.95x10^2	1.55x10^4	1.52x10^4	6.16x10^5	3.82x10^5
0.05	1.11x10^2	1.11x10^2	4.35x10^3	4.26x10^3	1.75x10^5	1.75x10^5

Table 7 Seismic response strength of secondary system (ζ_s=0.01, ζ_p=0.05, T_s=T_p=1.0s, type A)

γ	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0	5.70x10^2	5.70x10^2	2.24x10^4	2.20x10^4	8.88x10^5	8.88x10^5
0.01	9.61x10	9.61x10	3.74x10^3	3.67x10^3	1.50x10^5	1.50x10^5
0.02	5.30x10	5.30x10	2.04x10^3	2.00x10^3	8.26x10^4	8.26x10^4
0.05	2.32x10	2.32x10	8.64x10^2	8.47x10^2	3.62x10^4	3.62x10^4

Table 8 Seismic response strength of secondary system (ζ_s=0.01, ζ_p=0.05, T_s=T_p=1.0s, type B)

γ	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0	9.19×10^2	9.19×10^2	3.62×10^4	3.55×10^4	1.43×10^6	1.43×10^6
0.01	1.55×10^2	1.55×10^2	6.04×10^3	5.92×10^3	2.42×10^5	2.42×10^5
0.02	8.55x10	8.55x10	3.30×10^3	3.23×10^3	1.33×10^5	1.33×10^5
0.05	3.74x10	3.74x10	1.39×10^3	1.37×10^3	5.84×10^4	5.84×10^4

Table 9 Seismic response strength of secondary system (γ=0, ζ_s=0.01, ζ_p=0.05, type A)

T_s=T_p(s)	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.2	4.56	4.56	4.49×10^3	4.40×10^3	4.44×10^6	4.44×10^6
0.5	7.12x10	7.12x10	1.12×10^4	1.10×10^4	1.78×10^6	1.78×10^6
0.8	2.92×10^2	2.92×10^2	1.80×10^4	1.76×10^4	1.11×10^6	1.11×10^6
1.0	5.70×10^2	5.70×10^2	2.24×10^4	2.20×10^4	8.88×10^5	8.88×10^5

Table 10 Seismic response strength of secondary system (γ=0, ζ_s=0.01, ζ_p=0.05, type B)

T_s=T_p(s)	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.2	7.35	7.35	7.24×10^3	7.10×10^3	7.17×10^6	7.17×10^6
0.5	1.14×10^2	1.14×10^2	1.81×10^4	1.77×10^4	2.87×10^6	2.87×10^6
0.8	4.71×10^2	4.71×10^2	2.90×10^4	2.84×10^4	1.79×10^6	1.79×10^6
1.0	9.19×10^2	9.19×10^2	3.62×10^4	3.55×10^4	1.43×10^6	1.43×10^6

Table 11 Seismic response strength of secondary system (γ=0, ζ_s=0.01, ζ_p=0.05, T_p=1.0s, type A)

T_s(s)	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.5	2.48×10^{-1}	2.48×10^{-1}	2.13x10	2.00x10	6.19×10^3	6.19×10^3
0.8	1.44x10	1.44x10	7.98×10^2	7.75×10^2	5.49×10^4	5.49×10^4
1.0	5.70×10^2	5.70×10^2	2.24×10^4	2.20×10^4	8.88×10^5	8.88×10^5
1.5	7.71x10	7.71x10	1.45×10^3	1.43×10^3	2.37×10^4	2.37×10^4

Table 12 Seismic response strength of secondary system (γ=0, ζ_s=0.01, ζ_p=0.05, T_p=1.0s, type B)

T_s(s)	Displacement(m^2 s)		Velocity(m^2/s^2 s)		Acceleration(m^2/s^4 s)	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
0.5	4.00×10^{-1}	4.00×10^{-1}	3.43x10	3.22x10	9.98×10^3	9.98×10^3
0.8	2.33x10	2.33x10	1.29×10^3	1.25×10^3	8.85×10^4	8.85×10^4
1.0	9.19×10^2	9.19×10^2	3.62×10^4	3.55×10^4	1.43×10^6	1.43×10^6
1.5	1.24×10^2	1.24×10^2	2.33×10^3	2.31×10^3	3.83×10^4	3.83×10^4