13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 696

DISCRETE MODELING OF SLIDING SHEAR FAILURE IN CONTACTS BETWEEN RC MEMBERS WITH DIFFERENT THICKNESS

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SUMMARY

Shear-walls are one of the most effective structural components for providing lateral load resistance in buildings, dissipating large amounts of cyclic energy through the desirable ductile failure modes. For that purpose, the design in practice should ensure that the diagonal tension and diagonal compression failure modes are suppressed in order that energy dissipation take place by means of flexural yielding at the contact between the wall and the foundation beam. However, in that case the contact zone must be properly detailed, because of the phenomenon of shear sliding which prevents the ductile behavior after yielding of the main reinforcement. As observed experimentally by many researchers, in this case the applied shear is resisted mainly by friction and aggregate interlock between the cracked surfaces along the contact zone. For large deformations, dowel action of the longitudinal reinforcement also takes place. For the sake of behavioral simulation of these contact zones, a 3D integral FEM based constitutive model has been proposed, taking into account the slippage on the contact between the portions with different thickness, as well as the uplift and sinking of the members relatively to the basement using a discrete-crack approach with 3D contact elements. The model includes constitutive relations for aggregate interlock, dowel action of the reinforcement crossing the crack surface, and bond between the concrete and reinforcement. As a result, original software for 3D non-linear analysis of reinforced concrete structures, based on tangent stiffness approach has been developed. The proposed discrete model has been verified using basic experiments performed in Japan. Also, characteristic specimens of shear-walls have been used for comparison of the analytically obtained results, showing a good agreement with the experimental ones.

INTRODUCTION

It has long been recognized that contact zones between two components with different thickness of RC members can have a significant effect on the overall structural response, especially in

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cases of a very small amount of compressive axial force, so that the shear slip failure on contacts can occur. Also, most of the related investigations and experimental tests reported in the literature have concentrated either on studying the opening and closing behavior of cracks ([1], [2]), or on shear-transfer mechanisms across cracks ([3], [4]). Such tests have been usually carried out under constant crack dilation *w*, or under constant shear slip *s,* treating, practically the both phenomena independently. However, in real situations, the crack dilation *w* and shear slip *s* are interdependent and the results of such tests, although useful for many practical situations, cannot be considered as general constitutive laws. Within this context, it is worth noting the research conducted by Tassios & Vintzeleou [5], where a very important relation between the crack dilation and shear slip under monotonic and cyclic loading has been proposed. Later, based on intensive experimental tests, Okamura & Maekawa [6] proposed a relation for the frictional effects at a discrete crack of a plane concrete element, taking into account the ratio between shear slip and crack dilation. They also provided a very important constitutive law between strain and bond-slip of the reinforcement crossing the crack, necessary for evaluation of the bar pullout stress, important component of the contact mechanism.

Within this research, an attempt has been made to formulate an integral discrete contact model, which can be easily implemented using FEM procedures. For that purpose, an iso-parametric curved contact element has been proposed, with incorporated proposed constitutive laws for opening/closing and shear-slip behavior of the contact zones of RC members. This has been achieved by taking into account all factors influencing the response of the contact: bond-slip effects, dowel-action of the steel bars crossing the crack, as well as aggregate-interlock and other frictional effects appearing on the crack surfaces. The failure mechanisms along the contact zones (Fig. 1), apart from shear sliding and steel-bar pullout, also could be manifested by sinking of the columns into the foundations on the compression side. However, within the proposed integral constitutive model, the sinking effects will be considered by linear relation between the stresses and deformations for compression.

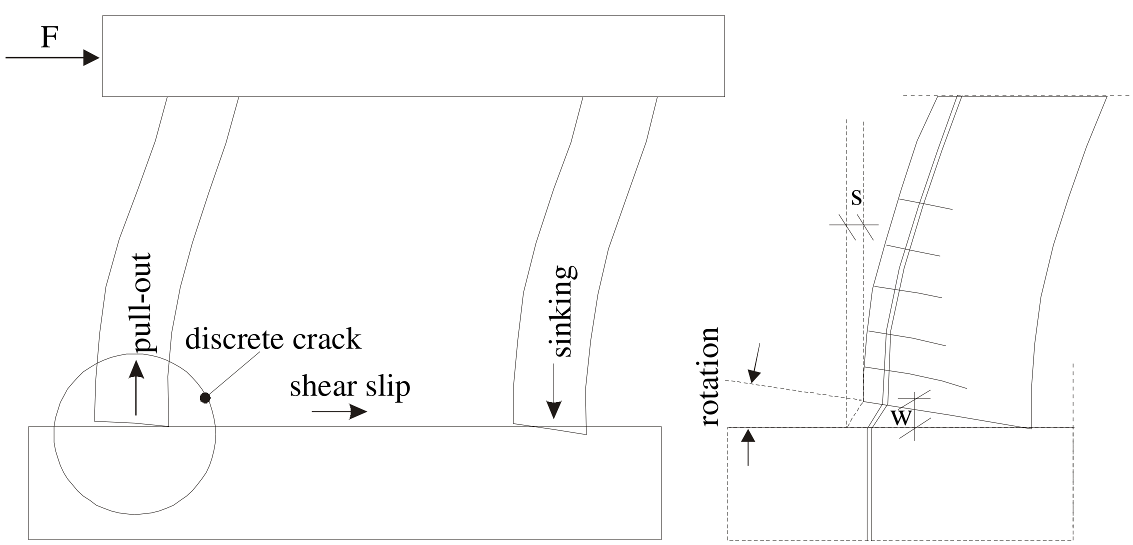

Fig. 1. Failure mechanisms along the contact zone between the wall and the foundation beam.

FORMULATION OF THE ANALITYCAL FEM MODEL

Basic model for the continuum

The basic model for the continuum, using curved 20-nodal iso-parametric finite elements, uses the rotating-crack concept. This concept can grasp the experimentally observed phenomenon of formation of new cracks in rotated direction, while previously formed ones close. Practically, it means that the rotating crack model is treated as a special case of fixed non-orthogonal crack model storing the information only for the latest formed cracks, so that an optimal finite element implementation is provided.

The mathematical model is based on hypo-elasticity (Noguchi [7]). The principal stresses and strains are allowed to rotate in coaxial directions during the loading process that is necessary condition for satisfying the form-invariance of the crack-induced orthotropic material behavior. The ultimate function adopted in the model is based on the Willam-Warnke yield surface, taking into account the influence of the strength reduction of cracked concrete in compression (Noguchi et al., [8]). More information regarding this basic model for the continuum, has been given in the paper by Hristovski and Noguchi [9].

3D CONTACT element

The behavior of the contact zones between reinforced concrete structural members with different thickness, taking into account the influence of the contact arising due to stiffness discontinuity and local phenomena, mentioned in the introduction. Since these phenomena are essentially three-dimensional, a discrete model becomes indispensable in order to capture the additional non-linear sources resulting from the contact areas. Hence, in addition to the proposed smeared-based constitutive relations, a discrete model via contact element has been herein proposed.

The proposed contact element can be geometrically described as a curved spatial surface, as follows:

$$
\begin{aligned}
x &= \sum_{i=1}^{8} N_i x_i = f_1(\xi,\eta) \\
y &= \sum_{i=1}^{8} N_i y_i = f_2(\xi,\eta) \qquad (1) \\
z &= \sum_{i=1}^{8} N_i z_i = f_3(\xi,\eta)
\end{aligned}
$$

where x_i, y_i and z_i (i=1,…,8) are coordinates of the double nodes (Fig. 2) and N_i are shape functions expressed in natural coordinates ξ and η :

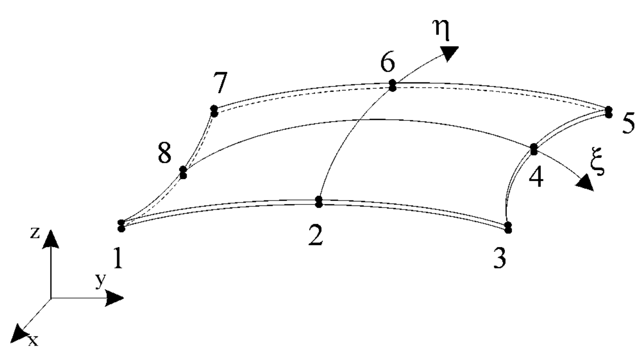

Fig. 2 Geometry of the CONTACT element

$$
\begin{aligned}
&N_i = \tfrac{1}{4}(1+\xi_o)(1+\eta_o)(\xi_o+\eta_o-1) \quad \textit{for nodes } i = 1,3,5,7 \\
&N_i = \tfrac{1}{2}(1+\xi_o)(1-\eta^2) \quad \textit{for nodes } i = 4,8 \\
&N_i = \tfrac{1}{2}(1-\xi^2)(1+\eta_o) \quad \textit{for nodes } i = 2,6 \\
&\eta_o = \eta\eta_i, \quad \xi_o = \xi\xi_i
\end{aligned} \tag{2}
$$

The basic constitutive equation for any point **M** (Fig. 3) on the described contact surface can be expressed as follows:

$$\{\sigma\} = [D]\{\varepsilon\} \tag{3}$$

where {σ} is stress vector and {ε} is strain vector in the considered point along the natural coordinate axes ξ, η and n:

$$\{\sigma\} = \begin{Bmatrix} \tau_\xi \\ \tau_\eta \\ \sigma_n \end{Bmatrix}, \quad \{\varepsilon\} = \begin{Bmatrix} s_\xi \\ s_\eta \\ w_n \end{Bmatrix} = [G]\sum_{i=1}^{8} N_i(\{u\}_k - \{u\}_{k+1}), \quad k = 2i-1 \tag{4}$$

In these equations [D] is constitutive matrix for the contact, [G] is transformation matrix from global x-y-z coordinates to local ξ–η–n coordinates and {u} are element nodal displacements in global coordinates. The members of the matrix [G] can be found as unit vector's coordinates of the local axes ξ, η and n at point **M**.

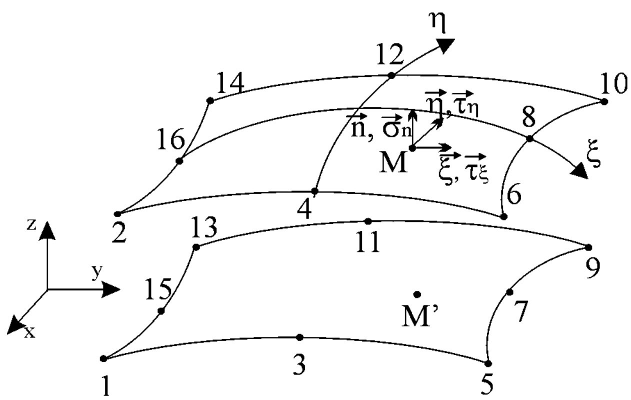

Fig. 3 Nodal convention and local stress field for the proposed CONTACT element

Applying Eq. (4) for strain vector, the following incremental relation can be obtained:

$$\{d\varepsilon\} = [B]\{du\} \tag{5}$$

where [B] is the strain matrix 3x48 which can be expressed as follows:

$$[B] = [[B]_1\ [B]_2\ [B]_3 \ldots [B]_k \ldots [B]_{16}] \tag{6}$$

with:

$$[B]_k = \pm N_i[G] \tag{7}$$

In Eq. (7) $i=k/2$ if k is even nodal number and $i=(k+1)/2$ if k is odd nodal number. The positive sign (+) applies if k is odd number and the negative sign (-) applies if k is even number. The element stiffness matrix can be numerically obtained by Gaussian integration (applying 3x3 integration rule), using the following equation:

$$[K]=\int_A [B]^T [D][B]dA \tag{8}$$

The infinitesimal surface dA can be found as an intensity of the normal vector $\vec{dn}$, using vector product as follows:

$$dA=\left|\vec{dn}\right|=[d\vec{\xi}, d\vec{\eta}]=\sqrt{\left(\frac{\partial y}{\partial \xi}\frac{\partial z}{\partial \eta}-\frac{\partial z}{\partial \xi}\frac{\partial y}{\partial \eta}\right)^2+\left(\frac{\partial x}{\partial \xi}\frac{\partial z}{\partial \eta}-\frac{\partial x}{\partial \eta}\frac{\partial z}{\partial \xi}\right)^2+\left(\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta}-\frac{\partial y}{\partial \xi}\frac{\partial x}{\partial \eta}\right)^2} \tag{9}$$

The internal nodal forces from the total element stresses $\{\sigma\}$ can be obtained according to the following relation:

$$\{f\}=\int_A [B]^T \{\sigma\}dA \tag{10}$$

CONSTITUTIVE RELATIONS OF THE INTEGRAL DISCRETE MODEL

The physical mechanism and the interdependence between the crack dilatancy and the slip can be described as follows: Under cracking forces $N=N_c$ a crack forms developing initial crack width w_o (Fig. 4a). In this stage the aggregate interlock mechanism and the dowel action are not yet mobilized. Further, a free slip s_f is needed in order that the mechanism of aggregate interlock be activated, however, dowel action mechanism is already mobilized by applying small slip $s>0$ (Fig. 4b). Finally, increasing the slip so that it becomes equal or greater than the value of the free-slip $s>s_f$ the contact is established and all resistance mechanisms are now mobilized (Fig. 5c). It is important to note that until $s<s_f$, there is no increase of the crack dilatancy: it starts to increase after full contact has been established. The increase of crack dilatancy δ_{n1} practically means activation of the pullout mechanism with gradually increasing length of the bond deterioration between the steel-bar and concrete. The tensile stresses in steel increase very fast up to the yielding strength, so that, as a reaction, a high compressive stresses tending to close the crack surfaces appear, contributing to the development of the horizontal frictional component τ (Fig 4c).

The componential constitutive relations, as well as the integral model based on the described physical model, have been discussed in the following sections for both monotonic and cyclic loading cases.

Influence of aggregate interlock and friction

The most important component of the shear transfer is the mechanism of aggregate interlock, as a manifestation of so called "general roughness" which depends on the size of the aggregates, as well as the mechanism of friction resulting from the "local roughness" of the crack interfaces (Fardis and Buyukozturk, [4]). In order to formulate a suitable constitutive model for monotonic loading (which further will serve as an envelope for the proposed cyclic model), the following basic assumptions have been adopted:

1. The steel-bars cross the crack normally. Only one equivalent steel-bar is taken into account, regardless of the real number of bars crossing the crack on the analyzed area.

2. Slip is not possible if the initial crack w_o is zero or has a negative value. If w_o is zero, it implies that there is full continuity in the material, without existence of crack. If $w_o<0$ it is considered as a compression, which is treated separately using linear elastic constitutive law.
3. The response exhibits a discontinuity and a singularity at the origin ($w=0$ and $s=0$) and near the origin. Theses stages, as well as the initiation of the crack should be carefully modeled using FEM procedures.

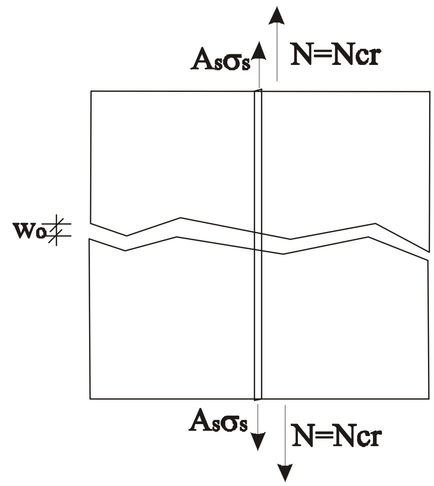

Fig. 4a Physical model of the discrete crack: Stage I – initial cracking due to pure tension

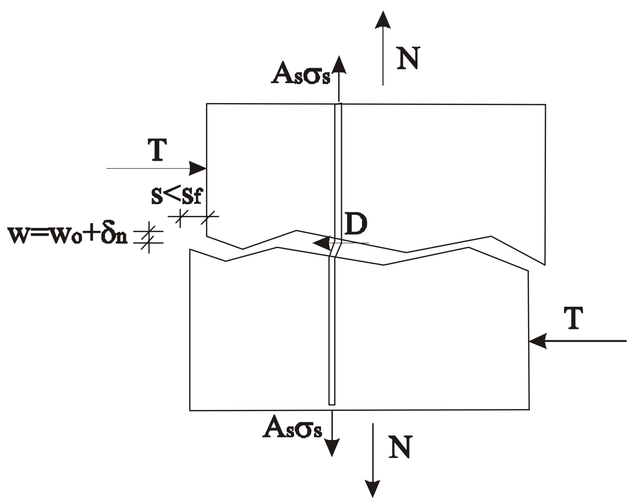

Fig. 4b Physical model of the discrete crack: Stage II– free slip with mobilized only dowel action

Using the simplified particular solution of the path-dependent probabilistic model (Contact Density Model) proposed by Li and Maekawa [6], the vertical resistant stress σ_c (Fig. 4c) and the horizontal shear stress τ can be calculated as follows:

$$\beta = \frac{s}{w} \tag{11}$$

$$f_{st} = 3.8 \,|\, f_c^{'} \,|^{1/3} \text{ in MPa} \tag{12}$$

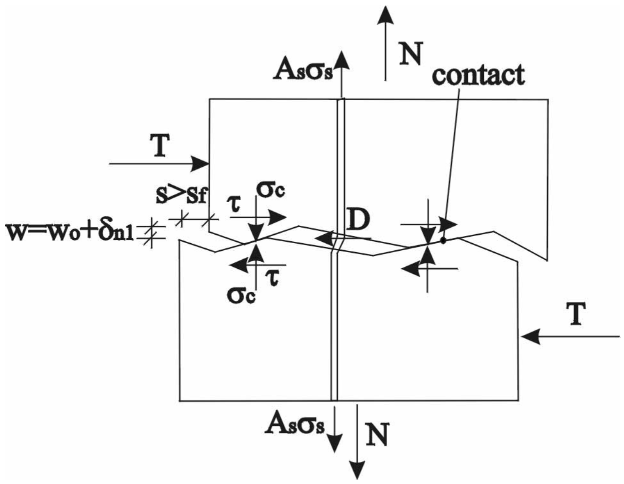

Fig. 4c Physical model of the discrete crack: Stage III– contact with mobilized shear transfer mechanism

$$\sigma_c = f_{st}\left[\frac{\pi}{2} - \arctan\left(\frac{1}{\beta}\right) - \frac{\beta}{1+\beta^2}\right] \tag{13}$$

$$\tau = f_{st}\frac{\beta^2}{1+\beta^2} \tag{14}$$

where f_c' is compression concrete strength and f_{st} is compression limit stress on the contact.

Pullout of the steel bar

After the concrete cracking, the progressive bond deterioration can result in bond-slip between the steel-bar and surrounding concrete, as mentioned before. These bond-slip (or pullout) deformations can be considered as a direct measure of the crack dilatancy *w*. In this stage, if shear slip *s* is equal to zero, it can be assumed that the response of the contact will not be influenced by friction or aggregate-interlock. In other words, treating this stage as a case of open crack due to monotonously increasing tensile force *N*, the envelope relation between the acting normal force *N* and the developed crack width *w* (or crack dilation) can be constructed (Figs. 5, 6).

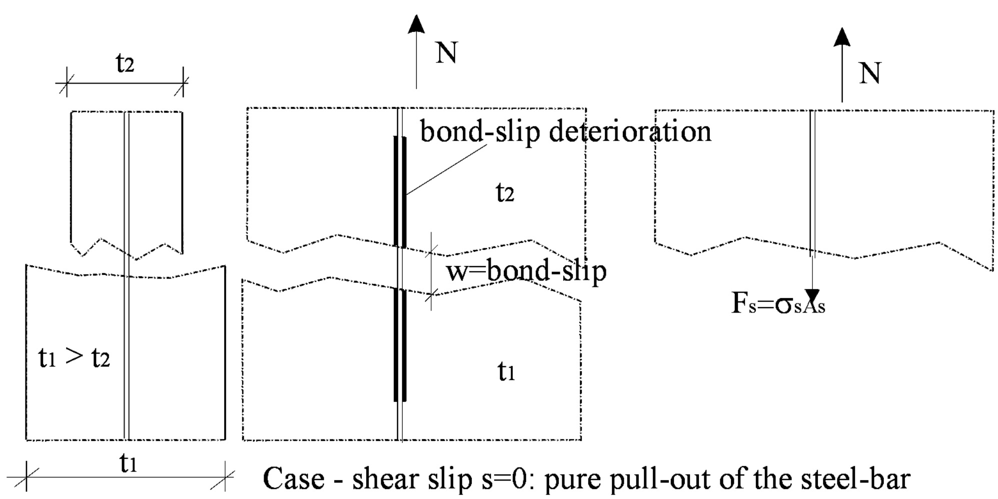

Fig. 5 Monotonic loading on tension (open crack): pullout of steel-bar

To this end, herein, slightly modified Okamura & Maekawa's relations (see [10]) between the bond-slip and strain in the steel-bar have been used. This model offers good simulation for cases of steel-bars embedded in the thicker member's part with long anchorage length (Fig. 5), assuming that the free-end slip is negligibly small. According to this model, the strain in the steel-bar ε_s can be calculated from the following relations:

$$w_s = \varepsilon_s (6 + 3500\varepsilon_s) b_1 \;\; for\; w_n \leq w_{ny} \tag{15}$$

$$\varepsilon_s = \frac{1}{2}(\varepsilon_{sy} + \varepsilon_{sh}) \;\; for\; w_n = w_{ny} \tag{16}$$

$$\varepsilon_s = \varepsilon_{sh} + \frac{w_n - w_{ny}}{0.2(f_{su} - f_{sy})} \;\; for\; w_{ny} < w_n \leq w_{sn} \tag{17}$$

$$\varepsilon_s = \varepsilon_{sh} + \frac{w_n - 0.5w_y - 0.06}{0.07(f_{su} - f_{sy})} \;\; for\; w_n > w_{sn} \tag{18}$$

where:

$$w_n = \frac{w}{d_b}\left(\frac{f_c^{'}}{20MPa}\right)^{\frac{2}{3}}, f_c^{'}\, in MPa \tag{19}$$

$$w_{ny} = \varepsilon_{sy}(6 + 3500\varepsilon_{sy}) b_1 \tag{20}$$

$$w_{sn} = w_{yn} + 0.2(f_{su} - f_{sy})(\varepsilon_{ss} - \varepsilon_{sh}) \tag{21}$$

$$\varepsilon_{ss} = \frac{0.06 - 0.5w_{ny}}{0.13(f_{su} - f_{sy})} + \varepsilon_{sh} \tag{22}$$

$$b_1 = 1.3 \tag{23}$$

In equations (15)-(23) w is actual crack width (dilatancy, or bond-slip), d_b is steel-bar diameter, ε_{sy} and ε_{sh} are yielding strain and strain on the onset of strain-hardening of the steel-bar, respectively, f_{su} and f_{sy} are yielding strength and ultimate strength of the steel-bar, respectively. Once ε_s has been calculated, the stresses σ_s in the steel-bar can be easily found. Hence, the total axial force N will be:

$$N = \sigma_s A_s \geq N_{cr} \tag{24}$$

where N_{cr} is the cracking axial force, which can be calculated, as follows:

$$N_{cr} = a_1 f_t A_{con} \;, a_1 = 0.43 \tag{25}$$

In Eq. (25) f_t is the uni-axial tensile concrete strength and A_{con} is the area of the concrete contact zone. The N-w diagram is shown in Fig. 6, where it can be noticed that the part 2-3 (up to the yielding of the steel-bar) is non-linear due to the influence of bond-deterioration. Also, the part 1-2 has been taken as a constant equal to cracking force N_{cr}, although, applying formulae (15)-(23), a line starting from zero force can be obtained. The reason for this approximation is the fact that the initial continuous de- formation (strain) in concrete before onset of the discrete cracking is neglected in the model. However, as e result of this initial concrete strain and the initial perfect bond between the concrete and steel, non-zero force in the steel-bar is initially induced, which is not taken into account in the model. The similar trend of the diagram *N-w* after cracking has been observed in many experiments, too. For this reason, the total contact normal force in this model is kept not less than N_{cr}.

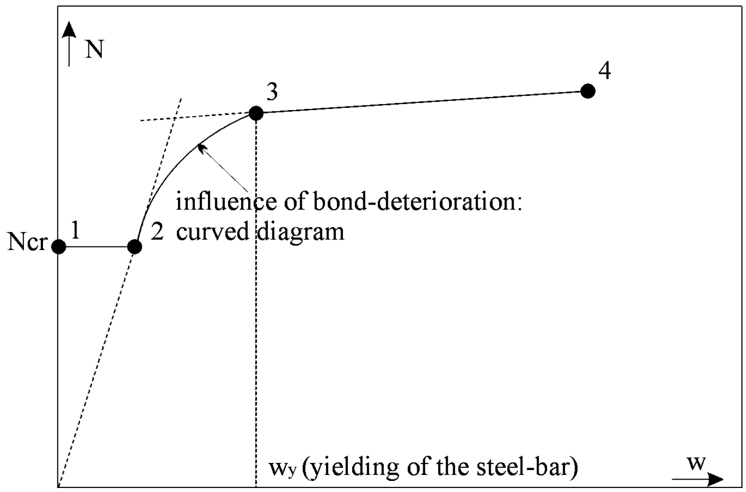

Fig. 6 ***N-w* diagram: monotonic loading on tension (open crack)**

Dowel-action of the steel-bar crossing the crack

The dowel-action effect within this integral discrete model is taken into account using the Vinzeleou-Tassios relations [11], as follows:

$$D = \begin{cases} 3D_u s & \text{for } s \leq 0.1 \text{ mm} \\ 0.7D_u \sqrt[4]{s} & \text{for } s \geq 0.1 \text{ mm} \end{cases} \tag{26}$$

$$D_u = 1.3 d_b^2 \sqrt{f_{cc} f_{sy}} \tag{27}$$

where s is given in mm, $f_{cc} = 1.17 f_{c'}$ and f_{sy} in MPa, and forces D and D_u in N.

NUMERICAL EXAMPLE

For the purpose of verifying the FEM analytical model including both the proposed contact element and constitutive relations for the discussed integral discrete model, numerical examples have been performed and compared with selected experimental tests of RC members (shear walls, columns). Within this paper, an example of a shear wall #1 tested by Aoyama et al. [12] is given. The JCI shear-wall specimen #1 consists of heavily reinforced base and top spreader beams for the purpose of the load transfer, as well as of columns with t=20 cm, cast integrally with the walls t=10 cm. As observed experimentally (the specimen was subjected to reversed cyclic loading), the failure mechanisms included flexural cracking of the tensioned column, shear-tension failure of the walls and crushing of the compressed column. The failure mechanisms have been reasonably simulated using the proposed analytical model (Fig. 7). In addition, shear-slip along the contact zone between the foundation beam and the wall have been experimentally observed, as also has been predicted by the analysis. The material properties for this specimen are given in Tab. 1.

Actually, two analyses have been performed: the first one using only the basic smeared model treating the contact between the foundation beam and upper structure (wall + columns) as continuous (denoted in Fig.

8 as “w/o”), and the second one modeling this contact by the proposed integral discrete model via CONTACT elements (denoted in Fig. 8 it is denoted as “w”).

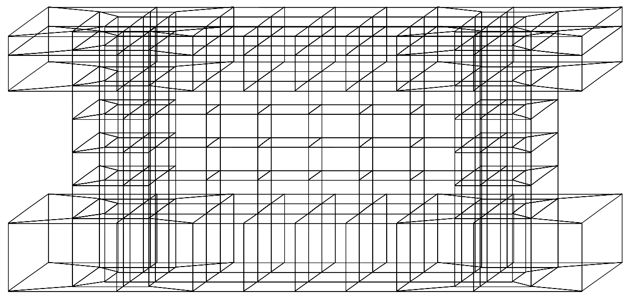

Fig. 7 3D FEM analytical model for the test specimen #1

Table.1 Material properties for the JCI Shear-Wall Specimen #1

CONCRETE		STEEL					
		columns		beams		wall	
		Diameter	f_y[MPa]	Diameter	f_y[MPa]	Diameter	f_y[MPa]
f_c'[MPa]	29.7	D13	368	D10	353	6ϕ 2x6ϕ/7.5 cm, vert. and horiz.)	363
E_c[GPa]	23.4	6ϕ	399	D22, D29	400 (default)		
f_t'[MPa]	2.36						

From the Fig. 8 it is obvious that the influence of the contact for this specimen is significant (see differences between analyses “w” and “w/o”). Using the basic model plus integral discrete model for the contact good agreement with the experimental force-displacement diagram has been obtained.

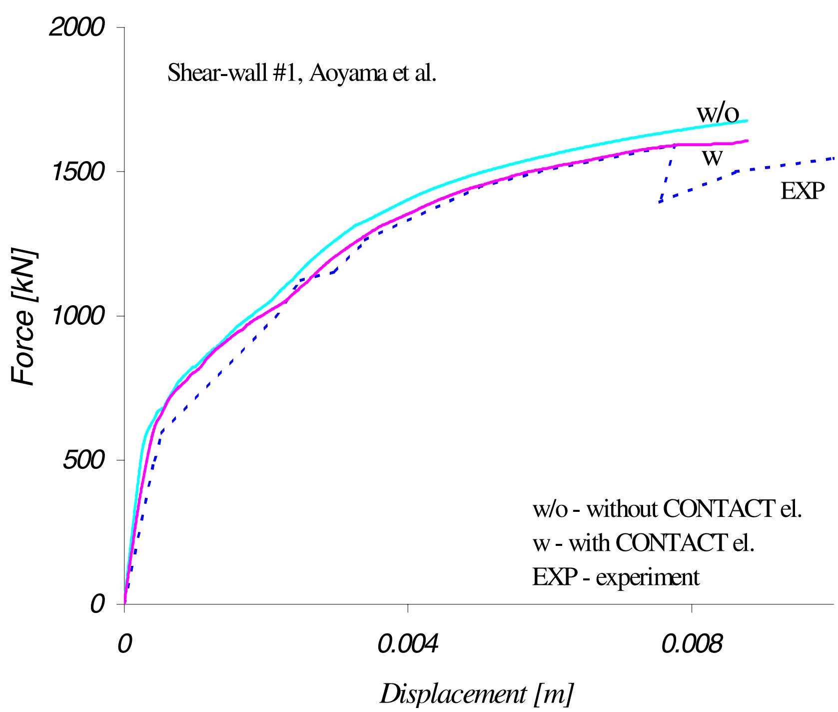

Fig. 8 Comparison between the experimental test and analysis

CONCLUSIONS

From the above investigation the following conclusions can be drawn:

1. Apart from the basic model using hypo-elastic formulation and smeared crack approach based on rotational concept, a 3D integral discrete formulation for contact problems arising on the boundaries between the members components with different thickness has been proposed via a contact element.
2. The discussed integral discrete model takes into account the influence of the contact phenomena like aggregate interlock, dowel action of the reinforcement crossing the crack, and pullout mechanism of the reinforcement via bond deterioration relationships.
3. The proposed physical model of the contact zone is based on the mechanism that explains the interdependence between crack dilatancy and slip along the surface. Separate stages of the response of the contact zone, depending on which mechanisms are mobilized, have been discussed.
4. The discussed constitutive relations for each particular mechanism contributing to the overall behavior of the contact have been integrated resulting in the proposed integral discrete model that has been implemented by using the curved iso-parametric contact element with 16 nodes.
5. For the purpose of verifying the FEM analytical model, analyses of experimentally tested RC specimens (shear-walls, columns, etc.) have been performed, showing a good agreement with the results obtained by the tests.

ACKNOWLDEGMENTS

The first author would like to express his appreciations to the Japan Society for the Promotion of Science (JSPS) that supported this research through the Post-doctoral Fellowship Program for Foreign Researchers.

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