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TIME-DOMAIN IDENTIFICATION SYSTEM OF DYNAMIC SOIL-STRUCTURE INTERACTION

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SUMMARY

Time-domain identification system of dynamic soil-structure interaction effects was developed. Kalman filter with weighted local iterations is applied to a soil-structure interaction system that was modeled by a single-degree-of-freedom system supported by a sway-rocking foundation model. By using this system, both stiffness and damping parameters are well identified.

INTRODUCTION

Soil-structure interaction during earthquakes has been a challenging research topic for the last fifty years. Many researchers have devoted themselves to this problem resulting in tremendous advances in the field. However, these advances are still not sufficiently reflected in earthquake resistant design codes. One of the reasons is that soil-structure interaction is mathematically complicated; the analysis has to take into account ground conditions as well as geometries of the foundations. The analysis becomes even more complicated if the stiffness of the soil deteriorates due to strong ground shaking. This compels researchers to idealize the model of soil-structure interaction so that they can handle the problems.

Application of system identification may be effective in these kinds of complications. It enables us to identify the soil stiffness from the observation records of soil-structure interaction systems. In this study, therefore, a time-domain identification system of soil-structure interaction effects is developed. The soil-structure interaction system is modeled by a single degree of freedom system supported by a sway-rocking foundation model, then, the governing equation is expressed by a state equation. In order to stably identify the soil stiffness in the time-domain, an extended Kalman filter with a weighted local iteration algorithm is applied to the interaction system. Observation records are artificially created in this study by computing the responses of the soil-structure interaction system, since the objective of this research is only to confirm the applicability of the identification scheme to the soil-structure interaction systems.

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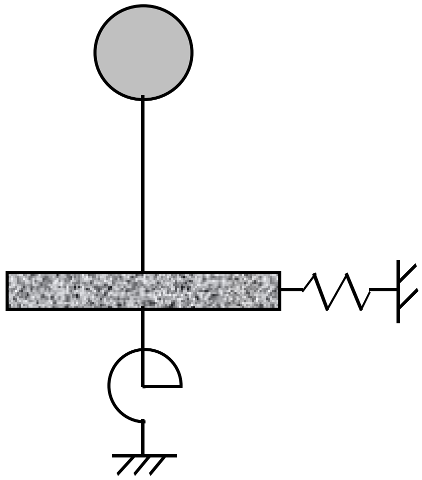

Fig.1 Sway-Rocking Model

MODELING OF SOIL-STRUCTURE INTERACTION SYSTEMS AND THEIR STATE SPACE EXPRESSIONS

Modeling of soil-structure interaction

Soil-structure interaction (SSI) analyses can be categorized as direct and substructure approaches. In a direct approach, the soil and structure are included within the same model and analyzed in a single step. Because assumptions of superposition are not necessary, it is possible for nonlinear analyses to be performed. However, the analyses remain quite expensive from a computational standpoint.

In a substructure approach, the SSI problem is broken into several parts, i.e. evaluation of foundation input motion, determination of impedance function and dynamic analysis of the structure based on these two procedures. Because each step is independent of the others, one can engage in a specific problem. In addition, results from each step are physically interpreted independently since the SSI analyses procedure in the substructure approach are physically separated. Thus, the results from the substructure method are more likely reflected in seismic design codes. This study aims to develop an identification system to determine the impedance functions of SSI.

The governing equation of superstructure-foundation-soil system is expressed as follows if the superstructure is modeled as a single-degree-of-freedom system.

$$\begin{bmatrix} m_S & m_S & m_S H \\ m_S & m_S + m_f & m_S H \\ m_S H & m_S H & J_f + m_S H^2 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{u}_f \\ \ddot{\theta}_f \end{Bmatrix} + \begin{bmatrix} c_S & 0 & 0 \\ 0 & C_{hh} & C_{hr} \\ 0 & C_{rh} & C_{rr} \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{u}_f \\ \dot{\theta}_f \end{Bmatrix} + \begin{bmatrix} k_S & 0 & 0 \\ 0 & K_{hh} & K_{hr} \\ 0 & K_{rh} & K_{rr} \end{bmatrix} \begin{Bmatrix} u \\ u_f \\ \theta_f \end{Bmatrix} = -\begin{Bmatrix} m_S \\ m_S + m_f \\ m_S H \end{Bmatrix} \ddot{u}_K - \begin{Bmatrix} m_S H \\ m_S H \\ J_f + m_S H^2 \end{Bmatrix} \ddot{\theta}_K \tag{1}$$

where u =horizontal displacement of the structure relative to the foundation, u_f =displacement of the foundation relative to free-field, θ_f =rocking angle of the foundation, m_S =mass of the structure, H =height of the structure, m_f =mass of the foundation, u_K =translation of massless foundation due to kinematic interaction, θ_K =base rocking of massless foundation slab due to kinematic interaction, $C_{hh}, C_{hr}, C_{rh}, C_{rr}$ =damping coefficients for foundation, $K_{hh}, K_{hr}, K_{rh}, K_{rr}$ =stiffnesses for foundation

For the modeling of SSI employed in this study, the effect of kinematic interaction is ignored, and the off-diagonal components of the stiffness and damping matrices are ignored considering the foundation geometry such that its depth is small enough compared with its width. Then, the equation (1) becomes as equation (2) which is shown schematically in Figure 1.

$$\begin{bmatrix} m_S & m_S & m_S H \\ m_S & m_S + m_f & m_S H \\ m_S H & m_S H & J_f + m_S H^2 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{u}_f \\ \ddot{\theta}_f \end{Bmatrix} + \begin{bmatrix} c_S & 0 & 0 \\ 0 & C_{hh} & 0 \\ 0 & 0 & C_{rr} \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{u}_f \\ \dot{\theta}_f \end{Bmatrix} + \begin{bmatrix} k_S & 0 & 0 \\ 0 & K_{hh} & 0 \\ 0 & 0 & K_{rr} \end{bmatrix} \begin{Bmatrix} u \\ u_f \\ \theta_f \end{Bmatrix} = -\begin{Bmatrix} m_S \\ m_S + m_f \\ m_S H \end{Bmatrix} \ddot{u}_g \tag{2}$$

Although the rigorous solutions of stiffnesses for circular foundations are frequency dependent functions (Veletsos and Wei [1]), stiffnesses at the dominant frequency of the SSI system are prescribed for the model. Based on equation (2), dynamic SSI analysis is performed.

State space expressions of the SSI model

To obtain a state-space expression, the equation (2) is expressed in a simple form.

$$[M]\{\ddot{U}(t)\}+[C]\{\dot{U}(t)\}+[K]\{U(t)\}=\{F(t)\} \tag{3}$$

Solving equation (3) by the linear acceleration method, the following equations are obtained.

$$\begin{aligned} \{Z_1(k+1)\} &= [A]^{-1}\left[\{F(k+1)\}-[C]\{a(k)\}-[K]\{b(k)\}\right] \\ \{Z_2(k+1)\} &= \{a(k)\}+\frac{1}{2}\Delta t\{Z_1(k+1)\} \\ \{Z_3(k+1)\} &= \{b(k)\}+\frac{1}{6}\Delta t^2\{Z_1(k+1)\} \end{aligned} \tag{4)-(6}$$

where k =time step, Δt =time increment, then time is expressed as $t_k = k\Delta t$. $\{\ddot{U}(k)\},\{\dot{U}(k)\},\{U(k)\}$ are expressed by state vectors $\{Z_1(k)),\{Z_2(k)\},\{Z_3(k)\}$.

$$\begin{aligned} [A] &= [M]+\frac{1}{2}\Delta t[C]+\frac{1}{6}\Delta t^2[K] \\ \{a(k)\} &= \{\dot{x}(k)\}+\frac{1}{2}\Delta t\{\ddot{x}(k)\} \\ \{b(k)\} &= \{x(k)\}+\Delta t\{\dot{x}(k)\}+\frac{1}{3}\Delta t^2\{\ddot{x}(k)\} \end{aligned} \tag{7)-(9}$$

$$F(k+1) = -\begin{Bmatrix} m_S \\ m_S+m_f \\ m_S H \end{Bmatrix} \ddot{u}_g(k+1) \tag{10}$$

These equations express forwarding process from time step k to $k+1$ except the input term $F(k+1)$. The input term includes noise because the term is substituted by observed records. Here, the input noise is extracted by using Sawada's method [2] which incorporates the acceleration difference

$$w(k) = \ddot{x}_g(k+1) - \ddot{x}_g(k) \tag{11}$$

This function $w(k)$ becomes acceleration difference when the input acceleration does not include any noise. However, the input usually accompanies noise, thus the function consists of the acceleration difference and the noise. Using this function, the transition of the input acceleration, velocity and displacement are expressed as follows:

$$\ddot{u}_g(k+1) = \ddot{u}_g(k) + w(k)$$
$$\dot{u}_g(k+1) = \dot{u}_g(k) + \Delta t \cdot \ddot{u}_g(k) + \frac{\Delta t}{2} w(k) \qquad (12)\text{-}(14)$$
$$u_g(k+1) = u_g(k) + \Delta t \cdot \dot{u}_g(k) + \frac{\Delta t^2}{2} \ddot{u}_g(k) + \frac{\Delta t^2}{6} w(k)$$

Statistical characteristics concerning the system noise are assumed as follows.

$$E[w(t)] = 0$$
$$E[w^2(t)] = [V(t)] \qquad (15), (16)$$

Here, $V(t)$ is a variance and it is approximately computed as follows.

$$V(t) = \frac{1}{T_a} \int_{t-T_a}^{t} w^2(u)du \qquad (17)$$

where T_a =averaging time

By expressing equations (12)-(14) in a matrix form,

$$\begin{Bmatrix} \ddot{u}_g(k+1) \\ \dot{u}_g(k+1) \\ u_g(k+1) \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \Delta t & 1 & 0 \\ \Delta t^2/2 & \Delta t & 1 \end{bmatrix} \begin{Bmatrix} \ddot{u}_g(k) \\ \dot{u}_g(k) \\ u_g(k) \end{Bmatrix} + \begin{Bmatrix} 1 \\ \Delta t/2 \\ \Delta t^2/6 \end{Bmatrix} w(k) \qquad (18)$$

This equation is simply expressed as below.

$$\{Z_6(k+1)\} = [B]\{Z_6(k)\} + [D]w(t) \qquad (19)$$

From equation (10) and (12)-(14), the term $F(k+1)$ included in equation (4) is expressed as follows.

$$\{F(k+1)\} = \begin{bmatrix} -m_S & 0 & 0 \\ -(m_S + m_f) & 0 & 0 \\ -m_S H & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_g(k+1) \\ \dot{u}_g(k+1) \\ u_g(k+1) \end{Bmatrix} \qquad (20)$$

This equation is also simply expressed as below.

$$\{F(k+1)\} = [E]\{Z_6(k+1)\} \qquad (21)$$

Substituting equation (19) into equation (21), the following equation is finally obtained.

$$\{F(k+1)\} = [E][B]\{Z_6(k)\} + [E]\{D\}w(k) \qquad (22)$$

With this manipulation, equations (4)-(6) become a purely forwarding process from time step k to $k+1$.

Including stiffnesses k_S, K_{hh}, K_{rr} into state vector $\{Z_4\}$ and damping coefficients c_S, C_{hh}, C_{rr} into state vector $\{Z_5\}$, transitions of these vectors are expressed as follows.

$$\begin{aligned} \{Z_4(k+1)\} &= \{Z_4(k)\} \\ \{Z_5(k+1)\} &= \{Z_5(k)\} \end{aligned} \qquad (23), (24)$$

Concerning mass parameters of the system, the mass of the structure and the mass and moment of inertia of the foundation are assumed to be known.

Taking into account accelerations, velocities and displacements at each degree of freedom and at the base, in addition to these spring constants and damping coefficients as state variables, the following state space expression is obtained.

$$\{Z(k+1)\} = G\{Z(k)\} + \{\Gamma(k)\}w(k) \qquad (25)$$

where $\{Z\}$ and $G\{Z(t)\}$ are composed of several vectors as follows:

$$\begin{aligned} \{Z\} &= [\{Z_1\}^T, \{Z_2\}^T, \{Z_3\}^T, \{Z_4\}^T, \{Z_5\}^T, \{Z_6\}^T]^T \\ G\{Z(t)\} &= [\{G_1\}^T, \{G_2\}^T, \{G_3\}^T, \{G_4\}^T, \{G_5\}^T, \{G_6\}^T] \end{aligned} \qquad (26), (27)$$

where

$$\begin{aligned} G_1 &= [A]^{-1}\{[E][B]\{Z_6\} - [C]\{a\} - [K]\{b\}\} \\ G_2 &= \{a\} + \frac{1}{2}\Delta t \cdot G_1 \\ G_3 &= \{b\} + \frac{1}{6}\Delta t^2 \cdot G_1 \\ G_4 &= \{Z_4\} \\ G_5 &= \{Z_5\} \\ G_6 &= B\{Z_6\} \end{aligned} \qquad (28)\text{-}(33)$$

$$\{\Gamma(k)\} = \begin{Bmatrix} [A]^{-1}[E]\{D\} \\ \Delta t[A]^{-1}[E]\{D\}/2 \\ \Delta t^2[A]^{-1}[E]\{D\}/6 \\ 0 \\ 0 \\ \{D\} \end{Bmatrix} \qquad (34)$$

SYSTEM FOR IDENTIFICATION IN THE TIME-DOMAIN

Time-domain identification system is established by using the state space expression of the governing equation. To apply Kalman filter for the system identification, the nonlinear equation (25) is to be linearized first, then an observation equation is prepared. With these and an iterated extended Kalman filter, time-domain identification system is finally developed.

Linearization of state equation

Kalman filter is originally developed for the identification of linear systems. Thus, an extended Kalman filter is applied for the nonlinear system identification assuming that nonlinear behavior of the system can be approximated as linear for a small perturbation.

Now, assuming $G\{Z(k)\}$ as a smooth function, Tayler expansion of $G\{Z(k)\}$ is found about the optimal estimation at the time step k. Ignoring the terms of Tayler expansion higher than second order, we obtain the truncated form of $G\{Z(k)\}$. Then,

$$\{Z(k+1)\} = G\{\hat{Z}(k \mid k)\} + \Phi(k+1 \mid k)\{Z(k) - \hat{Z}(k \mid k)\} + \{\Gamma(k)\}w(k) \tag{35}$$

where $\Phi(k+1 \mid k)$ is a transition matrix which is expressed as below.

$$\Phi(k+1 \mid k) = \left[\frac{\partial G\{Z(k)\}}{\partial Z_j}\right]_{Z(k)=\hat{Z}(k|k)} \quad (j = 1,\ldots,18) \tag{36}$$

where the components are computed as follows.

$$\frac{\partial G_1}{\partial Z_j} = [A]^{-1}\left(-\frac{\partial [A]}{\partial Z_j}G_1 + [E][B]\frac{\partial\{Z_6\}}{\partial Z_j} - \frac{\partial [C]}{\partial Z_j}\{a\} - C\frac{\partial\{a\}}{\partial Z_j} - \frac{\partial K}{\partial Z_j}\{b\} - K\frac{\partial\{b\}}{\partial Z_j}\right) \tag{37}$$

$$\frac{\partial G_2}{\partial Z_j} = \frac{\partial\{a\}}{\partial Z_j} + \frac{1}{2}\Delta t\frac{\partial G_1}{\partial Z_j} \tag{38}$$

$$\frac{\partial G_3}{\partial Z_j} = \frac{\partial\{b\}}{\partial Z_j} + \frac{1}{6}\Delta t^2\frac{\partial G_1}{\partial Z_j} \tag{39}$$

$$\frac{\partial G_4}{\partial Z_j} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 18} \tag{40}$$

$$\frac{\partial G_5}{\partial Z_j} = \begin{bmatrix} 0 & \ldots & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}_{3\times 18} \tag{41}$$

$$\frac{\partial G_6}{\partial Z_j} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & 0 \\ 0 & \cdots & 0 & \Delta t & 1 & 0 \\ 0 & \cdots & 0 & \Delta t^2/2 & \Delta t & 1 \end{bmatrix}_{3\times 18} \tag{42}$$

Observation equation

Taking velocities and displacements at each degree of freedom in addition to accelerations, velocities and displacements at the base, the observation equation is prepared.

$$\{Y(k)\}\} = [H]\{U(k)\} + \{v(k)\} \tag{43}$$

where $[H]$ is observation matrix expressed as below.

$$[H] = \begin{bmatrix} 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{44}$$

$\{v(k)\}$ is observation noise vector which is assumed to have the following characteristics.

$$\begin{aligned} E[\{v(t)\}] &= \{0\} \\ E[\{v(t)\}\cdot\{v(t)\}^T] &= [R(t)] \end{aligned} \qquad (45), (46)$$

where $[R(t)]$ is covariance matrix of observation noise.

Kalman filter with weighted iterations

The extended Kalman filter can be applied for the non-linear system identification if the non-linear behavior of the system can be approximated as linear for a small perturbation. However, the algorithm becomes unable to track the phenomenon by the linear approximation of the transfer matrix if the system shows strong non-linear behavior. In addition, if the observation information includes a strong non-stationary phenomenon, the ordinary Kalman filter is not applicable. One approach to compensate these deficiencies is to incorporate the forgetting factor for the obtuseness of the system noise. Another contractive method is the Kalman filtering with weighted local iteration (Sawada [3]), which is to be used in this study.

The algorithm of the Kalman filter with weighted local iteration is shown in Fig.2. This method first carries out an ordinary extended Kalman filtering algorithm using the known estimate $\hat{X}(k\,|\,k)$ and its error covariance matrix $P(k\,|\,k)$, then $\hat{X}(k+1\,|\,k+1)$ and $P_1(k+1\,|\,k+1)$ are obtained (Step1). Then a weighted local iterative extended Kalman filtering algorithm is conducted near the current time step. In this process, the error covariance matrix $P_1(k+1\,|\,k+1)$ is modified by multiplying the weighting factor r, that is $r\cdot P_1(k+1\,|\,k+1)$ (Step2). Starting from the time step $k+1$, an extended Kalman filtering is conducted m times in a forward $k+1+i \quad (i=1,2,3,...,m)$ and $2m$ times in a backward direction $k+1+m-i \quad (i=1,2,3,...,2m)$,

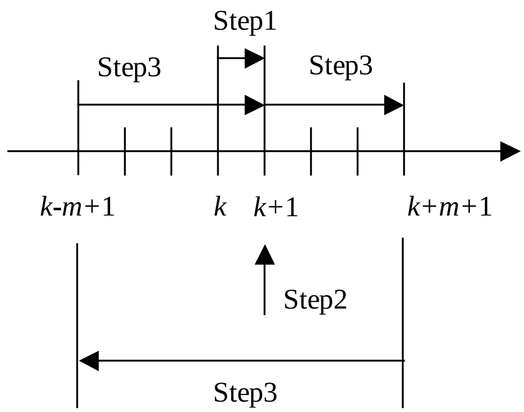

Fig.2 Weighted local iteration

Table 1 Specification of the SSI model

Parameters	Prescribed value
m_S	5.0×10^{5} (kg)
m_f	3.927×10^{5} (kg)
J_f	2.978×10^{6} (kg m^2)
H	7.0 (m)
k_S	3.16×10^{8} (N/m)
K_{hh}	1.035×10^{9} (N/m)
K_{rr}	2.2524×10^{10} (Nm)
c_S	1.257×10^{6} (kg/sec)
C_{hh}	1.9733×10^{7} (kg/sec)
C_{rr}	5.0653×10^{7} (kg m^2/sec)

and again m times in the forward $k+1-m+i$ $(i=1,2,3,\ldots,m)$, in turn (Step3) as shown in Fig.2. With this algorithm, non-stationary parameters are identified more easily and stably.

RESULTS OF IDENTIFICATION

Parameters of the SSI model

Parameters which specify the SSI model used in this study are shown in Table 1. Among these parameters, spring constants and damping coefficients are to be identified.

Fig.3 shows a horizontal acceleration record observed at Chiba experimental station, Institute of Industrial Science, University of Tokyo. This record in which maximum acceleration is about 60 gal is used as input to the SSI system and the computed responses are used as mock observation records.

Parameters for identification

For the identification of the SSI model, both the ordinary extended Kalman filter (EK) and the extended Kalman filter with iteration (EKI) approach are applied. Parameters that control the identification algorithm are shown in Table 2.

Results of identification

Figures 4 and 5 show displacement responses of SSI model identified by EK and EKI, respectively, in conjunction with the mock observation records. Wide discrepancy can be seen between the results identified by EK and the observed records, whereas the results identified by EKI completely coincide with observed ones. These results indicate that the EKI scheme is capable of identifying the observed records.

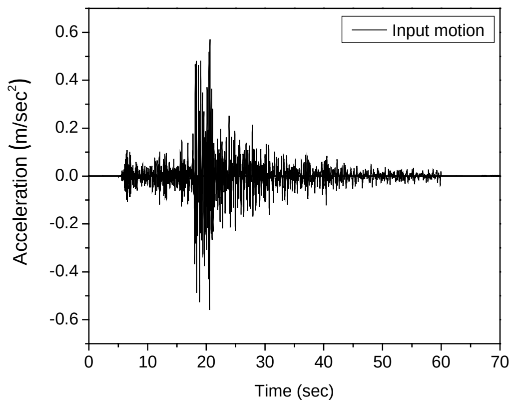

Fig.3 Input motion

Table 2 Parameters for identification

Parameters	considered value
Number of data for an iteration *m*	30
Number of local iteration *n*	0 (EK) 1 (EKI)
Weighting factor *r*	1.1

Figures 6 and 7 show identification results of stiffness and damping parameters, respectively. Both stiffness and damping parameters can be well identified by using EKI scheme with principal part of the observed wave. However, without any local iteration, prescribed stiffness and damping parameters could not be identified.

CONCLUSIONS

This study established a time-domain identification system of soil-structure interaction effects. A soil-structure interaction system was modeled by a single degree of freedom system supported by a sway-rocking foundation model; then the governing equation was expressed by a state equation. In order to stably identify the soil stiffness in the time-domain, an extended Kalman filter with a weighted local iteration algorithm was applied to the soil-structure interaction system. Observation records were artificially created by preliminarily computing the soil-structure interaction responses. The identified responses are identical with the artificially created observed records. Identified stiffness and damping parameters coincide with their prescribed values. With these results, applicability of the time-domain identification system of the soil-structure interaction was confirmed.

ACKNOWLEDGMENT

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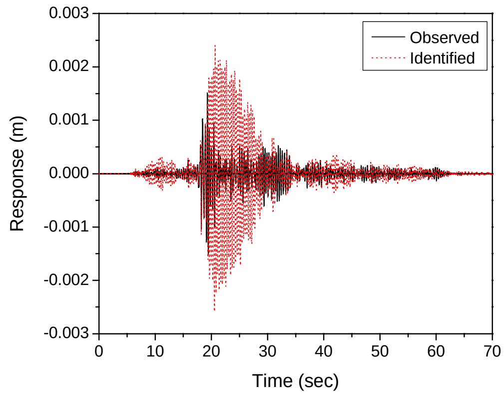
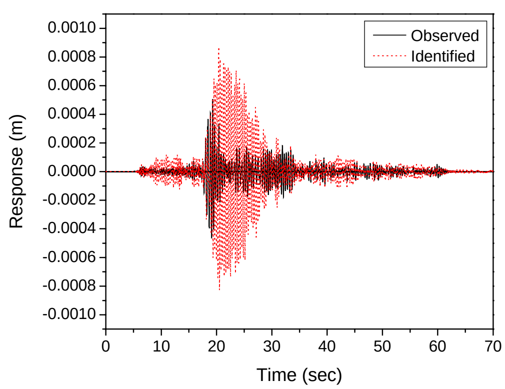
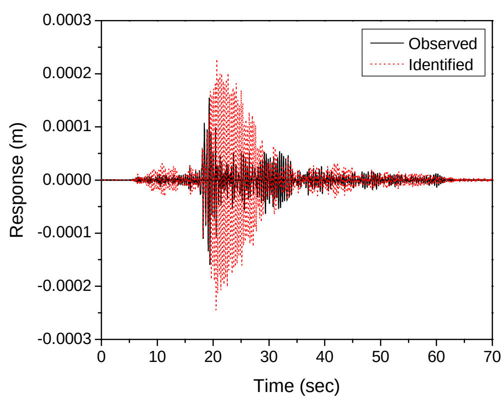


(a) Structure

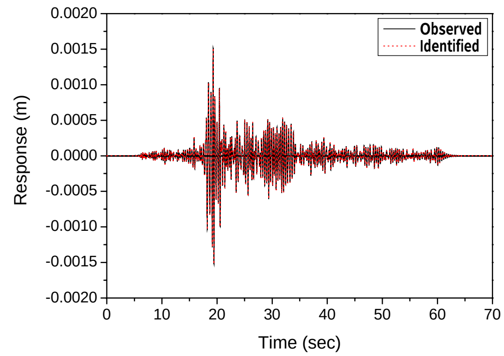


(b) Foundation (sway)

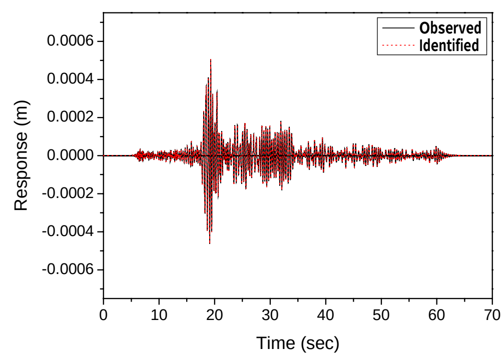


(c) Foundation (rocking)

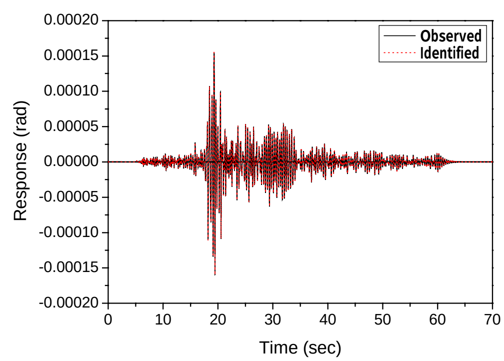
Fig.4 Identified results by EK



(a) Structure

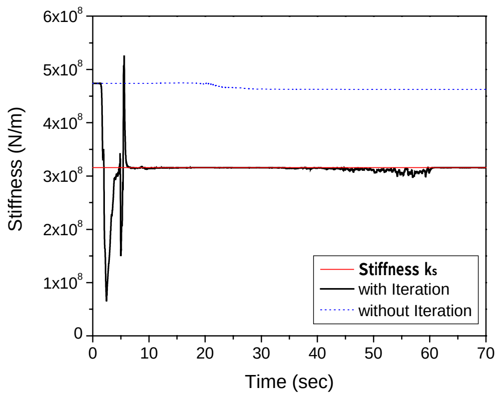


(b) Foundation (sway)

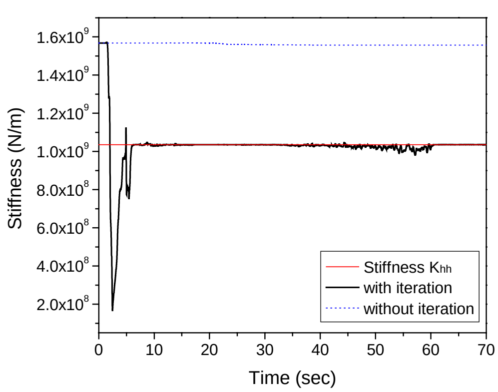


(c) Foundation (rocking)

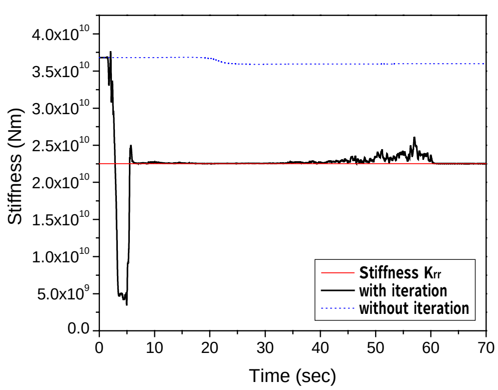
Fig.5 Identified results by EKI



(a) Structure

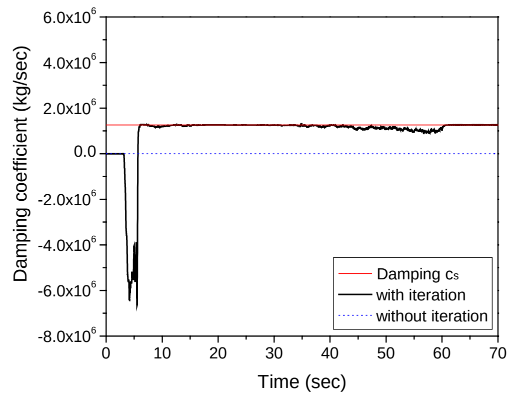


(b) Foundation (sway)

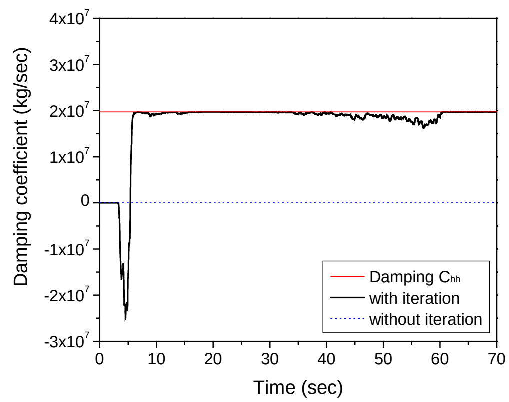


(c) Foundation (rocking)

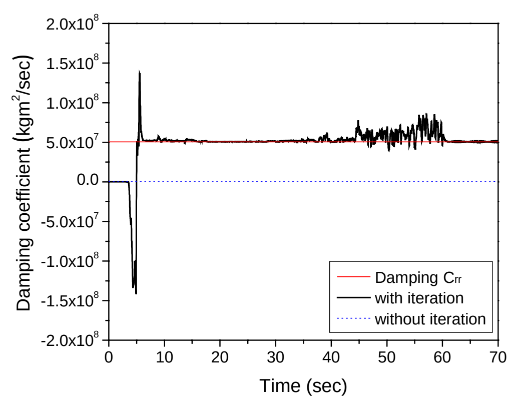
Fig.6 Identification of stiffness



(a) Structure



(b) Foundation (sway)



(c) Foundation (rocking)

Fig.7 Identification of damping