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LIMIT ANALYSIS OF RC INTERIOR BEAM-COLUMN JOINTS USING SINKING MECHANISM OF CONNECTING BEAMS

Shuichi UEHARA[1], Fumiya ESAKI[2] and Tomomi NISHIDA[3]

SUMMARY

A new model to estimate the strength of reinforced concrete (referred to as RC hereafter) interior beam-column joints is proposed using sinking mechanism of connecting beams. The strength of joints is estimated by the forces at the beam end sections connecting with the joints, where stress of compressive strut at beam ends is assumed to be equal to the stress of strut in joint, neglecting the effect of bond . The efficiency of the model was examined by analyzing 84 specimens in the literature. The results show that the proposed method overestimate by about 10% the strength of joints, however the standard deviations are very small. It is also shown that the estimation of J type failure specimens with single-layered reinforcement is good but not so good of ones with double-layered reinforcement. The error may come from the estimation of bond strength. The study in the paper as a whole shows the efficiency of the proposed method.

INTRODUCTION

The design of RC beam-column joints resisting earthquake load becomes more important recently because of adopting ductile frame design and using high strength material. The failure of RC beam-column joints has been assumed to be shear failure caused by the compressive failure of strut in joints[1]. Shiohara [2] however, insists that the failure of RC beam-column joints is flexural failure of joints, since shear stress at the horizontal shear plane in joints does not decrease, sometimes increase even after the failure of joints.

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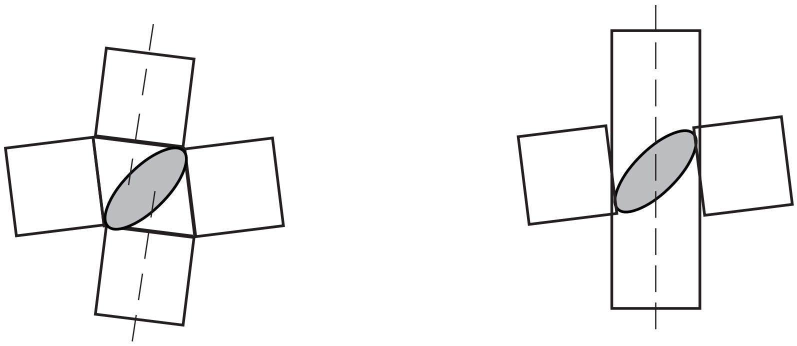
[1]Professor, Ariake National College of Technology, Omuta, Japan, Email: uehara@ariake-nct.ac.jp
[2]Professor, Kyushu Kyoritsu University, Kitakyushu, Japan, Email: ezaki@kyukyo-u.ac.jp
[3]Advanced Course Student, Ariake National College of Technology, Omuta, Japan



(a)Shear failure mechanism of a joint

(b)Sinking failure mechanism of a joint

Fig.1 Failure mechanisms of an interior beam-column joint

Horizontal shear failure plane is observed when beam-column joints strengthened with steel plates[3], however no horizontal shear failure plane is observed when beam-column joints with usual reinforcement fail. Most of beam-column joints with usual reinforcement fail when compressive strut expands out of panel[4]. Therefore, the failure may be treated as flexural failure if it is estimated as sectional failure of members. Since the failure of joints is different from the shear failure of columns, the hoops in joints may not work to strengthen the joints.

From the point of the view above, the failure of the joints are evaluated as sinking of connecting beams into joint in this paper. In short, the failure is assumed to occur when the compressive concrete portions of connecting beams sink into joints since the concrete in joints becomes weak because of cracks. The objective of this study is to evaluate the rationality of the proposed idea. Hereon, objective joints are ordinary ones with column broader than connecting beams.

AN ANALYTICAL METHOD USING SINKING MECHANISM

AN EXAMPLE OF BEAM SINKING OBSERVED IN THE LITERATURE

Fig.1 shows the differences between shear deformation mechanism and sinking deformation mechanism of joints. The gray portions of the joints are failure zones of struts. The deformation assumed generally is the one of Fig.1(a). Columns are displaced horizontally according to the model. Columns are kept almost straight in the sinking mechanism model since some parts of the column does not fail.

Fig.2 shows an example of beam sinking observed in a specimen in the literature. The specimen is reported to have failed with increasing deformation of the joint after flexural yielding of beams and failure of joints. Therefore the residual deformation is estimated to have occurred mainly by the deformation of the joint. The lines in Fig.2 indicate the deformation of the specimen is caused mainly by rotation of beams. The columns are almost straight as assumed in the sinking mechanism model.

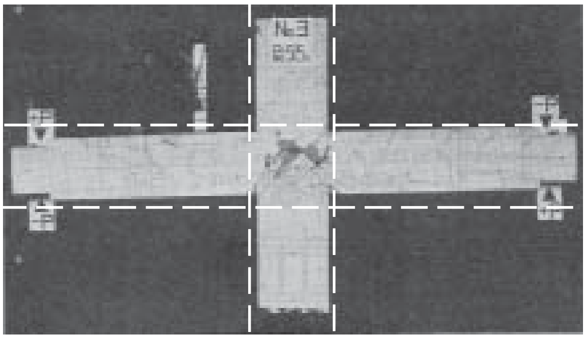


Fig.2 An example of joint failure specimen selected by JCI[4]

ESTIMATION OF THE STRENGTH OF JOINT FAILURE BEFORE BEAM YIELDING

Outline of the method and features

Since the beam ends sink into the joint in the proposed model as shown in Fig.1(b) , the horizontal story shear force is calculated using the sectional forces at beam ends.The authors have proposed a limit analysis method of RC columns considering interaction of combined forces[5][6]. The outline is as follows. The yield lines of member are assumed as slanting planes which cross the extreme compressive corners of column ends when the column fails under bending moment M, axial force N and shear force Q. The yield criterion of the RC slanting section is estimated by adding the yield criterion of reinforcement to the one of concrete considering the interaction of combined forces, M,N,Q. The horizontal shear capacity is the minimum value when the yield line inclination angle is varied.

The yield line is determined at critical section of beams when the limit analysis method is used for joint analysis. The features of the method are to consider of the effectiveness factor of concrete strength, the effect of shear force to bending strength and the effect of axial force in beams if necessary.

Estimation of the stress of concrete at beam ends

It is reported that bond stress is weakened when joint fails[2]. Fig.3 shows the relationship among compressive struts of beams, columns and joint neglecting the bond of longitudinal reinforcement of beams and columns. All the stresses of struts are same when the struts with the same thickness cross orthogonally at a connecting triangle element. It is proved by that the force polygon A'B'C 'of struts is similar to the connecting element ABC as shown in Fig.4. And the stresses of struts in beams can be assumed to be same with the stress of strut in the joint.

Therefore the stress of concrete at a critical beam section is assumed to be $\nu\sigma_B$, when the strut in joint fails at the stress of $\nu\sigma_B$ and the beam ends sink, where σ_B is compressive strength of concrete. The effectiveness factor of concrete compressive strength ν is estimated as follows. The shear strength of concrete in joints is shown in the reference[1]. The concrete compressive strength of the strut in a joint is assumed to incline by 45 degrees. In other words, major compressive strength is obtained by doubling the shear strength.

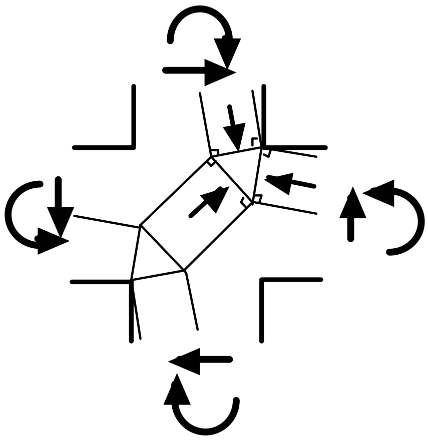
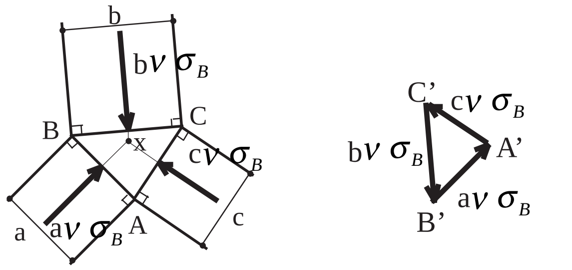


Fig.3 Compressive struts in a joint, connecting beams and columns



(a)Forces in struts **(b)Force polygon of struts**

Fig.4 Relationship of forces in struts

Eq. (1) is obtained by dividing the compressive strength by concrete cylinder strength. Here, safety factor 1/0.85 is removed.

$$\nu = 2 \times 1.0 \times 0.8 \times \sigma_B^{-0.3} \tag{1}$$

The equation is used to calculate the effectiveness factor of concrete in joints. At the same time it is used to estimate the concrete stress at beam ends.

Estimation of bond strength at beam ends

Since the joint failure before beam yielding accompanies with bond failure of longitudinal reinforcement in beams, the yield criterion of the reinforcement bases upon bond strength. So the beams rotate because of bond failure and slipping out of longitudinal reinforcement. As a result, the strut in a joint fails when compressive concrete at beam ends thrusts the strut in the joint. From the assumption above, bond strength can be treated as premature yielding.

Bond strength is assumed as Eq.(2)[1].

$$\tau_u = 0.7\sigma_B^{\frac{2}{3}} \times 1.25 \tag{2}$$

The safety factor in Eq.(2) is removed by multiplying 1.25. The term of axial force is also removed since it is assumed that axial force is not resisted by inner part of joint and is resisted at corners of core concrete. The yield criterion of longitudinal reinforcement in beams is decided using bond strength by Eq.(2).

The yield criterion of longitudinal reinforcement is shown by the thin line in Fig.5 in case of bond failure, where the sectional area of tensile reinforcement a_t and the one of compressive reinforcement a_c are assumed to be the same. Here, $m=M_f/\sigma_B b_b D_b^2$, $n=N_f/\sigma_B b_b D_b$, $q=Q_f/\sigma_B b_b D_b$, M_f, N_f, Q_f: bending moment, axial force and shear force, b_b: width of beam, D_b: depth of beam. And the peak point on the m-axis is shown by

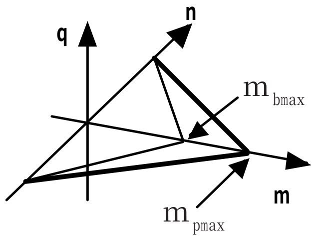


Fig.5 Yield criterion of logitudinal reinforcement in beams considering bond failure

$$m_{b\max} = \frac{\Sigma\psi_{tpr}\tau_u}{b_b D_b{}^2\sigma_B}\frac{D_c}{2}D_{b1}' \qquad (3)$$

where ψ_{tpr} : the sum of perimeter of tensile reinforcement, D_c: depth of column.

The yield criterion of longitudinal reinforcement is shown by the thick line in Fig.5 in case of plastic yielding and the peak point on the m-axis is shown by

$$m_{p\max} = \frac{a_t\sigma_y}{b_b D_b{}^2\sigma_B}D_{b1}' \qquad (4)$$

where σ_y : yield point of longitudinal reinforcement , D_{b1}'=D_b'/D_b, D_b': distance between tensile reinforcement and compressive reinforcement.

Estimation of story shear force when joint failure before beam yielding

An analytical nondimensional story shear force of joint failure q_{cal} can be shown by Eq.(5) using nondimensional shear force of beams.

$$q_{cal} = q_{bcal}\frac{L}{H}\frac{b_b D_b}{b_c D_c} \qquad (5)$$

In which L: length of span, H: height of story, b_c: breadth of column. In addition.

$$q_{bcal} = \frac{-B+\sqrt{B^2-4AC}}{2A} \qquad (6)$$

where

$$A = \frac{1}{2\nu}\frac{1}{\alpha_q^2} \qquad B = \frac{1}{\alpha_m}\left(\frac{h_b}{D_b}\right)$$

$$C = \frac{1}{2\nu}\left\{\frac{1}{\alpha_n^2}\left(n_b - \frac{\nu}{2}\right)^2 - \left(\frac{\nu}{2}\right)^2\right\} \quad (7)$$

h_b: distance between the beam end section and the inflection point, $n_b=N_b/\sigma_B b_b D_b$, N_b:Axial force of beams, however all the specimens used in the paper were not loaded with axial force in beams.

$$\alpha_m = \frac{\frac{\nu}{8} + m_{b\max}}{\frac{\nu}{8}} \quad (8)$$

where $\psi_l = a_g/(b_b D_b \sigma_B)$, a_g :sectional area of longitudinal reinforcement in beams. In addition, variables α_n , α_q , α_m are expanding factors of yield criterion of concrete for yield criterion of RC section. In detail, refer to [5].

Estimation of story shear force when joint failure after beam yielding

Story shear force when joint failure occurs after beam yielding (referred to as BJ specimens hereafter) can be estimated as J failure specimens were done. However, m_{pmax} is used instead of m_{bmax} in Eq.(8) since longitudinal reinforcement in beams yields.

DISCUSSION

ANALYZED SPECIMENS

Eighty four specimens were collected from the proceedings of the Japan Concrete Institute during 1991 to 2000 and were analyzed at every type of failure. However, the specimens that failed at beam ends (referred to as B type hereafter) and did not fail at joint were not analyzed in the present paper. Table 1 shows the list of all specimens. The collected specimens are ordinary ones and precasted specimens and specimens strengthened with special metal were removed.

ANALYSIS OF J TYPE FAILURE SPECIMENS

Fig.6 shows the comparison between experimental values q_{ex} and calculated ones q_{cal} of nondimensional story shear force of J type failure specimens. The calculated values overestimate the experimental values a little. However the standard deviation is small. The calculated values of the specimens with double-layered reinforcement are smaller than the ones with single-layered reinforcement. The error may come from smaller bond strength of double layered-reinforcement compared with single-layered reinforcement. If it is true, the accuracy of analytical values can be improved by more accurate values of bond strength.

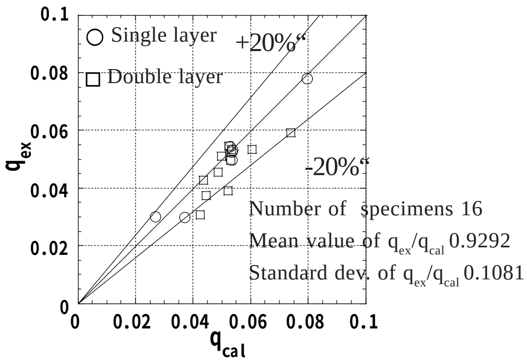


Fig.6 Analytical values of story shear force of J type specimens q_{cal} and experimental values q_{ex}

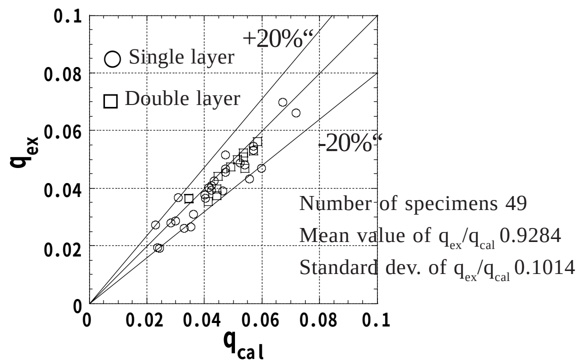


Fig.7 Analytical nondimensional story shear force of BJ type specimens q_{cal} and experimental values q_{ex}

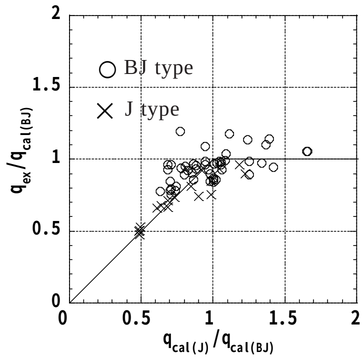


Fig.8 Analytical values and experimental values of story shear force

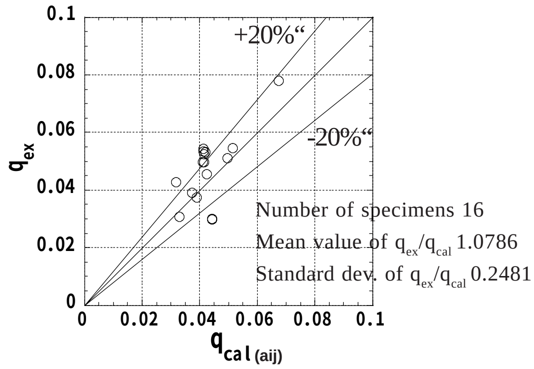


Fig.9 Analytical nondimensional story shear force of BJ type specimens $q_{cal(aij)}$ by conventional method and experimental values q_{ex}

ANALYSIS OF BJ TYPE FAILURE SPECIMENS

Fig.7 shows the comparison between q_{ex} and q_{cal} of BJ type failure specimens. The experimental values are a little smaller than calculated values as a whole. The standard deviation is also very small. The difference between the double-layered and the single-layered is not recognized since the strength of longitudinal reinforcement in beams depends on plastic yielding strength but not on bond strength.

PREDICTION OF FAILURE TYPES BY THE PROPOSED METHOD

Fig.8 shows comparison between the predicted failure types by the analysis and the failure types by experiment. A value less than 1 on the x-axis predicts that the failure type of the corresponding specimen is J failure type. The results show that the prediction is not good by the proposed method. More improvement of the strength estimation formulae is necessary.

STRENGTH ESTIMATION BY A CONVENTIONAL METHOD

Fig.9 shows that the comparison of J type specimens between q_{ex} and $q_{cal(aij)}$ calculated by the method by

Table 1 Specimens used in the analysis

No	Proceedings of JCI and pages	Name	exp. value	cal. Value	raito	Failure	No	Proceedings of JCI and pages	Name	exp. value	cal. Value	raito	Failure
			qex	qcal	qex/qcal					qex	qcal	qex/qcal	
1	1991,pp.495-500	I5	0.0240	-	-	B	43	1993,pp.583-588	NO47	0.0323	0.0366	0.8814	BJ
2	1991,pp.519-524	J1	0.0500	-	-	B	44		NO48	0.0377	0.0408	0.9244	BJ
3		J2	0.0464	-	-	B	45		NO49	0.0469	0.0610	0.7679	BJ
4		J3	0.0462	-	-	B	46		NO50	0.0368	0.0312	1.1763	BJ
5	1992,pp.379-384	A6	0.0524	-	-	B	47		HN08	0.0266	0.0286	0.9282	BJ
6	1993,pp.583-588	NO43	0.0251	-	-	B	48		HN09	0.0279	0.0288	0.9701	BJ
7		NO44	0.0279	-	-	B	49		HN10	0.0272	0.0233	1.1659	BJ
8		NO45	0.0336	-	-	B	50	1997,pp.987-992	NO1	0.0364	0.0410	0.8873	BJ
9		NO46	0.0251	-	-	B	51		NO3	0.0391	0.0471	0.8308	BJ
10	1995,pp.309-314	J1	0.0570	-	-	B	52	1997,pp.993-998	BN-3	0.0530	0.0582	0.9102	BJ
11		J2	0.0738	-	-	B	53		BN-5	0.0661	0.0732	0.9033	BJ
12	1997,pp.993-998	BN-2	0.0592	-	-	B	54	1997,pp.1011-1016	I8C	0.0500	0.0525	0.9524	BJ
13		BN-4	0.0534	-	-	B	55	1997,pp.505-510	B15-13	0.0402	0.0419	0.9587	BJ
14		BN-1	0.0491	-	-	B	56		B16-16	0.0481	0.0547	0.8791	BJ
15	1997,pp.1011-1016	I6C	0.0376	-	-	B	57		B16-13	0.0469	0.0549	0.8541	BJ
16	1997,pp.505-510	B15-16	0.0339	-	-	B	58		B16-10	0.0441	0.0455	0.9677	BJ
17		B15-10	0.0306	-	-	B	59	1998,pp.517-522	AIJ	0.0486	0.0529	0.9189	BJ
18	1998,pp.535-540	NO5	0.0292	-	-	B	60		HRP	0.0532	0.0577	0.9219	BJ
19		NO6	0.0289	-	-	B	61		CSP	0.0424	0.0438	0.9679	BJ
20	1991,pp.475-478	OKJ-1	0.0375	0.0452	0.8304	BJ	62		JCR	0.0545	0.0577	0.9445	BJ
21		OKJ-4	0.0397	0.0452	0.8786	BJ	63		HBS	0.0562	0.0591	0.9500	BJ
22	1991,pp.495-500	I6	0.0287	0.0303	0.9479	BJ	64	1998,pp.535-540	NO1	0.0431	0.0568	0.7579	BJ
23	1991,pp.507-512	NO.34	0.0455	0.0482	0.9452	BJ	65		NO2	0.0408	0.0433	0.9425	BJ
24		NO.35	0.0467	0.0482	0.9687	BJ	66		NO3	0.0392	0.0433	0.9069	BJ
25		NO.36	0.0522	0.0546	0.9570	BJ	67		J1	0.0393	0.0433	0.9087	BJ
26		NO.37	0.0508	0.0546	0.9298	BJ	68	1999,pp.643-548	B1	0.0309	0.0365	0.8473	BJ
27		NO.38	0.0515	0.0482	1.0699	BJ	69	1991,pp.475-478	OKJ-2	0.0373	0.0445	0.8389	J
28	1991,pp.513-518	HC	0.0362	0.0351	1.0340	BJ	70		OKJ-3	0.0308	0.0424	0.7250	J
29		HLC	0.0365	0.0353	1.0352	BJ	71		OKJ-5	0.0391	0.0522	0.7481	J
30	1991,pp.519-524	I2	0.0474	0.0492	0.9644	BJ	72		OKJ-6	0.0454	0.0486	0.9345	J
31		I4	0.0354	0.0421	0.8406	BJ	73	1991,pp.513-518	LA1	0.0510	0.0499	1.0219	J
32	1992,pp.397-400	MJK-1	0.0191	0.0248	0.7709	BJ	74		A1	0.0545	0.0525	1.0375	J
33		MJK-2	0.0266	0.0359	0.7408	BJ	75	1992,pp.379-384	I1	0.0427	0.0435	0.9814	J
34		MJK-3	0.0192	0.0240	0.8013	BJ	76	1997,pp.987-992	NO2	0.0299	0.0372	0.8023	J
35		MJK-4	0.0261	0.0334	0.7816	BJ	77		NO4	0.0299	0.0269	1.1144	J
36	1992,pp.401-404	PL	0.0398	0.0479	0.8308	BJ	78	1998,pp.517-522	KSC	0.0780	0.0798	0.9776	J
37		PH	0.0395	0.0467	0.8465	BJ	79	1999,pp.679-684	NO1	0.0525	0.0533	0.9845	J
38	1993,pp.553-558	NO1	0.0453	0.0406	1.1148	BJ	80		NO2	0.0497	0.0534	0.9316	J
39	1993,pp.559-564	J11A	0.0613	0.0547	1.1210	BJ	81		NO3	0.0532	0.0538	0.9871	J
40		J12A	0.0741	0.0690	1.0732	BJ	82		NO4	0.0544	0.0529	1.0286	J
41		J31A	0.0698	0.0685	1.0190	BJ	83		NO5	0.0533	0.0533	0.9993	J
42		J32A	0.0775	0.0802	0.9666	BJ	84		NO6	0.0497	0.0532	0.9330	J

Type B:Failed in beams(19 specimens)

Type BJ:Failed in joint after beam yielding(49 specimens)

Type J:Failed in joint before beam yielding(16 specimens)

AIJ[1]. The safty factor is removed in the analysis. The standard deviation of the AIJ method is larger than the one by the proposed method shown in Fig.6.

CONCLUSIONS

An strength estimation method of RC interior beam-column joints is proposed using sinking mechanism of connecting beams and the efficiency of the method was examined by analyzing 84 specimens in the literature. The following conclusions are derived from the study.

(1) The proposed method overestimate by about 10% the strength of joints, however the standard deviations

are very small.
(2)The estimation of J type failure specimens with single-layered reinforcement is good but not so good of specimens with double-layered reinforcement. The error may come from the estimation of bond strength.
The study in the paper shows the efficiency of the proposed method to estimate the strength of joints using sinking mechanism of connecting beams. Recently, authors confirmed that connecting beams sink into joints considerably when joints fail in experiment. The results will be shown elsewhere.

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