

BIDIRECTIONAL PSEUDODYNAMIC TECHNIQUE FOR TESTING A THREE-STOREY REINFORCED CONCRETE BUILDING

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SUMMARY

Within the project SPEAR, a full-size three-storey non-symmetric RC building has been seismically tested at the ELSA laboratory by means of the PsD method. Since the induced torsion was one of the major concerns about the response of this structure, the tests included bidirectional excitation and allowed for three in-plane DoFs per floor: two displacements plus one rotation. The equation of motion is formulated and integrated in generalised DoFs, which are those displacements and rotation defined at the CM of each floor. At every step of integration the experimental restoring forces are statically transformed into generalised forces and, once the new generalised displacements are computed, the target displacement for the transducers on the structure are obtained by a cinematic transformation. The implemented PsD technique allows for the use of more than three actuators per floor in order improve the load distribution and avoid possible saturation of the actuators. The first series of test on the un-retrofitted structure was successfully performed giving most profitable information about the seismic behaviour of such structure.

INTRODUCTION

Through the hybrid technique of pseudodynamic (PsD) method, the seismic response of large-size specimens can be obtained by means of the on-line combination of experimental restoring forces with analytical inertial and seismic-equivalent forces (Takanashi [1]). Thanks to the use of quasistatic imposed displacements, the accuracy of the control and hence the quality of such test is normally better than for a shaking-table test, especially for heavy and tall specimens.

For PsD testing of building specimens in one direction without torsion, normally two hydraulic actuators acting in that direction are used per floor. A first bi-directional PsD test was successfully performed on a real-size frame in 1997 at the European Laboratory for Structural Assessment (ELSA) by using two actuators acting on each direction at each of its three floors (Molina [2]). For every integration step of the equation of motion, the developed procedure included, firstly, the transformation of measured restoring forces from the four actuator coordinates to the three generalised degrees of freedom (DoFs) of every floor, secondly, the finite-difference prediction of the new displacement and, thirdly, its transformation into actuator displacements. Those new target displacements were then imposed to the structure through a

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ramp phase that might last several seconds. This technique may be called the step wise or classic PsD method.

However, after that test, a major improvement has been attained in the PsD tests performed at ELSA in the last years by the introduction of the continuous PsD method for mono-directional tests (Magonette [3]). The main difference between the step wise method and the continuous one is that the latter requires usually only some two milliseconds of real time per step and the displacement ramp phase is not required anymore. In this case the number of executed steps could be much higher but the duration of the test will be shorter and the experimental errors are diminished at the same time (Molina [4]).

Thanks to the recent introduction of new computing and interfacing hardware at ELSA, the bidirectional continuous PsD testing technique might become possible in the near future. However, for the tests described in this paper, that hardware had just been adopted and the applied testing technique was still the classic one.

The testing technique described here was applied for obtaining the experimental bidirectional response of a reinforced concrete (RC) three-storey building within project SPEAR. The project SPEAR (Seismic Performance Assessment and Rehabilitation) focuses on existing buildings because of the current economic and social relevance of their seismic performance. The experimental campaign on this structure foresees rounds of seismic tests for the original not retrofitted structure as well as for two retrofitted configurations. This paper is devoted to the description of the used testing technique and set-up; more information about the project, the specimen and the test results are given by Negro [5] and Mola [6].

TESTING TECHNIQUE

This section of the paper covers all the aspects of the applied PsD testing technique, starting with the formulated general equation of motion (including three DoFs per floor) and time integration algorithm and following with the particular force and displacements coordinate transformations for the adopted control system and strategy. Details on the implemented step-by-step testing procedure and characteristics of the testing hardware and software are also given. Most of the fundamentals of this testing technique developed at ELSA were already described by Molina [2].

Equation of motion

The mass of the building will be assumed to be concentrated at the rigid floors and for every floor the equation of motion in the horizontal plane will be

$$\mathbf{ma} + \mathbf{r} = -\mathbf{mja}_{g} \tag{1}$$

where the floor mass matrix

$$\mathbf{m} = \begin{bmatrix} \mathbf{m} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$
(2)

contains the floor mass m and moment of inertia I at the Centre of Mass (CM) and

$$\mathbf{a} = \begin{bmatrix} a_x & a_y & a_\theta \end{bmatrix}^{\mathbf{T}} = \frac{d^2}{dt^2} \mathbf{d} = \frac{d^2}{dt^2} \begin{bmatrix} d_x & d_y & d_\theta \end{bmatrix}^{\mathbf{T}}$$
(3)

is the vector of relative accelerations, which is the second derivative with respect to time of the floor relative displacements d_x , d_y and rotation d_θ at the CM (Figure 1).

There also, the vector of conjugated generalised restoring forces

$$\mathbf{r} = \begin{bmatrix} r_x & r_y & r_\theta \end{bmatrix}^{\mathbf{T}}$$
(4)

contains the resultant forces and moment at the CM which are a function of the deformations of the building and the vector

$$\mathbf{a}_{g} = \begin{bmatrix} a_{gx} & a_{gy} \end{bmatrix}^{\mathrm{T}}$$
(5)

appearing at the right-hand side of equation (1) contains the ground accelerations in directions x and y and

$$\mathbf{j} = \begin{bmatrix} 1 & 0\\ 0 & 1\\ 0 & 0 \end{bmatrix} \tag{6}$$

is the influence matrix.



Figure 1. Floor axis, actuators and displacement transducers

Thus, the system of equations of motion for the multi-storey structure can be written as

$$\mathbf{M}\mathbf{A} + \mathbf{R} = -\mathbf{M}\mathbf{J}\mathbf{a}_{\sigma} \tag{7}$$

with the matrices

$$\mathbf{M} = \begin{bmatrix} \ddots & 0 \\ & \mathbf{m} \\ 0 & \ddots \end{bmatrix}; \ \mathbf{A} = \begin{bmatrix} \vdots \\ \mathbf{a} \\ \vdots \end{bmatrix}; \ \mathbf{R} = \begin{bmatrix} \vdots \\ \mathbf{r} \\ \vdots \end{bmatrix}; \ \mathbf{J} = \begin{bmatrix} \vdots \\ \mathbf{j} \\ \vdots \end{bmatrix}$$
(8)

taking into account the different floors of the building.

Time integration algorithm

Equation (7) is now generalised as

$$\mathbf{M}\mathbf{A} + \mathbf{C}\mathbf{V} + \mathbf{R} = \mathbf{F} \tag{9}$$

where C is a viscous damping matrix, V is the vector of relative velocities and F is a general external force vector including either seismic equivalent forces or directly applied forces. For classic materials

such as steel or concrete, most of the damping is hysteretic, which is included in the velocity-independent restoring forces \mathbf{R} , so that \mathbf{C} matrix is taken null for the PsD model.

Two different time integration algorithms are used at ELSA for PsD testing (Molina [2]):

- the Explicit Newmark algorithm (equivalent to central differences)
- the α Operator Splitting algorithm

The second method requires and estimation of the stiffness matrix of the structure which may affect the results if not properly chosen or updated. For this reason, the Explicit Newmark algorithm is the preferred one whenever a time increment Δt may be chosen such that the algorithm is stable, i.e.,

$$\Delta t \le T_{\min} / \pi \tag{10}$$

where T_{\min} is the minimum period of the structure, and the test duration is feasible.

The Explicit Newmark algorithm was adopted for the current test and will be described here. It is based on the Newmark scheme, which can be stated as:

$$\mathbf{M}\mathbf{A}_{n+1} + \mathbf{C}\mathbf{V}_{n+1} + \mathbf{R}_{n+1} = \mathbf{F}_{n+1}$$

$$\mathbf{D}_{n+1} = \mathbf{D}_n + \Delta t \mathbf{V}_n + \Delta t^2 \left[\left(\frac{1}{2} - \beta \right) \mathbf{A}_n + \beta \mathbf{A}_{n+1} \right]$$

$$\mathbf{V}_{n+1} = \mathbf{V}_n + \Delta t \left[(1 - \gamma) \mathbf{A}_n + \gamma \mathbf{A}_{n+1} \right]$$
(11)

with

$$\beta = 0; \ \gamma = \frac{1}{2} \tag{12}$$

Contrarily to an integration based on a finite element model of the structure, in a PsD test, the restoring forces at every time step are not computed but directly measured by imposing the corresponding displacements on the specimen:

$$\mathbf{R}_{n+1} = \mathbf{R}(\mathbf{D}_{n+1}) \tag{13}$$

Starting from known initial values $\mathbf{D}_1, \mathbf{V}_1, \mathbf{A}_1$, at time step n=1 (t=0), the successive computation stages of the method at every time step are:

Stage 1) Compute at instant n+1 the values of displacements \mathbf{D}_{n+1} using equation (11).

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Stage 2) Impose to the structure the new displacement \mathbf{D}_{n+1} and measure the associated restoring force (13).

Stage 3) Compute the acceleration at new time n+1 from the equilibrium equation

$$\mathbf{A}_{n+1} = \left[\mathbf{M} + \Delta t/2\mathbf{C}\right]^{-1} \left[\mathbf{F}_{n+1} - \mathbf{R}_{n+1} - \mathbf{C}\left(\mathbf{V}_n + \Delta t/2\mathbf{A}_n\right)\right]$$
(14)

Stage 4) Increment the step counter $n \leftarrow n+1$ and go back to stage 1 until the final time is reached.

Cinematic transformation from generalised to transducer displacements

At every step of the PsD test, the computed generalised displacement of the floor is imposed through the actuators with feedback from displacement transducers. Thus, in order to determine the target displacements at transducer level, a geometric transformation is needed. A minimum number of 3 high-resolution linear displacement transducers are attached to each floor for control purposes.

As shown in Figure 2, each one of these control displacement transducers consists of a slider G1-G2 on which a body M translates and gives a measure of its relative position on the slider. This slider is attached to a fixed reference frame while the mobile body M is connected to the measuring point D on the structure through a pin-jointed rod.



Figure 2. Control displacement transducer

For every floor, starting from a known set of generalised displacements d (3), the position of the floor CM is updated as

$$\mathbf{S}_{\mathrm{C}} = \mathbf{S}_{\mathrm{C}}^{\mathbf{0}} + \mathbf{d} \tag{15}$$

where

$$\mathbf{S}_{\mathbf{C}} = \begin{bmatrix} x_C & y_C & \boldsymbol{\theta}_C \end{bmatrix}^{\mathbf{T}}$$
(16)

are its global coordinates and $\mathbf{S}_{\mathbf{C}}^{\mathbf{0}}$ are their reference value for zero displacement. Then, the position of the measuring point D is likewise updated as

$$\mathbf{S}_{\mathbf{D}} = \begin{bmatrix} x_D \\ y_D \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \mathbf{T}_{\theta_C} \hat{\mathbf{S}}_{\mathbf{D}}$$
(17)

where

$$\hat{\mathbf{S}}_{\mathbf{D}} = \begin{bmatrix} \hat{x}_D \\ \hat{y}_D \end{bmatrix}$$
(18)

are the coordinates of point D in a reference system local to the floor, centred at its CM, and

$$\mathbf{T}_{\theta_{\mathrm{C}}} = \begin{bmatrix} \cos\theta_{\mathrm{C}} & -\sin\theta_{\mathrm{C}} \\ \sin\theta_{\mathrm{C}} & \cos\theta_{\mathrm{C}} \end{bmatrix}$$
(19)

is a rotation matrix. Assuming an infinitely rigid floor, the local coordinates (18) are constant. Now, in order to express the position of body M (Figure 2) of the transducer along its slider G1-G2, the relative position of the measuring point D with respect to the slider origin G1 is computed as

$$\widetilde{\mathbf{S}}_{\mathbf{D}} = \overline{G_{\mathbf{I}}D} = \mathbf{S}_{\mathbf{D}} - \mathbf{S}_{\mathbf{G}\mathbf{I}}$$
(20)

and the projection of vector (20) along the slider is computed as

$$q = \overline{G_1 D'} = \widetilde{\mathbf{S}}_{\mathbf{D}}^{T} \mathbf{e}_{\mathbf{G}}$$
(21)

where $\mathbf{e}_{\mathbf{G}}$ is the constant unit vector defined in the direction of positive measurement along the slider. The position of body M along the slider is then given by

$$\overline{\overline{S}}_{D} = \overline{G_{1}M} = \overline{G_{1}D'} - \overline{MD'} = q - \operatorname{sign}(q)\sqrt{l_{D}^{2} - \overline{DD'}^{2}} = q - \operatorname{sign}(q)\sqrt{l_{D}^{2} - \widetilde{\mathbf{S}}_{\mathbf{D}}^{T}\widetilde{\mathbf{S}}_{\mathbf{D}} + q^{2}}$$
(22)

being l_D the constant length of the rod. Finally, the corresponding measure at the transducer is

$$d_D = \overline{\overline{S}}_D - \overline{\overline{S}}_D^0 \tag{23}$$

where \overline{S}_D^0 is a reference position for zero displacement.

Cinematic transformation from measured to generalised displacements

The actual displacements and rotation at the CM will differ from the desired ones and this is due to geometry and control errors and to the flexibility of the floor. In order to get an estimate of the actual generalised displacements, the measures given by all the control displacement transducers may be exploited. Since the obtained relations between generalised and transducer displacements are non-linear and cannot easily be inverted in closed form, the solution is achieved through a Newton-Raphson iteration procedure starting from a first estimate of the generalised displacements, e.g.,

$$\mathbf{d}^{EST} = \mathbf{0} \tag{24}$$

Then, the non-linear equations giving the associated transducer displacements $\mathbf{d}_{\mathbf{D}}$ are applied by substituting \mathbf{d} by \mathbf{d}^{EST} into equation (15) and applying formulae (17) to (23)

$$\mathbf{d}_{\mathbf{D}}^{EST} = \mathbf{d}_{\mathbf{D}} \left(\mathbf{d}^{EST} \right) \tag{25}$$

Now, the difference between this estimate and the measured transducer displacements $\mathbf{d}_{\mathbf{D}}$ is used to provide an new estimate of the generalised displacements

$$\mathbf{d}^{EST} \leftarrow \mathbf{J}^{-1} \left(\mathbf{d}_{\mathbf{D}} - \mathbf{d}_{\mathbf{D}}^{EST} \right) + \mathbf{d}^{EST}$$
⁽²⁶⁾

where \mathbf{J} is the Jacobian matrix

$$\mathbf{J} = \frac{\partial \mathbf{d}_{\mathbf{D}}}{\partial \mathbf{d}} \Big|_{\mathbf{d}^{EST}}$$
(27)

computed at \mathbf{d}^{EST} .

Having into account expression (22), every single row of J can be obtained as

$$\frac{\partial d_D}{\partial \mathbf{d}} = \left[\frac{\partial d_D}{\partial \mathbf{S}_{\mathbf{D}}}\right] \left[\frac{\partial \mathbf{S}_{\mathbf{D}}}{\partial \mathbf{d}}\right] = \left[\frac{\partial \overline{GD'}}{\partial \mathbf{S}_{\mathbf{D}}} - \frac{\partial \overline{MD'}}{\partial \mathbf{S}_{\mathbf{D}}}\right] \left[\frac{\partial \mathbf{S}_{\mathbf{D}}}{\partial \mathbf{d}}\right] = \left[\mathbf{e}_{\mathbf{G}}^{\mathrm{T}} - \frac{q\mathbf{e}_{\mathbf{G}}^{\mathrm{T}} - \mathbf{S}_{\mathbf{D}}^{\mathrm{T}}}{\overline{MD'}}\right] \left[\frac{1}{0} \quad 0 \quad 1 \quad \frac{\partial \mathbf{T}_{\theta_{\mathrm{C}}}}{\partial \theta_{\mathrm{C}}} \hat{\mathbf{S}}_{\mathbf{D}}\right]$$
(28)

If the number of control transducers on the floor exceeds 3, equation (25) must be solved in a least squares sense, in which case the inverse of \mathbf{J} in (26) is replaced by the pseudo-inverse

$$psinv(\mathbf{J}) = \left[\mathbf{J}^{\mathrm{T}}\mathbf{J}\right]^{-1}\mathbf{J}^{\mathrm{T}}$$
⁽²⁹⁾

Then, steps (25) and (26) are iteratively repeated until a specified tolerance is reached.

Static transformation from actuator to generalised forces

In the PsD, the restoring forces are experimentally obtained from the specimen. Once the prescribed displacements are imposed, the acting axial force at every actuator is measured by its load cell. However, in order to express these forces as resultant generalised forces at the CoM of the floor, a static transformation is needed. Since the ends of the actuator are pin-jointed, it is assumed that it acts with a purely linear force along the line PR (Figure 3) connecting the ends of the actuator. Starting from the load-

cell measure r_P of this force and assuming that the current position of the floor CoM is known, the global position of the loading point P is obtained as

$$\mathbf{S}_{\mathbf{P}} = \begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_C \\ y_C \end{bmatrix} + \mathbf{T}_{\theta_C} \hat{\mathbf{S}}_{\mathbf{P}}$$
(30)

where $\mathbf{T}_{\theta_{c}}$ is given by equation (19). Similarly to expression (18), $\hat{\mathbf{S}}_{\mathbf{P}}$ contains the coordinates of point P in the local reference system to the floor.

Then, the global components of the piston force are computed as

$$\mathbf{p}_{\mathbf{P}} = r_{\mathbf{P}} \mathbf{e}_{\mathbf{P}} \tag{31}$$

where $\mathbf{e}_{\mathbf{p}}$ is a unit vector in the direction of \overrightarrow{PR} . Now, the floor generalised restoring forces (4) are obtained by summing up the effects of all pistons acting on the floor

$$\mathbf{r} = \sum_{\mathbf{P}} \mathbf{T}_{\mathbf{P}} \mathbf{p}_{\mathbf{P}}$$
(32)

(33)

where



Figure 3. Force applied by the actuator

Optimal distribution of piston loads

When using a number of actuators larger the number of DoFs at a floor, it is convenient to maintain an acceptable distribution of loads among all the pistons. This can be done by implementing an algorithm capable of optimising the distribution of piston loads for a known set of generalised floor loads. Even distribution of forces is also desirable because it leads to a better approximation of the distributed inertial forces of a real dynamic event.

For our purposes, piston forces will be defined as 'optimal' if statically equivalent to specified generalised loads and while minimising a penalty function that becomes infinite when any piston force reaches its working limit. The adopted expression for the penalty function is

$$f(\mathbf{r}_{\mathbf{P}}) = \sum_{P} \frac{1}{M_{P}^{2} - r_{P}^{2}}$$
(34)

where M_p is the working limit of the absolute value of the piston load r_p . Clearly, minimising the function (34) will guarantee that all piston loads are kept far from their limit. Using expressions (31) and (32), the conditions of static equivalence of the piston loads with the known set of generalised loads give a constraint on the minimisation problem which can be written in the vector form

$$\mathbf{g}(\mathbf{r}_{\mathbf{p}}) = \sum_{p} \mathbf{T}_{p} \mathbf{e}_{p} \mathbf{r}_{p} - \mathbf{r} = \mathbf{0}$$
(35)

where \mathbf{r} is the known set of generalised forces (4). The constrained minimisation problem is then

$$\begin{cases} f(\mathbf{r}_{\mathbf{p}}) & minimum \\ \mathbf{g}(\mathbf{r}_{\mathbf{p}}) = \mathbf{0} \end{cases}$$
(36)

By introducing a set of Lagrange multipliers associated to the constraint equations, it can be restated in unconstrained form, leading now to the stationarity condition

$$\frac{\partial h}{\partial \mathbf{x}} = \mathbf{0} \tag{37}$$

of the augmented functional

$$h(\mathbf{x}) = f(\mathbf{r}_{\mathbf{p}}) + \mathbf{g}^{\mathrm{T}}(\mathbf{r}_{\mathbf{p}})\boldsymbol{\lambda}$$
(38)

The solution vector

$$\mathbf{x} = \begin{bmatrix} \mathbf{r}_{\mathbf{p}} \\ \lambda \end{bmatrix}$$
(39)

contains as unknowns the piston loads and Lagrange multipliers

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}^{\mathrm{T}}$$
(40)

The system (37) may also be written as

$$\mathbf{A}(\mathbf{x}) = \mathbf{0} \tag{41}$$

where

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \begin{bmatrix} \vdots \\ 2r_p \\ (M_p^2 - r_p^2)^2 \\ \vdots \end{bmatrix} + \mathbf{R}^{\mathrm{T}} \boldsymbol{\lambda} \\ \mathbf{R}\mathbf{r}_{\mathbf{P}} - \mathbf{r} \end{bmatrix}$$
(42)

and, from expression(35),

$$\mathbf{R} = \begin{bmatrix} \dots & \mathbf{T}_{\mathbf{P}} \mathbf{e}_{\mathbf{P}} & \dots \end{bmatrix}$$
(43)

Since the obtained equations (41) are not linear, their solution may be afforded by the Newton-Raphson method in a similar way as it was done for obtaining the generalised displacements. Thus, starting from an initial estimate \mathbf{x}^{EST} of the unknowns, every iteration would consist in computing

$$\mathbf{B}^{\mathrm{EST}} = \mathbf{A} \left(\mathbf{x}^{\mathrm{EST}} \right) \tag{44}$$

from formula (42) and updating the estimated optimum parameters by doing $\mathbf{x}^{\text{EST}} \leftarrow \mathbf{I}^{-1} (\mathbf{0} - \mathbf{R}^{\text{EST}}) + \mathbf{v}^{\text{EST}}$

$$S^{T} \leftarrow \mathbf{J}^{-1} \left(\mathbf{0} - \mathbf{B}^{\text{EST}} \right) + \mathbf{x}^{\text{EST}}$$
 (45)

where the Jacobian matrix is now

$$\mathbf{J} = \frac{\partial \mathbf{A}}{\partial \mathbf{x}}\Big|_{\mathbf{x}^{\text{EST}}} = \begin{bmatrix} \ddots & \mathbf{0} & \\ & \frac{2M_{p}^{2} + 6r_{p}^{2}}{\left(M_{p}^{2} - r_{p}^{2}\right)^{3}} & \mathbf{R}^{\text{T}} \\ \mathbf{0} & & \ddots & \\ & \mathbf{R} & & \mathbf{0} \end{bmatrix}$$
(46)

Piston internal displacements

A further step for indirectly imposing a desired load at the redundant pistons mentioned at the preceding subsection is the calculation of the displacement at their internal transducer. This is because using that internal displacement as feedback leads to a stable control.

Every actuator has an internal displacement transducer whose measure will be called d_P and represents the excursion of the piston. This displacement is a function of the floor generalised displacements **d**. Once the position of the CM (16) is known and the position $\mathbf{S}_{\mathbf{P}}$ of the point P (Figure 3) of attachment of the piston to the floor is calculated by expression (30), the current length of the piston is

$$\overline{\overline{S}}_{P} = \sqrt{\left(\mathbf{S}_{\mathbf{P}} - \mathbf{S}_{\mathbf{PR}}\right)^{T} \left(\mathbf{S}_{\mathbf{P}} - \mathbf{S}_{\mathbf{PR}}\right)}$$
(47)

and the internal displacement is

$$d_P = \overline{\overline{S}}_P^{\ 0} - \overline{\overline{S}}_P \tag{48}$$

where $\overline{\overline{S}}_{P}^{0}$ is the reference length and a positive sign of d_{P} means a reduction of the actuator length.

Marching procedure

The application of the described time integration method to the 3-DoF-per-floor model was done by the following experimental step-by-step procedure. The algorithm starts from known initial values $\mathbf{D}_1, \mathbf{V}_1, \mathbf{A}_1$ and the operations are organised in the following stages:

Stage 0) Let n = 0.

Stage 1) Transform the generalised displacements into target displacements $\mathbf{d}_{\mathbf{D}}$ at the control transducers

$$\left(\mathbf{d}_{\mathbf{D}}\right)_{n+1}^{TARGET} = \mathbf{d}_{\mathbf{D}}\left(\mathbf{D}_{n+1}\right)$$
(49)

computed from expression (23).

Stage 2) Send these target displacements (49) to the controllers which impose them to the specimen by performing a ramp from the previous position.

Stage 4) Measure the new displacements $(\mathbf{d}_{\mathbf{D}})_{n+1}^{MEAS}$ and restoring loads $(\mathbf{r}_{\mathbf{P}})_{n+1}$ at the controllers, $\mathbf{r}_{\mathbf{P}}$ being the forces applied by the pistons.

Stage 5) From the measured control displacements, estimate the generalised displacements on each floor

$$\mathbf{D}_{n+1}^{MEAS} = \mathbf{D}\left(\left(\mathbf{d}_{\mathbf{D}}\right)_{n+1}^{MEAS}\right)$$
(50)

using the iterative least-squares solution (23).

Stage 6) From the measured piston forces, compute the new generalised restoring forces

$$\mathbf{R}_{n+1} = \mathbf{R}\left(\left(\mathbf{r}_{\mathbf{P}}\right)_{n+1}, \mathbf{D}_{n+1}^{MEAS}\right)$$
(51)

using the transformation formulae (32).

Stage 7) If n > 1, compute the new accelerations

$$\mathbf{A}_{n+1} = \mathbf{A}_{n+1} \left(\mathbf{F}_{n+1}, \mathbf{R}_{n+1}, \mathbf{V}_n, \mathbf{A}_n \right)$$
(52)

using expression (14).

Stage 3) Let $n \leftarrow n+1$.

Stage 8) Predict the generalised displacement at the next time increment

$$\mathbf{D}_{n+1} = \mathbf{D}_{n+1}(\mathbf{D}_n, \mathbf{V}_n, \mathbf{A}_n)$$
(53)

using the finite-difference approximation (11) of the integration algorithm. Stage 9) Go back to stage 1) until reaching the final time.

Control strategy

Let us assume that PID controllers are used and that, at each floor, three actuators are controlled using as feedback one displacement transducer on the structure. In that case, there are for us, in principle, three options in the choice of the feedback transducer to control the remaining redundant actuators: a) the aligned displacement transducer on the structure (as for the other three actuators), b) the actuator load cell force or, c) the actuator internal displacement transducer.

The first option is not practicable in general because the control system becomes unstable even for very low controller gains because of the high stiffness of the floor slab in comparison with the one of the pistons. The second option may lead to a stable control system, but, usually, force control strategies result in poor accuracy. It could be acceptable for a cyclic test, but not for a PsD test in which relatively small control errors may result in a large distortion of the integrated response (Shing [7], Molina [4]).

The third option may give at the same time an accurate and stable control system thanks of being a displacement control strategy, as in the first option, but associated to a transducer which sees a more flexible subsystem. In fact, the displacements measured by the internal transducer of the actuator are considerably larger than the those coming from the flexibility of the floor slabs since they comprise also the deformation of the reaction wall and actuator attachments. Thus, we have adopted the actuator internal transducers as feedbacks for the redundant pistons. In practice, the computed target for the redundant pistons is slightly adjusted at every integration step in order to control the distribution of loads on the floor at the same time. So, instead of using expression (49), their target is computed as

$$\left(d_{P}\right)_{n+1}^{TARGET} = d_{P}\left(\mathbf{D}_{n+1}\right) + \left(\overline{d}_{P}\right)_{n+1}$$
(54)

where the first term on the right-hand side is the theoretical elongation of the piston (48) and the second one is a correction introduced in order to modify the force. The latter is updated at every time step in the cumulative form

$$\left(\overline{d}_{P}\right)_{n+1} = \left(\overline{d}_{P}\right)_{n} + \frac{\left(r_{P}^{OPTIMUM} - r_{P}^{MEASURED}\right)_{n}}{K_{P}}$$
(55)

where the term added to the correction is the difference between the computed optimum force (45) of the piston and the measured one at the former step, divided by a stiffness parameter K_p empirically selected. In general terms, the smaller the parameter, the faster is the convergence of the force to the optimum value. However, using too small a value may result in instability, which would make the force to oscillate out of control in very few steps.

Hardware set-up

The servo-control units used were MOOG actuators with ± 0.5 m stroke and load capacity of 0.5 MN. The control displacement transducers on the structure were optical HEIDENHAIN sensors with a stroke of ± 0.5 m and a 2 µm resolution. Every actuator was equipped with a load cell, a TEMPOSONICS internal displacement transducer and a PID controller based on a MSM486DX CPU operating the digital control loop at a sampling period of 2 ms. The four controllers for the four pistons of each floor are governed by a powerful master CPU which is able to transmit and receive the controllers signals at real time (every 2 ms)

through a high-speed communication channel based on dual port RAM boards. The three master units for the three floors have a common clock signal for the 2 ms interrupt.



Figure 4. Controller set-up for the PsD tests.

This master controller units are programmed in C and have been designed to be able to apply complex algorithms in which the signals of up to four actuators are combined in real time. The sampling period of 2 ms has shown to be fast enough for stable digital control of structural specimens by hydraulic jacks but could be too short for cases in which the computations for those algorithms may be cumbersome. That means that this hardware may permit the implementation of a PsD test with three DoFs per floor on a real time continuous basis only if the algorithm computations for each floor at every step can be performed in less than 2 ms.

Unfortunately, at the time of performing the PsD tests object of this paper, the described hardware had just arrived at ELSA and the software developments for a bidirectional continuous implementation on it were not ready to guarantee the required speed. For this reason, this PsD test was performed on a classic step-by-step basis. For this classic implementation of the PsD method, all the master units were connected to a Windows workstation by means of ETHERNET. On such workstation, an application developed at ELSA and called STEPTEST runned the PsD test. This application is written in interpreted language (MATLAB) and implements the described model and marching procedure with interactive capabilities for monitoring and controlling the test (Figure 4).

PERFORMED TESTS

Within project SPEAR, a full-size RC building is being tested at ELSA. The tested mock-up is representative of non-seismic construction of the 60-70's in southern European countries, irregularities in plan and detailing included [5]. The structure is a simplification of an actual Greek three-storey building and was designed for gravity loads alone using the Greek design code applied from 1954 to 1995. As seen in Figure 5, it has a doubly non symmetric plan configuration, but it is regular in elevation. It is made of three 2-bay frames spanning from 3 to 6 m in each direction with a storey height of 3 m.



Figure 5. Plan and 3D views of the SPEAR RC building specimen

For practical reasons, the specimen was built outside of the laboratory, starting from a thick RC base slab from which the ground columns were anchored. Afterwards, this base was made roll over PVC rolls (Figure 6) in order to transport the built specimen up to its final position on the strong floor of the laboratory, where it was clamped.

As shown in Figure 1, for the bidirectional testing of the structure, four actuators with four associated control displacement transducers were used. In principle, these actuators were connected to the floor in positions not too close to the structural joints. Additional RC stiffeners were created in the floor slabs (Figure 6) in order to properly distribute the local force applied by the pistons. The clamping of the piston attachments included post-tensioning bars. The piston body was supported by the reaction wall or a supplementary reaction structure while the control displacement transducers were fixed on unloaded reference frames (Figure 7).

Apart from the load cells and control displacement transducers, additional instrumentation was installed on the specimen. Damage was expected mostly on top and bottom of the columns of the first two stories. Accordingly, clinometers were installed at the columns on up to three levels of each one. Additionally, some potentiometer extensimeters were also installed on some localised areas. For the first time at ELSA, photogrammetry techniques were applied to a bidirectional test and, at two locations, a couple of cameras were used in order to record the stereo images and estimate the bidimensional displacements of marked targets.

The CM, total mass and moment of inertia for the PsD equations were obtained from the theoretical mass distribution for the seismic assessment of the building according to Eurocode 8. Since the mass of the model was lower than that, additional water containers (Figure 7) were set on the floors of the specimen in the laboratory in order to have realistic gravity loads in the members. The input signals for the test were semi-artificial records modulated consistently with Eurocode 8. After some preliminary small tests to verify the testing system and tune the control parameters, two big PsD tests were performed for peak ground acceleration of 0.15 and 0.20 g, respectively. For each test, the intensity of the excitation was equal in the horizontal x and y components.

The tests were successfully executed in January 2004 and damaged the structure up to a level that is susceptible for repairing and retrofitting in the next phases of the project. Details on the obtained response and damage are given by Negro et al. [5] and Mola et al. [6]. During the strongest test, the obtained displacements at the third floor were in the order of 100 mm in both directions and 20 mrad of torsion. This level of torsion was higher than the predicted one using regulated methods.



Figure 6. Trasportation of the building specimen inside of ELSA.

The tests were performed with very low experimental errors. For example, the errors produced at the controller loops were lower than 0.1 m and those verified in the generalised displacement at the CM were lower than 0.3 mm. The latter are mostly due to the existing flexibility of the floors since three structural displacements were used for the control while all four of them entered in the estimation of the generalised displacement. An overall way to assess the quality of a PsD in relation to the existing control errors is through the computation of the energy of error (Molina [4]), which amounted some 50 J, in comparison to the total absorbed energy in the structure, which was around 130 kJ for the second test for example. A way to look more in detail at the effects of the control errors on each mode consists in the identification of the eigen frequency and damping from the measured forces and either the measured and the reference displacements (Molina [4]). This has been done for example for the second test and the results for the first three modes at different time instants are shown in Figure 8. The results for the other estimated six modes are not much worse. The conclusion is that the control errors verified during the test have not affected these clue parameters for the response of the structure.

CONCLUSIONS

In the framework of the activity of the SPEAR research project, a round of bi-directional PsD tests was carried out on a full-scale three-storey plan-wise irregular RC frame structure. The description of the applied PsD technique has been the object of this paper.

The technique considers three DoF per floor and includes rigorous geometrically non-linear transformation of coordinates for the actuator forces and displacements transducers attached to the floors. It also allows for using more than three pistons and displacements transducers per floor, in which case it guarantees and even distribution of piston forces.

The performed tests have been quite successful from the point of view of the low experimental errors verified. The obtained results are expected to be of high relevance for the understanding of the torsional behaviour of this kind of structures.



Figure 7. View of the structure ready for the PsD tests.

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A bi-directional pseudodynamic test on a full-scale complete building is a highly innovative and challenging experiment, and whenever doing experimental work, many difficulties can be encountered. In the test described in this paper, difficulties were much more severe than expected: the test, which was a common effort of the whole ELSA staff, was made possible by the expertise, dedication, as well as forbearance and friendship of all staff.



Figure 8. Identified frequencies and damping ratios from PsD results.

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