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SPECIFIC FRACTURE ENERGY APPROMOXATION OF DAM CONCRETE

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SUMMARY

The numerical nonlinear simulation of seismic dam behavior requires nonlinear fracture parameters of dam concrete such as specific fracture energy. But experimental determination of these parameters needs large scale specimens depending on maximum aggregate size. In this study a series of tests was carried out based on the size effect due on a number of geometrically similar notched specimens of various sizes. Experimental tests include three-point bending tests. The specimens were of square cross section with span to depth ratio of 3. Three different specimens with depth of 200, 250 and 300mm were considered for the purpose of test. The maximum aggregate sizes of concrete mixes were 50mm and 75mm, and two values of relative notch depths of 0.1 and 0.2 were used. According to specific fracture energy definition based on cohesive crack model and size effect, the total energy dissipated by fracture per unit area of the crack plane can be approximated between 325 and 375 N/m.

INTRODUCTION

The numerical simulation of concrete dams requires nonlinear properties of dam concrete. Some of important nonlinear parameters are rigorously determined and need to be investigated. One of the most important parameters is specific fracture energy. The specific fracture energy of dam concrete is a basic material characteristic needed for a rational prediction of concrete dams fracture behavior. The specimen sizes in dam concrete tests are larger than the specimen sizes in common concrete tests. Thus the most difficulty in determination of specific fracture energy of dam concrete lies in large specimen tests. Fracture energy determination depends on testing and the used concrete crack model. Although in principle, the specific fracture energy as a material property should be constant, and its value should be independent of the method of measurement, various test methods, specimen shapes, and sizes yield very different results-sometimes differing even by hundred percent [1-7]. There are few researches in determination of specific fracture energy of dam concrete. In some researches, the specific fracture energy of dam concrete is estimated two or three times larger than the specific fracture energy of normal concrete. The material

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properties of aggregates used in dam concrete have the most important effect on specific fracture energy of dam concrete. This indicates that the high specific fracture energy of dam concrete is rather the result of the nature and material properties of aggregates that of the aggregate size [8]. In dam concrete, the fracture process takes place over a relatively large fracture process zone whose size is, for the laboratory specimens, of the same order of magnitude as the size of the specimen itself.

Dam concrete is a material characterized by blunt fracture. The material behavior in the fracture process zone may be described by a strain-softening relation or, to some extent equivalently, by a stress-displacement relation with softening that characterizes the fracture process zone over its full width. This behavior indicates that the fracture energy is not the only controlling parameter and that the size and shape of the fracture process zone as well as the shape of the softening stress-strain diagram have a significant influence.

COHESIVE CARCK MODEL

Fracture Characteristics

The simplest model to characterize the behavior of a fracture process zone is the cohesive crack model. The basic hypothesis of the cohesive crack model is that, for mode I fracture, the fracture process zone of a finite width can be described by a fictitious line crack that transmits normal stress $\sigma(x)$ and that this stress is a function (monotonically decreasing) of the separation w called also the opening displacement or opening width:

$$\sigma = f(w) \tag{1}$$

By definition, $f(0) = f_t'$ = direct local tensile strength of concrete (in ACI notation). The terminal point of the softening curve $f(w)$ is denoted as w_f ; $f(w_f) = 0$

Function $f(w)$ first descends very steeply and then, roughly at $\sigma \approx 0.15 f_t' - 0.33 f_t'$, the descent becomes slow. Following Wittmann [9], the CEB-FIP code uses $0.15 f_t'$ [10], and Rokugo proposed $0.25 f_t'$ [11] for starting of the slow part of the slope.

Following Petersson [12], a simple bilinear form has generally been adopted (Fig. 1). The stress at slope change is generally considered to be between $0.15 f_t'$ and $0.33 f_t'$.

The area under the entire softening stress-separation curve $f(w)$ represents the total energy dissipated by fracture per unit area of the crack plane, G_F (dimension J/m^2), as the crack faces are completely separated at a given point. From this, it is often inferred, without any proof, that the area under $f(w)$ also represents the energy dissipated per unit area of crack plane as the fracture process zone moves forward which is crack propagation length. According to this concept, the specific fracture energy G_F is the energy required for crack growth in unit surface of the fracture process zone.

In the light of the generally accepted bilinear approximation (Fig.1) of the softening curve, $f(w)$, the cohesive crack model is characterized by two fracture energies:

$$G_F = \int_0^{w_f} f(w)\,dw \tag{2}$$

$$G_f = \frac{f_t'^2}{2\sigma_0'} = \frac{w_0^2 \sigma_0'}{2} \tag{3}$$

$$\sigma_0' = \frac{df(0)}{dw} \tag{4}$$

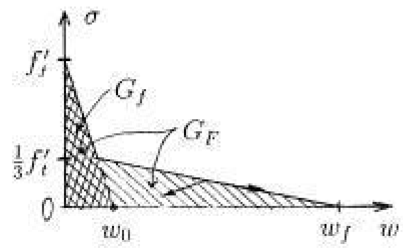

Fig.1: Bilinear softening stress-separation law

G_F Correspond to the area under the entire curve $f(w)$, while G_f to the area under initial tangent of slope σ'_0 , i.e. under the initial steep segment extended down to the w axis. Within a realistic size range, it is solely G_f which controls the maximum loads of structures and thus the size effect, as noticed by plans [13].

For the scaling and size effect, it is important to realize that the fracture energy and material strength imply, according to dimensional analysis, the existence of a fracture characteristic length as a material property[14]. In view of the bilinear approximation of $f(w)$, concrete possesses two fracture characteristic lengths:

$$l_1 = \frac{EG_f}{f_t'^2} \tag{5}$$

$$l_{ch} = \frac{EG_F}{f_t'^2} \tag{6}$$

This expression for l_{ch} is in the sense of the length of fracture process zone. While some other parameters of the dimension of length can be formed, a systematic use of l_{ch} for concrete was initiated by Hillerborg [15]. Typically:

$$G_F \approx 2.5G_f \tag{7}$$

$$l_{ch} \approx 2.5l_1 \tag{8}$$

Which was concluded by Planas [13] and Guinea [16,17].

SIZE EFFECT LAW

Size Effect Fracture Parameters

The structural size effect is probably the most important manifestation of fracture phenomena, at least from the engineering point of view. Therefore, it is important to relate the size effect behavior to the fracture properties of the material, such as specific fracture energy. As previously described the G_f value of a microscopically heterogeneous quasi-brittle material such as concrete is the energy required for crack growth in unit surface of an infinitely large specimen. The fracture energy determination would be exact if we know the exact form of the size-effect law to be used for extrapolation to infinite size. Unfortunately, we know this law only approximately. In the large size range, all the equivalent crack models satisfy at peak load, more over, for large specimen sizes ($D \to \infty$), the critical equivalent crack extension Δa_{ec} tends to a limiting constant value C_f ($\Delta a_{ec} \to C_f$ for $D \to \infty$)

To find the size effect law in the large size range, Bazant [18] proposed the following expression:

$$\sigma_{NU} = \frac{K_{IC}}{\sqrt{D\,\mathrm{k}^2(\alpha_0 + \frac{C_f}{D})}} \tag{9}$$

in which $\sigma_{NU} = \frac{P_u}{bd}$ where P_u, b and d are maximum load, specimen thickness, characteristic dimension of the specimen or structure, respectively. α_0 Is relative notch length and equal with $\frac{a_0}{d}$ where a_0 is notch depth, K_{IC} is critical intensity factor.

The function $k^2(\alpha_0 + \frac{C_f}{D})$ is approximated by its two-term Taylor series expansion at α_0:

$$k^2(\alpha_0 + \frac{C_f}{D}) \approx k_0^2 + 2k_0 k_0' \frac{C_f}{D} \tag{10}$$

Where $k_0 = k(\alpha_0)$ and $k_0' = k'(\alpha_0)$ stands for the values of $k(\alpha)$ and its first derivative for the initial crack length. Inserting this approximation into equation (9) will result in:

$$\sigma_{NU} = \frac{K_{IC}}{\sqrt{k_0^2 D + 2k_0 k_0' C_f}} = \frac{K_{IC}}{\sqrt{2k_0 k_0' C_f}\sqrt{1 + \frac{D}{(2k_0' C_f / k_0)}}} = \sqrt{\frac{EG_f}{g_0' C_f + g_0 D}} \tag{11}$$

in which $g_0 = g(\alpha_0) = k_0^2$ and $g_0' = g_0'(\alpha_0) = 2k_0 k_0'$

Bf_t' and D_0 can be defined as:

$$Bf_t' = \frac{K_{IC}}{\sqrt{2k_0 k_0' C_f}} = \frac{K_{IC}}{\sqrt{g_0 C_f}} \tag{12}$$

$$D_0 = \frac{2k_0'}{k_0} C_f = \frac{g_0'}{g_0} C_f \tag{13}$$

Introducing Bf_t' and D_0 into equation (11) results the classic form of Bazant's size effect law as following:

$$\sigma_{NU} = \frac{Bf_t'}{\sqrt{1 + \frac{D}{D_0}}} \tag{14}$$

The equation (14) might be more appropriately called the size-shape effect law, because the function $k(\alpha)$ introduces the effect of geometry (shape).

The relations (12) and (13) are fundamental of the experimental determination of the fracture properties of concrete based on size effect; the equation (13) reveals the basic characteristics of the transitional size D_0 . It is proportional to the effective length of the fracture process zone C_f, which in turn, is approximately proportional to the in homogeneity size of the material and also to the characteristic size $l_1 = K_{IC}^2 / f_t'^2$. It is also proportional to the ratio $2k'(\alpha_0)/k(\alpha_0)$ or, equivalently $g'(\alpha_0)/g(\alpha_0)$, which is independent of material properties and introduces the effect of structure geometry.

Equation (12) shows the basic structure of B. It can be written in terms of l_1 as:

$$B = \frac{1}{\sqrt{2k_0 k_0'}}\sqrt{\frac{l_1}{C_f}} = \frac{1}{\sqrt{g_0'}}\sqrt{\frac{l_1}{C_f}} \tag{15}$$

Which shows that B also consists of a product of geometrical and material functions. The material parameter is $\beta = \frac{C_f}{l_1}$ and is related to the softening behavior of the material in which for concrete its value can be estimated in the range of 2 to 5.

SPECIFIC FRACTURE ENERGY APPROXIMATION

Experimental Techniques

Results of previous research show that different experimental techniques or different analysis may lead to different values of fracture parameters such as specific fracture energy. These parameters can be uniquely defined by extrapolating their values for infinite size specimen. Since specimen failure is dictated by the material characteristics, it must be possible to determine these characteristics from size effect measurements. In order to determine specific fracture energy from the test on specimens, D_0 and Bf_t' can be determined from experiment then K_{IC} and C_f can be obtained from equations (12) and (13):

$$K_{IC} = Bf_t'\sqrt{D_0}\,k_0 \tag{16}$$

$$C_f = \frac{k_0}{2k_0'} D_0 \tag{17}$$

In linear Elastic Fracture Mechanics (LEFM), for infinite size specimens according to Irwin's [19] relationship specific fracture energy can be obtained as:

$$G_f = \frac{K_{IC}^2}{E} \tag{18}$$

$$G_f = \frac{(Bf_t')^2 D_0 k_0^2}{E} \tag{19}$$

Using bilinear cohesive crack model will result in:

$$G_F \approx 2.5G_f \tag{20}$$

Test Procedures in Size Effect Law

In the size effect method, a number of geometrically similar notched specimens of various sizes are tested for the peak loads. The nominal stress is then computed and its plot vs. the size is drawn. The parameters Bf_t' and D_0 are obtained by optimal least-square fitting of the size effect law to the experimental results. Finally specific fracture energy is obtained from equation (20). In this study the three-point bent specimens are used to determine specific fracture energy of dam concrete. The three-point bent test is a procedure, in which roughly half of the cross section is subjected to tension and half to compression and the fracture process zone is of medium size. All the specimens are of the same external shapes as previously used in a shear fracture study [20]. The cross sections of the specimen are rectangular, and the span to depth ratio is 3:1 for all specimens. The cross section heights of the specimens are d=200,250,300mm. The width of the bending specimens is b=200,250,300mm respectively (square cross sections).

Two concrete mixes are used. The first concrete mix has a water-cement ratio 0.42 and aggregate-cement ratio of 8.51 (all by weight). At this concrete mix, the maximum aggregate size is $d_a = 50$mm. The second concrete mix has a water-cement ratio 0.53 and aggregate-cement ratio of 12.30 (all by weight). The maximum aggregate size is $d_a = 75$mm.

The aggregate consisted of crushed stones from quarries and river sand. To determine the strength, companion cylinders of 250mm diameter and 500mm length are cast from each batch of concrete. After the standard 90-day moist curing, the mean compression strength is $f_c' = 44.4$ MPa for first concrete mix of 50mm M.S.A. and $f_c' = 37.9$ MPa for second concrete mix of 75mm M.S.A.

The tensile strength is obtained from splitting tensile test. To determine the tensile strength, cylinders of 150mm diameter and 300mm length are used for each concrete mix. After the standard 90-day moist curing the mean tension strength is $f_t' = 3.43$ for concrete mix of 50mm M.S.A. and $f_t' = 3.23$ for concrete mix of 75mm M.S.A.

Young's modulus is estimated as $E_c = 31500$ MPa for concrete mix of 50mm M.S.A. and $E_c = 29500$ MPa for concrete mix of 75mm M.S.A. For concrete mix of 50mm maximum aggregate size, two values of relative notch depths of 0.1 and 0.2 are used. For concrete mix of 75mm maximum aggregate size, only one value of relative notch depth of 0.1 is used.
The loading mechanism for three-point bent specimens designed to apply three concentrated loads onto the specimen with one load through a hinge and two through rollers. The steel surface was carefully machined so as to minimize the friction of the rollers. A universal joint was provided at the top and bottom connections to the testing machine to minimize in-plane and out-of-plane bending effect. The specimens were loaded at constant loading rate.

Analyses of Test Results

Size effect law in equation (6) can be algebraically rearranged to a linear regression plot:

$$Y = AX + C \tag{21}$$

In which $X = D$, $Y = (\frac{1}{\sigma_{NU}^2})$, $Bf_t' = \frac{1}{\sqrt{C}}$ and $D_0 = \frac{C}{A}$

The specific fracture energy can be obtained from following equation:

$$G_f = \frac{g(\alpha_0)}{EA} \tag{22}$$

Values of $g(\alpha_0)$ for present three-point bent specimens can be calculated by linear elastic fracture and dimensional analysis. According to Tada et. al.[21]:

$$g(\alpha_0) = (s/d)^2 \pi\alpha_0 (1.635 - 2.603\alpha_0 + 12.30\alpha_0^2 - 21.27\alpha_0^3 + 21.86\alpha_0^4)^2 \tag{23}$$

Linear regression plots of the test results were obtained. The results are shown in table (1).

TABLE (1): RESULTS OF LEAST-SQUARE OPTIMIZATION FOR OBTAINING SPECIFIC FRACTURE ENERGY

Specimen type	a/d	M.S.A. (mm)	α_0	$g(\alpha_0)$	A	E_c (MPa)	ω_A %	G_f (N/m)	G_F (N/m)
1	0.1	50	0.1	7.25	0.0018	31500	6.38	127	317
2	0.2	50	0.2	14.36	0.0035	31500	7.02	130	325
3	0.1	75	0.1	7.25	0.0017	29500	9.95	147	367

In table (1) the resulting specific fracture energy values for concrete mix of 50mm M.S.A. with relative notch depth of 0.1, concrete mix of 50mm M.S.A. with relative notch depth of 0.2 and concrete mix of 75mm M.S.A. with relative notch depth of 0.1 are 127,130,147 N/m respectively. The relevant data scatter is characterized by the standard deviation A of the regression line slope and the corresponding coefficient of variation ω_A, which are defined as:

$$\omega_A = \frac{S_A}{A} \tag{24}$$

$$S_A = \frac{S_{\eta x}}{S_x\sqrt{n-1}} \tag{25}$$

Where n is the number of all data points in the linear regression, S_x is the coefficient of variation of the X values for all the points and $S_{\eta x}$ is the standard deviation of the vertical deviations from the regression line. The values of ω_A are listed in table (1).

Research finding that the size-effect law yields approximately unique specific fracture energy values regardless of size and geometry makes it meaningful to base on this law a nondimensional characteristics[22] that indicates whether the behavior of a given specimen or structure is closer to limit analysis or to linear elastic fracture mechanics. The relative structure size $\lambda = \frac{d}{d_a}$ cannot serve as an objective indicator of this behavior. An objective indicator is the proposed Bazant's brittleness number β [22]. It is defined as:

$$\beta = \frac{D}{D_0} \tag{26}$$

And can be calculated after D_0 has been determined either experimentally or by finite element analysis. For $\beta < 1$, the behavior is close to plastic limit analysis, and for $\beta > 1$ it is closer to linear elastic fracture mechanics. For $\beta \le 0.1$, the plastic limit analysis may be used as an approximation. For $\beta \ge 10$, linear elastic fracture mechanics may be used as an approximation and for $0.1 < \beta < 10$ nonlinear fracture analysis must be used. The brittleness number values are listed in table (2) for various concrete mixes. It can be concluded that behavior of concrete mixes is close to nonlinear fracture mechanics.

TABLE (2): SIZE-EFFECT PARAMETERS

Specimen type	a/d	M.S.A. (mm)	α_0	A	C	$\beta f'_t$ (MPa)	D_0 (mm)	β_{min}	β_{max}
1	0.1	50	0.1	0.0018	0.7163	1.181	397	0.50	0.75
2	0.2	50	0.2	0.0035	0.1515	0.931	329	0.60	0.91
3	0.1	75	0.1	0.0017	1.0049	0.997	591	0.33	0.5

Some researchers have tried to characterize the effect of structure size on the qualitative fracture behavior by means of some nondimensional combination of G_f, f'_t, E_c. This is, however, insufficient because these parameters cannot reflect differences in structure geometry. For example Carpinteri[23] characterized the effect of structure size on its brittleness by the nondimensional ratio $s = \frac{G_f}{bf'_t}$ and Hillerborg et. al.[24] by the nondimensional ratio of some structural dimension to the characteristic length $l_1 = \frac{E_c G_f}{f'^2_t}$.

CONCLUSIONS

Based on the test results obtained in this study for different specimen sizes. It can be found that specific fracture energy of dam concrete mix of 75mm M.S.A. is 367 N/m and its value of concrete mix of 50mm M.S.A. is 325 N/m. Thus G_F value of dam concrete mix of 75mm M.S.A. is larger than G_F value of dam concrete mix of 50mm M.S.A. one of the reasons of this behavior probably lies in ratios of specimen depth to maximum aggregate size for two dam concrete mixes. Elastic modulus of dam concrete with 75mm maximum aggregate size is smaller than its value of dam concrete with 50mm maximum aggregate size. Thus it can affect the specific fracture energy values obtained for two types of dam concrete mixes. Considering D_0 values of various concrete mixes, it is found that crack growth length in specimens with 75mm maximum aggregate size is 48 percent larger than its value in specimens with 50mm maximum aggregate size. Results of the test show that specific fracture energy values don't depend on relative notch depths. Plastic stress value in limiting case, Bf_t', for dam concrete mix of 50mm M.S.A. is larger than its value for concrete mix of 75mm M.S.A. According to Bazant's brittleness number, all specimen brittleness numbers are between 0.1 and 10. Thus fracture behavior of all dam specimens is closer to nonlinear fracture mechanics.

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