

# DEVELOPMENT OF A SOFTWARE FRAMEWORK FOR MUST-SIM: AN INTEGRATED COMPUTATIONAL AND EXPERIMENTAL SIMULATION FACILITY

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## SUMMARY

A unique advantage of the multi-axial full scale sub-structured testing and simulation (MUST-SIM) facility is the ability to simulate and test an entire soil-foundation-structure system at full scale. This is accomplished through the development of a software framework that integrates computational and experimental simulations (ICES framework). The ICES framework seamlessly combines distributed networked simulations of numerical and pseudo-dynamic experimental components of the overall system. The software framework described in the paper is modular and uses object oriented programming concepts. Examples of application are presented in terms of a two-bay steel frame and two multi-span reinforced concrete bridges, one of which includes soil-structure-foundation interaction.

## INTRODUCTION

Testing of full-scale structures is the most reliable method for assessment of seismic performance of structures. However, physical testing of such structures including soil-foundation-structure interaction (SFSI) remains extremely space- and equipment-intensive as well as costly.. As part of the George E. Brown Jr. Network for Earthquake Engineering Simulation (NEES), the Multi-axial full scale sub-structured testing and simulation (MUST-SIM) facility is currently under construction at the University of Illinois at Urbana-Champaign. The MUST-SIM facility will provide an integrated testing-analysis-simulation-visualization environment, Figure 1. The physical facility consists of three 6-DOF modular Loading and Boundary Condition Box (LBCB), an L-shaped reaction wall, multiple dense arrays of non-contact measurement devices and advanced visualization for integrated tele-operation. Further information on MUST-SIM facility can be found in a companion paper on the MUST-SIM facility, Elnashai et al. [1].

A unique feature of the MUST-SIM facility is the integrated computational and experimental simulation (ICES) software framework being developed in conjunction with the physical facility. The ICES framework allows the seamless integration of multiple physical and numerical simulations of structural

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and geotechnical components within a unified simulation of a full-scale system such as a bridge or a building with SFSI effects.



Figure 1 MUST-SIM NEES Facility

The ICES framework combines physical testing of selected components with accurate numerical simulations of other components to obtain the response of the overall structural system that may include soil, foundation, and structure alongside their interaction effects. The overall structure of interest such as a bridge or a building is represented virtually in a simulation controller referred to as ICES control module. Selected components are simulated either numerically or experimentally in separate modules; computational, structural or geotechnical experimental module. The ICES framework can utilize multiple experimental facilities throughout the NEES system and represents an important step towards realizing the major objective of NEES. The ICES software framework is general and extensible. The current implementation uses a pseudo-dynamic testing procedure. In the future, given the appropriate network bandwidth and computational speed, fast rate hybrid test as well as real time dynamic tests can be incorporated. A future development of the software framework will include intelligent on-line updating of material constitutive response in the numerical simulation based on physical component testing results, Ghaboussi et al. [2].

#### ICES FRAMEWORK FOR PSEUDO-DYNAMIC TESTING

In its simplest application, the ICES framework is capable of performing pseudo-dynamic (PSD) testing of a selected structure component. However, as the framework is capable of virtually representing an entire structural system, it is used to pseudo-dynamically test more than one component at the MUST-SIM facility or a combination of several NEES facilities. Currently, the ICES controller incorporates two numerical time integration algorithms;  $\alpha$ -Operator Splitting method, Nakashima et al. [3] and new Predictor-Corrector method, Ghaboussi et al. [4]. A schematic of the ICES software framework is illustrated in Figure 2 and is applied to sub-structured PSD test considering soil-foundation-structure interaction problems.

The current ICES framework consists of the following components:

- a. ICES Control Module: The module plays a key role in setting up the overall structure to be simulated and initializing all other modules. It identifies the parts of the structure that will be simulated in physical experiments and in numerical simulations. The controller also acts as a coordinator between computational, structural experimental and geotechnical experimental modules. It controls overall testing and simulation by providing scenario of communication between modules. ICES control module can control more than one component of either computational or experimental module at MUST-SIM facility or several NEES sites
- b. Experimental Module(s): This module can be any physical experimental site where component of the overall structure is tested. The MUST-SIM facility as well as other NEES and non-NEES sites can be an experimental module.
- c. Computational Module(s): Portions of the structure that are not tested in an experimental simulation are simulated in a computational module. The ICES framework is designed to incorporate any number of available simulation codes. The current implementation includes ABAQUS, ZEUS-NL [5] and MATLAB. Other codes including OPENSEES will be implemented in the near future.

The ICES framework uses communication protocols that are compatible with the NEES Point-of-Presence (NEESpop) system to perform the distributed numerical and experimental simulation. MUST-SIM facility will be run as a NEESgrid infrastructure system which includes NEESpop, DAQ, NSDS and CHEF. The NEESgrid system is under continuous development as part of NEES System Integration project. The ICES framework will eventually be used through CHEF collaborative environment and connected to the data repository using standardized NEESML.

In the future, the ICES controller capabilities will be extended so that measurements from experiments can be used for updating material constitutive relationships on-line.



Figure 2 Integrated computational and experimental simulation framework

## SUB-STRUCTURED PSEUDO-DYNAMIC TEST WITH ICES CONTROLLER

PSD test technique is a hybrid computational and experimental method that has been developed ([6], [7]) whereby the inertia and damping forces in the equations of motion are numerically modeled and static forces are replaced with restoring forces obtained from experiments. In each time step, the equation of motion is solved using a numerical time integration method while updating the restoring forces. This testing and simulation procedure is repetitive throughout whole time step. In particular, sub-structured PSD test is conducted by sub-dividing the overall system into computational and experimental parts. The

computational parts which shows linear elastic response are numerically simulated and experimental part which undergo severe inelastic behavior are tested quasi-statically in the laboratory.

Due to several constraints in PSD testing such as time lag effect and error propagation, time integration method for PSD test is required to be non-iterative and accurate. For instance, time lag in computational module due to iteration can lead to unfavorable effect on overall behavior of structural systems. Inaccuracy of time integration method can lead to significant distortion in the responses of structural system. ICES framework currently employs two numerical time integration algorithms that are appropriate for PSD test;  $\alpha$ -Operator Splitting method ( $\alpha$ -OSM), Nakashima [3] and new Predictor-Corrector method (P-CM), Ghaboussi et al [4].

#### $\alpha$ -Operator Splitting method

The details on  $\alpha$ -OS method can be found in reference, Nakashima [3]. In  $\alpha$ -OS method, the amount of adjustment of restoring forces is calculated at predictor stage of displacement and velocity to avoid iterative procedure. The main advantages of this method are that it requires no iteration and ensures unconditionally stability if the nonlinearity is of softening type, Nakashima et al. [3]. However, initial stiffness values are required for calculating restoring force adjustment before beginning the test.  $\alpha$ -OS method is based on approximation of restoring forces as shown in the following equations.

$$\mathbf{r}_{n+1}(\mathbf{x}_{n+1}) = \mathbf{K}^{\mathrm{L}}\mathbf{x}_{n+1} + \tilde{\mathbf{r}}_{n+1}(\tilde{\mathbf{x}}_{n+1}) - \mathbf{K}^{\mathrm{L}}\tilde{\mathbf{x}}_{n+1}$$
(1)

The equation of motion and difference equations are expressed as follows.

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{x}}_{n+1} + (1+\alpha)\mathbf{C}\dot{\mathbf{x}}_{n+1} - \alpha\mathbf{C}\dot{\mathbf{x}}_{n} + (1+\alpha)\mathbf{K}^{\mathrm{L}}\mathbf{x}_{n+1} - \alpha\mathbf{K}^{\mathrm{L}}\mathbf{x}_{n} \\ + (1+\alpha)\tilde{\mathbf{r}}_{n+1}\left(\tilde{\mathbf{x}}_{n+1}\right) - \alpha\tilde{\mathbf{r}}_{n}\left(\tilde{\mathbf{x}}_{n}\right) - (1+\alpha)\mathbf{K}^{\mathrm{L}}\tilde{\mathbf{x}}_{n+1} + \alpha\mathbf{K}^{\mathrm{L}}\tilde{\mathbf{x}}_{n} = (1+\alpha)F_{n+1} - \alpha F_{n+1} \end{aligned}$$
(2)

$$\tilde{\mathbf{x}}_{n+1} = \mathbf{x}_n + \Delta t \dot{\mathbf{x}}_n + \Delta t^2 \left(\frac{1}{2} - \beta\right) \ddot{\mathbf{x}}_n \tag{3}$$

$$\mathbf{x}_{n+1} = \tilde{\mathbf{x}}_{n+1} + \Delta t^2 \beta \ddot{\mathbf{x}}_{n+1} \tag{4}$$

$$\dot{\mathbf{x}}_{n+1} = \dot{\mathbf{x}}_n + \Delta t \Big[ (1 - \gamma) \ddot{\mathbf{x}}_n + \gamma \ddot{\mathbf{x}}_{n+1} \Big]$$
(5)

$$\beta = (1 - \alpha)^2 / 4 \quad \gamma = (1 - 2\alpha) / 2 \tag{6}$$

In the above equations M, C,  $K^L$  are mass, viscous damping and linear stiffness matrices and  $\tilde{x}_{n+1}$  and  $x_{n+1}$  are predicted displacement and corrected displacement vector at time step n+1, respectively.  $\dot{x}_{n+1}$  and  $\ddot{x}_{n+1}$  are velocity and acceleration vector. In the ICES framework, restoring forces are calculated in computational module using linear elastic stiffness obtained through preliminary test and treated as external loading.

#### New Predictor-Corrector method

For sub-structured PSD test of broad applicability, new P-C method is incorporated into ICES controller. It is based on implicit-explicit time integration method. At each time step, restoring forces are measured based on targeted displacements calculated explicitly and they replace static force terms in equation of motion. The unique advantages of new P-C method are that it requires no iteration procedure for nonlinear problem and no initial stiffness evaluation for use in the numerical procedure, which allows broader applications to complex structural and geotechnical system. This is accomplished by considering interaction effect between computational model and experimental model as a nonlinear external loading. The equation of motion and difference equations for multi DOF system are expressed as follows.

$$M\ddot{x}_{n+1} + C\dot{x}_{n+1} + K^{L}x_{n+1} + K^{NL}\tilde{x}_{n+1}^{b} = F_{n+1}$$
(7)

$$\mathbf{x}_{n+1} = \mathbf{x}_{n} + \Delta t \dot{\mathbf{x}}_{n} + \Delta t^{2} \left[ \left( \frac{1}{2} - \beta \right) \ddot{\mathbf{x}}_{n} + \beta \ddot{\mathbf{x}}_{n+1} \right]$$
(8)

$$\dot{\mathbf{x}}_{n+1} = \dot{\mathbf{x}}_n + \Delta t \Big[ (1 - \gamma) \ddot{\mathbf{x}}_n + \gamma \ddot{\mathbf{x}}_{n+1} \Big]$$
(9)

$$\tilde{\mathbf{x}}_{n+1}^{b} = \mathbf{x}_{n}^{b} + \Delta t \dot{\mathbf{x}}_{n}^{b} + \Delta t^{2} \left(\frac{1}{2} - \beta\right) \ddot{\mathbf{x}}_{n}^{b}$$
(10)

$$\beta = 1/4 \quad \gamma = 1/2 \tag{11}$$

In the above equations M, C,  $K^L$ ,  $K^{NL}$  are mass, viscous damping linear stiffness and tangent stiffness matrices respectively.  $\tilde{x}_{n+1}^{b}$  is predictor displacement vector at boundary between computational and experimental model. In this method, the stiffness matrix is divided into linear and tangent stiffness,  $K^L$  and  $K^{NL}$  respectively. The detail of stability analysis and accuracy in terms of period distortion and numerical damping can be found in the reference, Ghaboussi et al [4]. Table 1 compares  $\alpha$ -OS method and new P-C method. In this table, Newmark method means average acceleration method ( $\alpha$ =0,  $\beta$ =1/4,  $\gamma$ =1/2).

	α-Operator Splitting method	New Predictor-Corrector method
Stability	Unconditionally stable(softening type) Conditionally stable(hardening type)	$\label{eq:crit} \begin{array}{l} \mbox{Conditionally stable} \\ \Omega_{\mbox{crit}} < 2 \sqrt{1/(1-\chi)}, \ \chi = k^{\rm L}  /  k_{\rm T} \end{array}$
Period distortion	Larger than Newmark method	Less than Newmark method
Numerical damping	Amplitude change*	No amplitude change
Initial stiffness	Needed	Not needed
Iteration	No iteration	No iteration
		*nonlinear case

Table 1 Comparison of time integration algorithms

The main advantages of these algorithms are that they do not need iterative procedure and provide broad applications to complex structural systems. In particular, the new Predictor-Corrector method is more versatile than any other numerical algorithm since it does not need initial stiffness value for tested part and shows no numerical damping and less period distortion.

## **IMPLEMENTATION OF ICES FRAMEWORK**

In order to render the software framework development flexible and efficient, object-oriented programming concepts were used for development of the ICES framework. For example, ICES control module can be extended to include additional integration algorithms. Additional computational modules can be added to provide higher fidelity simulations. Any number of experimental facilities employing NEESpop can be incorporated within a simulation. The ICES controller was developed using C++. Its object-oriented structure allows easier code maintenance and also reduces the possibility of errors during upgrades.

#### **ICES control module**

The overall configurations of testing and simulation are determined in this module through intelligent script. For instance, structural system is subdivided into as many computational parts and experimental parts as users want and then each component is represented by a group of boundary nodes in intelligent script. During simulation, ICES control module prepares streams of string data to be sent to each module with action commands. For a PSD test, ICES control module receives target displacements at control points from computational modules and distributes them to each associated experimental module at every

time step. ICES control module retrieves measured restoring forces at control points from experimental modules and distributes them to associated computational modules. At each time step, restoring forces or restoring force balances are updated for solving equation of motion at next time step.

#### **Computational module**

Computational module is an independent program that includes numerical algorithms and interfaces with external computational tool. The current implementation includes ABAQUS, ZEUS-NL and MATLAB as computational engines or tools. ABAQUS provides a general FE tool for time integrator based on implicit time integration scheme. In order to use ABAQUS as time integrator, additional computational algorithms are implemented at the interface of computational module. There are two time integration algorithms incorporated into computational module. At each time step, computational module gets measured restoring forces at control points and sends targeted displacements at control points to associated experimental modules through ICES control module.

#### Virtual Structural/Geotechnical experimental module

Virtual structural and geotechnical modules are responsible for numerical experiments of structural and geotechnical experimental model instead of conducting real experiments. Static analysis with ABAQUS is conducted with targeted displacements from ICES control module. Numerical experiments can be conducted in displacement control mode or force control mode for a specific purpose.

#### Data protocol and management

One of the major features of ICES controller is the ability to manage many control points between multiple modules. This is possible since it uses efficient data protocol and data structures. In data protocol, each boundary points are identified by control point number which is equal to node number of numerical model. Boundary points of each component are classified into two groups; boundary points connected to experimental modules and boundary points connected to computational modules as in beam element with two nodes. However, each point group can have variable nodes depending on the size of interface between numerical models.

## VIRTUAL DISTRIBUTED SIMULATIONS USING ICES FRAMEWORK

A series of example are presented to demonstrate (i) the versatility of the ICES framework and (ii) the performance of the PSD time integration ( $\alpha$ -OS and new P-C) algorithms. The examples range from a two bay steel frame to soil-foundation-structure interaction analysis of a bridge. All the experiments are virtual experiments whereby the experimental modules are represented by an equivalent static numerical analysis.

#### Multi-site On-line Simulation Test (MOST) at the University of Illinois at Urbana Champaign

Distributed simulation (physical and analytical) of a steel frame structure is described in a companion paper by Spencer et al [8] and is illustrated in Figure 3. One of the El Centro earthquake records is used as the input ground motion.



Figure 4 Elastic case with 5.0% viscous damping

The distributed simulation consists of two experimental modules and one computational module. Figure 4 shows a plot of horizontal displacement response at the top of the left column. The figure shows the results for three simulations. In the analytical simulation the entire frame is simulated in a single numerical analysis and represents the correct or reference solution. The  $\alpha$ -OS and the PC methods represent the results of the distributed simulations within the ICES framework. The distributed simulation results are nearly identical to those for the reference analytical simulation.

#### Santa Monica Freeway (I-10) Collector-Distributor 36

A ramp structure of the I-10 Santa Monica Freeway, referred to as Distributor-Collector 36, suffered severe damage in the Northridge earthquake of 17 January 1994, [9].). The structure is a prestressed concrete box girder deck supported cast-in-place piers. It has four piers with a structural hinge between piers 6 and 7. Piers 6, 7 and 8 have circular sections and pier 9 is a rectangular RC wall. For multi-site PSD simulation, piers 7 and 8 are represented as virtual experiments. The deck and the rest of the piers are numerically simulated in computational module using dynamic analysis as illustrated in Figure 5. The ground motion input for this simulation is the NS Santa Monica City Hall record (~10km from the ramp structure) with peak ground acceleration of 0.370 g and applied in the transverse direction. The bridge deck and piers are modeled using 3D beam elements in ABAQUS.



Figure 5 Configuration of multi-site sub-structured PSD simulation

Table 2 shows material and sectional properties used in this simulation, taken from reference [9].. The material is assumed to be linear elastic. In this simulation, fixed base conditions are assumed without soil-foundation-structure interaction effect and no damping is included, as commonly used in pseudo-dynamic testing.

	Young's modulus(kN/m <sup>2</sup> )	Poisso n ratio	Unit weight(kN/m <sup>3</sup> )	Sectional Properties
Deck	2.9x10 <sup>7</sup>	0.2	2.5	$\begin{array}{c} A=3.74285(m^2)\\ I_{11}=1.8953(m^4)\\ I_{22}=17.0622(m^4)\\ J=5.49012(m^4) \end{array}$
Pier	2.9x10 <sup>7</sup>	0.2	2.5	R=0.6095(m)

Table 2 Material and	sectional	properties	of Santa	Monica	freeway

Figure 6 and Figure 7 plot the computed time history of displacement, velocity and acceleration from multi-site on-line PSD simulation and analytical simulation.



Figure 6 Horizontal responses at the top of pier7( $\alpha$ -OS method  $\Delta t$ =0.02 sec)



Figure 7 Horizontal responses at the top of pier 7(new P-C method  $\Delta t$ =0.005 sec)

Since the new P-C method is conditionally stable ( $\Omega_{crit} = \omega \Delta t < 2\sqrt{1/(1-\chi)}$ ), time step size was reduced to 0.005 sec. According to numerical test results, ICES controller can be used with confidence for multi-site sub-structured PSD test. Almost exact responses can be obtained from sub-structured PSD simulation using ICES controller.

## **Bridge with Soil-Foundation Representation**

The purpose of this simulation is to test the capabilities of the ICES controller for soil-foundationstructure interaction problems. A hypothetical two-span bridge with approach embankments is developed for virtual PSD simulation. Owing to the generality of the ICES controller, the bridge can be represented with multi-components of either computational or experimental modules. Figure 8 shows the schematic of this bridge with its approach embankments. Both abutments are assumed to consist of reinforced concrete pier wall connected by a curtain wall. The deck and abutment mass are assumed to be rigidly connected. As show in Figure 8, piles and soil system is replaced with frequency-independent soil springs. Inertia forces due to abutments and foundation of pier are considered through mass element. For an input ground motion, El Centro ground motion is imposed to the transverse direction. Employing sub-structuring technique, pier foundation/soil layer and deck and abutments system are represented by computational module and pier itself is represented by experimental module.



Figure 8 Bridge for soil-foundation-interaction problem and its configuration in ICES controller

To evaluate the stiffness of soil springs under the pier foundations, three-dimensional finite element analysis has been conducted using beam and eight-node solid element in ABAQUS. Table 3 shows the spring stiffness computed and assumed in this example.

	Abutment(kN/m)	Pier(kN/m)	
Longitudinal	$2.20 \times 10^5$	$2.04 \times 10^5$	
Transversal	$1.87 \ge 10^5$	1.94 x 10 <sup>5</sup>	
Vertical	6.66 x 10 <sup>5</sup>	$3.50 \times 10^5$	
Rocking around longitudinal	$2.20 \times 10^5$	$1.44 \text{ x } 10^6$	
Rocking around transversal	$7.38 \times 10^5$	6.69 x 10 <sup>5</sup>	

Table 3 Stiffness of soil spring

In the numerical model, damping effect is considered by assuming modal damping ratio 5.0% in the first and the second mode. To evaluate changes of dynamic characteristics due to soil-foundation-structure interaction (SFSI) effect, eigenvalue analysis is conducted on this model. Figure 13 shows comparison of its first three natural frequencies between model with no SFSI effect and model with SFSI effect. From this comparison, it can be judged that the SFSI effect should bring significant change of dynamic characteristics and lead to different responses under excitation such as earthquake ground motion.



Figure 9 Natural frequencies of model with no SFSI effect and with SFSI effect

To test the capabilities of ICES controller, virtual sub-structured PSD simulation considering SFSI effect has been conducted and compared with a single-model analytical simulation results computed by ABAQUS. Figure 14 plots horizontal responses of displacement, velocity and acceleration at the interface point between pier and bridge deck. The influence of SSI follows the expected trends of elongated periods and increased displacement amplitude. The ICES distributed simulation is shown to be capable of replicating the results of the single-model analysis.



Figure 10 Horizontal responses at the interface point between bridge deck and pier

Assessment of the above demonstration examples confirms that the ICES framework is applicable to the simulation of complex structural-geotechnical system in a sub-structured PSD testing environment. In the future, ICES controller will also include many innovative features utilizing information from real

experiments. For instance, the ICES controller can provide on-line updates of material constitutive models during testing and simulation based on measurement from experiments.

#### CONCLUSION

The MUST-SIM facility at the University of Illinois at Urbana-Champaign, part of the George E.Brown Network for Earthquake Engineering Simulation (NEES), is the next generation testing and simulation facility. The heart of MUST-SIM facility is the 'Integrated Computational and Experimental Simulation' (ICES) concept. The unique advantage of the ICES controller is the ability to simulate and test soil-foundation-structure system at full scale. The ICES controller consists of three main modules; a simulation coordinator, computational, structural and geotechnical experimental modules.. It has been developed using object-oriented programming approaches in C++. Currently, the ICES framework supports simulation using ABAQUS, ZEUS NL and MATLAB, deployed as computational or virtual experimental components of the system under investigation. Two numerical time integration algorithms are implemented in the ICES controller; the conventional  $\alpha$ -OS method and new Predictor-Corrector method. The latter method has distinct advantages in terms of accuracy and stability as well as eliminating the needed for calculating an initial stiffness.

Three simulation examples are presented in the paper; a two bay steel frame and two RC bridge structures, one of which includes soil-foundation-structure interaction. The accurate results from ICES distributed simulations, compared to a single-model analysis, confirm that the system is fully-functional and is capable of running any number of experimental and/or analytical components.

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