13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 769

A MODAL APPROACH TO OPTIMALLY PLACE DAMPERS IN FRAMED STRUCTURES

Bruno PALAZZO[1], Luigi PETTI[1], Massimiliano DE IULIIS[1]

SUMMARY

The study presents a new design approach to optimally locate dampers in framed structures. The optimal location and placing of damping devices is investigated by assessing the power balance of structures subjected to seismic actions described by a response spectrum. More specifically, through describing the amount of power dissipated by means of a performance index, the problem of optimization is solved by its maximization through a modal analysis in the state space. The proposed design methodology has been tested by analyzing the seismic response of two structural steel-framed typologies subjected to several recorded seismic excitations. The results demonstrate the effectiveness of the design procedure.

INTRODUCTION

In recent years new performance criteria have been developed to define more detailed and explicit seismic limit states both for structural and non-structural elements. The concept of "performance objectives" arises from the need to characterize, both in the design of new structures and in the retrofit of existing ones, the overall behavior performance for specific levels of seismic excitation (ATC 40 – 1996).

Among the performance levels defined by ATC 40, the "Operational" one requires the complete functionality of the building. Such a strict performance level, required for strategic structures (e.g. hospitals, barracks, etc.) at the occurrence of seismic events characterized by a high return period (e.g. higher than 475 years), could be straightforwardly achieved by using passive control strategies. Amongst other issues, technical improvements and structural behavior in the light of recent seismic events have resulted in a remarkable development of extra-structural dissipation strategy effectiveness. Therefore, this seismic protection strategy may be considered together with the need to control structural damage both in new and existing buildings. The damping devices typically work by means of mechanisms based on the transformation of the seismic input energy in heat (fig. 1) (Ribakov Y. and Reinhorn M., 2003, Sigaher A.N., Constantinou M.C., 2003), e.g. the inelastic response of metallic elements (hysteretic dissipators) (Whittaker et al., 1989, Aiken et al., 1990), friction between surfaces (friction dissipators) (Pall A.S. and Marsh C. 1982), viscoelastic dissipation, typical of copolymer materials (viscoelastic dissipators) (Constantinou and Symans 1993, Soong and Constantinou 1994).

This study proposes a new design methodology to optimally locate energy dissipation devices in framed structures by taking into account the spectral seismic response demand and modal structural behavior. The

[1] Department of Civil Enginnering – University of Salerno (Italy)

optimal damping allocation problem has been significantly investigated in relevant scientific literature (Dolce M. 1994, Vulcano 1993, Ciampi et al. 1992, Paolacci et al. 1998, Zhang and Soong 1992, Takewaki 1997, Lopez Garcia 2001, Singh and Moreschi 2001, Wei Liu 2002).

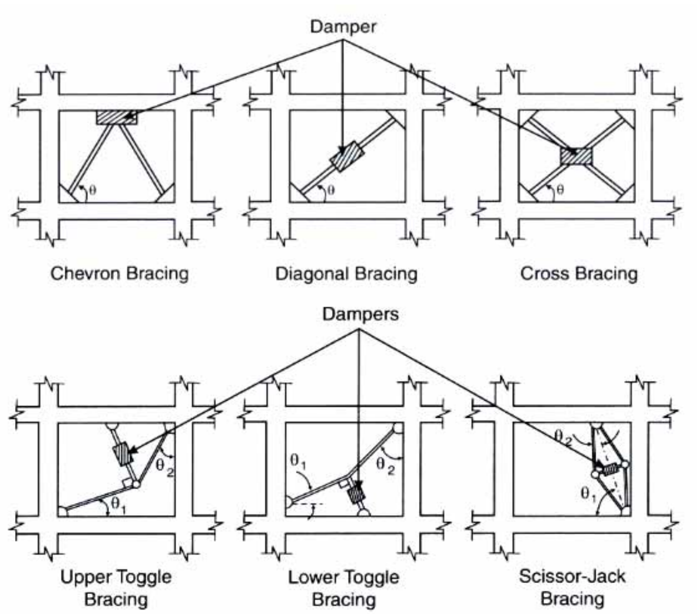

Figure 1: Convenient placement of dissipation devices in framed structures

Optimal damping device allocation is herein investigated by assessing the power balance of structures subjected to seismic actions described by a response spectrum. In particular, the dissipated power, through a modal analysis in the state space, is evaluated by varying the configuration of the damping devices. The greater the dissipated power the better the location of the devices.
Such a methodology has been tested by comparatively evaluating the seismic responses of two framed steel structures, with different configurations of damping devices and subjected to a set of recorded seismic excitations.

DISSIPATION DEVICES OPTIMAL DISTRIBUTION FORMULATION PROBLEM

Let us consider an equivalent linear multi-degree of freedom system (MDOF) described by:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{I}\ddot{u}_g \tag{1}$$

where $\mathbf{M}$, C and $\mathbf{K}$ respectively represent the mass, overall damping and stiffness matrices; $\mathbf{x}$ the base relative displacement vector; $\mathbf{I}$ the unitary vector and, finally, $\ddot{u}_g$ the seismic input acceleration. Note that C and $\mathbf{K}$ are, in this case, diagonal matrices.
By considering the space state vector $\mathbf{X} = [\mathbf{x} \quad \dot{\mathbf{x}}]^T$, equation 1 can be rewritten as:

$$\dot{\mathbf{X}} = \begin{Bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{Bmatrix} = \begin{bmatrix} \mathbf{O}_{NxN} & \mathbf{I}_{NxN} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{Bmatrix} + \begin{Bmatrix} \mathbf{O}_{Nx1} \\ \mathbf{I} \end{Bmatrix} \ddot{u}_g = \mathbf{A}\mathbf{X} + \mathbf{B_g}\ddot{u}_g \tag{2}$$

where $\mathbf{O}_{NxN}$ and $\mathbf{I}_{NxN}$, respectively represent the $N \times N$ zero and identity matrices and $\mathbf{O}_{Nx1}$ the zero vector of length *N*.
System dynamic behavior can be assessed through knowledge of eigenvalues λ_i and eigenvectors $\mathbf{\Phi}_i$ of matrix $\mathbf{A}$ (Perko L., 1993, Chopra A.K., 1995) which lead to:

$$\mathbf{X} = \sum_{i=1}^{N} \left(\mathbf{\Phi}_i p_i(t) + \overline{\mathbf{\Phi}}_i \overline{p}_i(t) \right) \tag{3}$$

where $p_i(t)$ represents the i-th modal coordinate, $\overline{p}_i(t)$ its complex conjugate.
By considering that the last N components of the space state vector $\mathbf{X} = [\mathbf{x} \quad \dot{\mathbf{x}}]^T$ represent relative velocity and defining the matrix $\mathbf{C}^*$:

$$\mathbf{C}^* = \begin{bmatrix} \mathbf{O}_{NxN} & \mathbf{O}_{NxN} \\ \mathbf{O}_{NxN} & \mathbf{C} \end{bmatrix}$$

the dissipated power by the overall system is given by:

$$P(t) = \mathbf{X}^T \mathbf{C}^* \mathbf{X} \tag{4}$$

being $\mathbf{C}$ a diagonal matrix, equation 4 can be rewritten as:

$$P(t) = \sum_{l=N+1}^{2N} c^*_{l,l} \left[\sum_{i=1}^{N} X_i^l \right]^2 = \sum_{l=N+1}^{2N} c^*_{l,l} \left[\sum_{i=1}^{N} \left(\Phi_i^l p_i(t) + \overline{\Phi}_i^l \overline{p}_i(t) \right) \right]^2 \tag{5}$$

where $c^*_{l,l}$ represent the *l-th* components of the matrix $\mathbf{C}^*$ and X_i^l the l-th components of the i-th modal form.
In the case of seismic demand described by a response spectrum, the maximum dissipated power can be estimated by means of CQC (Complete Quadratic Combination - Wilson, Der Kiureghian and Bayo, 1981) composition rule:

$$P_{\max} = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij} P_{i,\max} \cdot P_{j,\max}} \tag{6}$$

where $P_{i,max}$ and $P_{j,max}$ are respectively the maximum value of the dissipated power considering the contribution of the *i-th* and *j-th* modal form. The modal forms correlation coefficient ρ_{ij} can be expressed as:

$$\rho_{ij} = \frac{8\sqrt{\xi_i \xi_j}(\xi_i + \beta_{ij}\xi_j)\beta_{ij}^{3/2}}{(1-\beta_{ij}^2)^2 + 4\xi_i\xi_j\beta_{ij}(1+\beta_{ij}^2) + 4(\xi_i^2 + \xi_j^2)\beta_{ij}^2} \tag{7}$$

ξ_i, ξ_j, $\beta_{ij} = \omega_i / \omega_j$ being respectively the damping ratio coefficients and the natural frequency ratio of the modal forms *i* and *j*.
Therefore, a measure of the maximum dissipated power, for a fixed disposition of the additional dissipation devices, is given by:

$$P_{\max} = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{ij}\left(\sum_{l=N+1}^{2N} c_{l,l}^{*}(X_{i,\max}^{l})^{2}\right)\left(\sum_{l=N+1}^{2N} c_{l,l}^{*}(X_{j,\max}^{l})^{2}\right)} \tag{8}$$

Considering that the response of the system (2), using (3), can be rewritten as:

$$X_i^l(t) = 2\left[\left(re(\Phi_i^l)\cdot re(p_i(t)) - im(\Phi_i^l)\cdot im(p_i(t))\right)\right] \tag{9}$$

being $re(p_i)_{\max} = re(g_i)\cdot S_d$, $im(p_i)_{\max} = im(g_i)\cdot S_d$ and g_i the participation factor of the *i-th* modal form, we obtain:

$$X_{i,\max}^l = 2S_d(\omega_i)\left[re(\Phi_i^l)\cdot re(g_i) - im(\Phi_i^l)\cdot im(g_i)\right] \tag{10}$$

By combining (10) and (8), a measure of the maximum power is provided in consideration of additional damping, mode correlation factors, values of the spectra and participation coefficients of the single modal forms.

$$\begin{aligned} I_P = 4sqrt[&\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{ij}S_v^2(\omega_i,\xi_i)S_v^2(\omega_j,\xi_j)\left(\sum_{l=N+1}^{2N} c_{l,l}^{*}(re(\Phi_i^l)\cdot re(g_i) - im(\Phi_i^l)\cdot im(g_i))^2\right) \\ &\left(\sum_{l=N+1}^{2N} c_{l,l}^{*}(re(\Phi_j^l)\cdot re(g_j) - im(\Phi_j^l)\cdot im(g_j))^2\right)] \end{aligned} \tag{11}$$

The objective of passive control methodology is to maximize the maximum dissipated power through an optimal allocation of a fixed amount of additional damping to the system in accord to the following constraints:

$$\begin{cases} \sum_{l=1}^{n} c_{l,l}^{*} \leq c_{tot} \\ 0 \leq c_{l,l}^{*} \leq \bar{c}_{l,l} \qquad l = N+1, N+2, \ldots, 2N \end{cases} \tag{12}$$

Equations 12 represent the economic (c_{tot} = maximum value of the available dissipation resource) and technological constraints ($\bar{c}_{l,l}$ = maximum value of the dissipation resource for the l-th level) in an implicit form. The solution to the problem can be found by a non-linear programming evolutive algorithm (Palazzo, Petti, De Iuliis, 2001).

NUMERICAL EXPERIMENTATION

Proposed design methodology has been tested on two different framed steel structures: the first one "*structure A*" designed according to Eurocode 8 (ENV 1998-1-1) (fig. 2, tab. 1); the second "*structure B*" having as its reference model the one described in "Next generation Benchmark Control Problem for Seismically Excited Buildings" (Spencer B.F. - Christenson R.E. - Dyke S.J. 1998), (fig. 3, tab. 2) with modified seismic masses. Such a modification has been provided to investigate seismic response in the case of recorded seismic excitations characterized by high energetic content on periods *T=2-3 sec*.

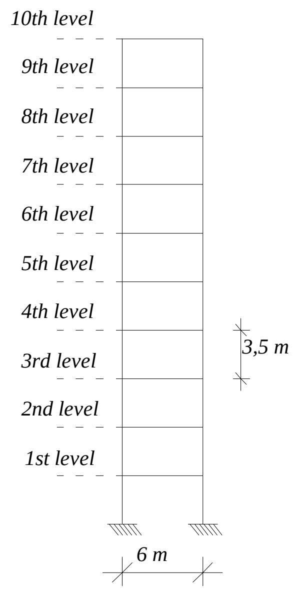

Figure 2: Model of structure A

Structure A

Beams	
Level 1-4	*IPE 600-O*
Level 5-10	*IPE 550-O*
Columns	
Level 1-2	*HE 450-B*
Level 3-5	*HE 360-B*
Level 6-8	*HE 320-B*
Level 9-10	*HE 260-B*
Constraints	
Columns are locked to the base	
Dimensions	
Inter-floor height	*3,50 m*
Span length	*6,00 m*
Seismic masses	
28,807 Kg_p cm/sec^2	*all levels*

Table 1: Frequencies and modal forms for the un-damped structure A

mode	1 T_1=0.455 sec	2 T_2=0.187 sec	3 T_3=0.109 sec	4 T_4=0.076 sec	5 T_5=0.063 sec
component					
Level 1	0.0282	0.0658	-0.120	0.139	0.198
Level 2	0.0787	0.1772	-0.298	0.302	0.384
Level 3	0.1312	0.2762	-0.395	0.296	0.270
Level 4	0.1873	0.3520	-0.362	0.085	-0.112
Level 5	0.2400	0.3810	-0.194	-0.186	-0.389
Level 6	0.2928	0.3549	0.072	-0.334	-0.274
Level 7	0.3525	0.2563	0.376	-0.206	0.213
Level 8	0.4013	0.1066	0.490	0.132	0.431
Level 9	0.4840	-0.3016	0.123	0.634	-0.479
Level 10	0.5271	-0.5701	-0.411	-0.431	0.203

Both the structures are considered to have 5% viscous damping on the first modal form. For such structures the optimal placement of viscous extra-structural dissipation devices has been designed to respectively achieve a 10% and 20% damping ratio by considering only the first modal form (n=1 in tabs. 3-4) and the first three ones (n=3 in tabs. 3-4). Extra-structural damping design has been carried out by taking into account the response spectra A and C, defined by EuroCode 8 (ENV 1998), and that of Mexico City (1985).

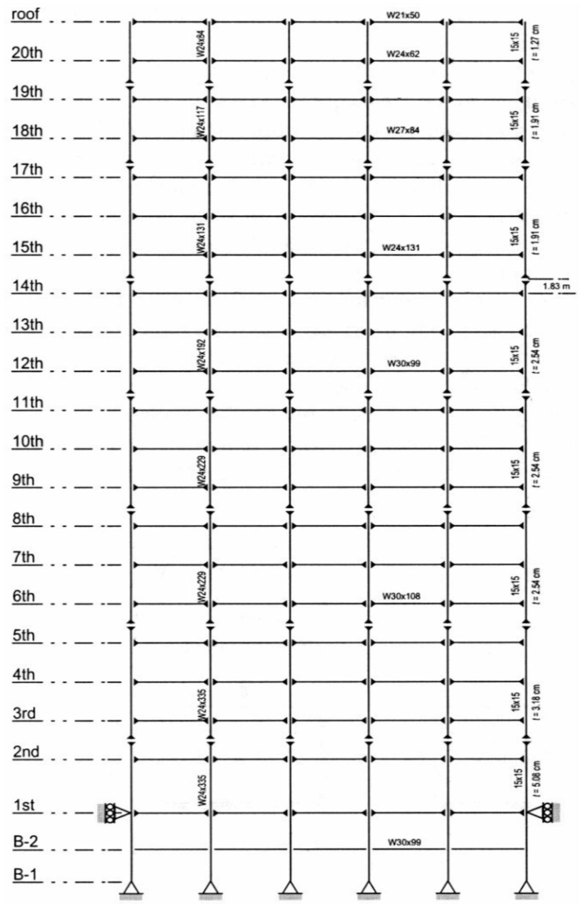

Figure 3: Model of structure B

Structure B

Beams (248 MPa)	
Level B2-5	W30x99
Level 6-11	W30x108
Level 12-17	W30x99
Level 18-19	W27x84
Level 20	W24x62
Roof	W21x50
Columns (345 MPa)	
The corner columns are square (ASTM A500) with different thicknesses; the central ones are type W varying in height.	
Constraints	
The columns are considered hinged at the base; the first level is horizontally constrained.	
Dimensions	
Base levels height	3,65 m
First level height	5,49 m
Level 2-20 height	3,96 m
Span length	6,10 m
Seismic masses	
First level	$1,33 \times 10^5$ Kg
Second level	$1,41 \times 10^5$ Kg
Level 3-20	$1,38 \times 10^5$ Kg
Roof	$1,46 \times 10^5$ Kg

Table 2: Frequencies and modal forms for the un-damped structure B

mode	1 T_1=2.493 sec	2 T_2=0.898 sec	3 T_3=0.554 sec	4 T_4=0.407 sec	5 T_5=0.323 sec
component					
Level 1	-0.032	0.0845	0.1313	0.1721	-0.2047
Level 2	-0.054	0.1381	0.2058	0.2535	-0.2764
Level 3	-0.075	0.1854	0.2573	0.2829	-0.2573
Level 4	-0.096	0.2255	0.2819	0.2563	-0.1554
Level 5	-0.116	0.2572	0.2772	0.1785	-0.0009
Level 6	-0.136	0.2781	0.2403	0.0598	0.1584
Level 7	-0.156	0.2873	0.1761	-0.072	0.2652
Level 8	-0.174	0.2844	0.0922	-0.190	0.2818
Level 9	-0.192	0.2695	-0.002	-0.267	0.2025
Level 10	-0.208	0.2432	-0.097	-0.288	0.0524
Level 11	-0.225	0.2048	-0.186	-0.249	-0.1252
Level 12	-0.241	0.1544	-0.256	-0.146	-0.2639
Level 13	-0.256	0.0956	-0.295	-0.005	-0.3024
Level 14	-0.270	0.0283	-0.298	0.1461	-0.2210
Level 15	-0.283	-0.045	-0.256	0.2671	-0.0287
Level 16	-0.294	-0.118	-0.174	0.3149	0.1804
Level 17	-0.304	-0.192	-0.054	0.2697	0.3196
Level 18	-0.313	-0.262	0.0912	0.1208	0.2872
Level 19	-0.321	-0.328	0.2545	-0.119	0.0439
Level 20	-0.327	-0.381	0.4041	-0.391	-0.3566

Table 3: Optimal placement for extra-structural dissipation devices (Structure A)

	C_{tot} = 11 KNsec/m				C_{tot} = 36.3 KNsec/m			
modal forms	n=1		n=3		n=1		n=3	
spectrum	**A**	**C**	**A**	**C**	**A**	**C**	**A**	**C**
Level 1	0	0	0	0	0	0	0	0
Level 2	0	0	0	0	0	0	0	0
Level 3	0	0	0	0	0	0	0	0
Level 4	4.7	5.0	5.2	5.5	15.4	15.4	15.3	15.3
Level 5	0	0	0	0	0	0	0	0
Level 6	0	0	0	0	8.3	9.5	8.5	9.4
Level 7	4.6	4.4	4.3	4.1	9.4	8.4	9.2	8.4
Level 8	0	0	0	0	0	0	0	0
Level 9	1.7	1.6	1.5	1.4	3.2	3.0	3.2	3.1
Level 10	0	0	0	0	0	0	0.1	0.1
Damping	ξ_1 = 10%				ξ_1 = 20%			

Results show how dissipation device placement is clearly related to the dynamic characteristics of the structure and response spectrum shape. For structure B, in the case of spectrum class A, by taking into account the first 3 modal forms, optimal placement leads to high damping amounts in higher storeys due to the significant contribution of the higher modes to the dynamic response.
The structure's dynamic behavior to recorded seismic events (tab. 5), with and without optimal extra-structural devices, has been numerically evaluated.

Table 4: Optimal placement for extra-structural dissipation devices (Structure B)

	C_{tot} = 610 KNsec/m				C_{tot} = 2205 KNsec/m			
modal forms	n=1		n=3		n=1		n=3	
spectrum	**A**	**MC**	**A**	**MC**	**A**	**MC**	**A**	**MC**
Level 1	180	170	150	170	275	270	200	270
Level 2	170	165	160	170	380	340	205	345
Level 3	145	140	165	150	380	340	200	340
Level 4	115	115	45	120	380	335	220	335
Level 5	0	20	0	0	365	325	175	325
Level 6	0	0	0	0	325	320	5	320
Level 7	0	0	0	0	100	275	0	270
Level 8	0	0	0	0	0	0	0	0
Level 9	0	0	0	0	0	0	0	0
Level 10	0	0	0	0	0	0	60	0
Level 11	0	0	80	0	0	0	350	0
Level 12	0	0	0	0	0	0	0	0
Level 13	0	0	0	0	0	0	0	0
Level 14	0	0	0	0	0	0	290	0
Level 15	0	0	0	0	0	0	270	0
Level 16	0	0	10	0	0	0	230	0
Level 17	0	0	0	0	0	0	0	0
Level 18	0	0	0	0	0	0	0	0
Level 19	0	0	150	170	275	270	200	270
Level 20	0	0	160	170	380	340	205	345
Damping	ξ_1 = 10%				ξ_1 = 20%			

MC = Mexico City seismic excitation spectru

Table 5: Dynamic characteristics of the seismic events under consideration

Seismic event	T[sec]	PGA[cm/s^2]
Imperial Valley (1940)	53.8	341.82
Kern County (1952)	54.42	175.90
Loma Prieta (1989)	40.00	270.36
Mexico City (1995)	180.1	167.91
Petrovac (1979)	19.62	437.60
Pacoima (1971)	41.90	1148.10
Parkfield (1966)	26.18	269.60

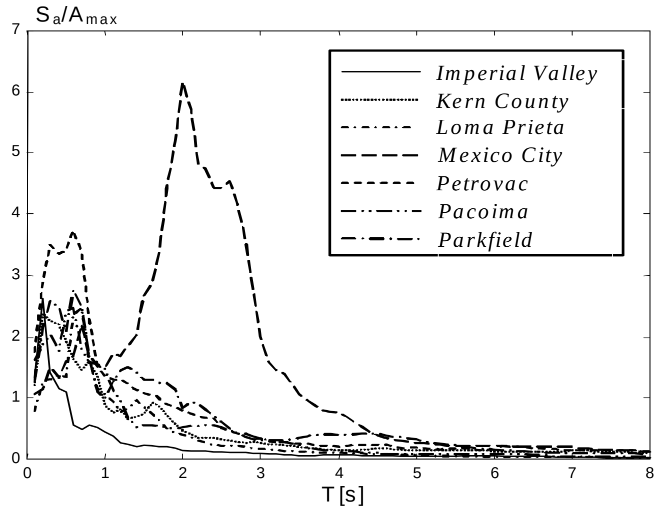

Figure 4: Seismic events under consideration: response spectra

Seismic response comparisons between uniform and optimal placement of extra-structural damping, in the case of structure B subjected to the Mexico City seismic events, are plotted in figs 5-8.

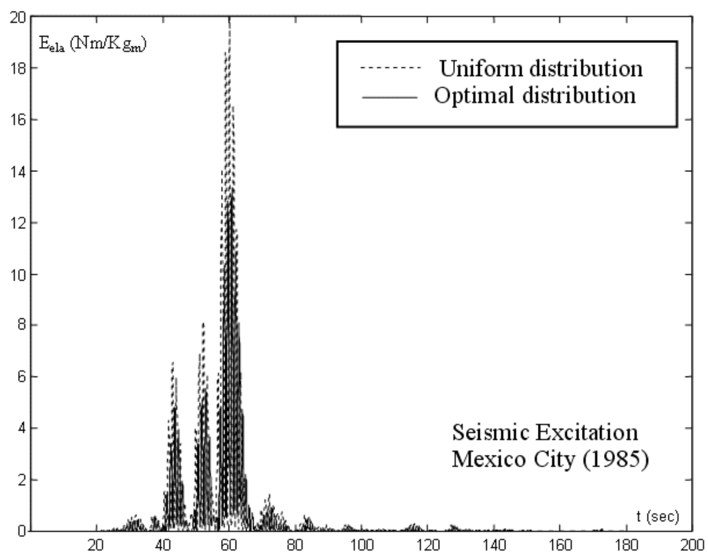

Figure 5: Elastic energy: comparison between uniform and optimal distribution of extra-structural dissipation (Structure B)

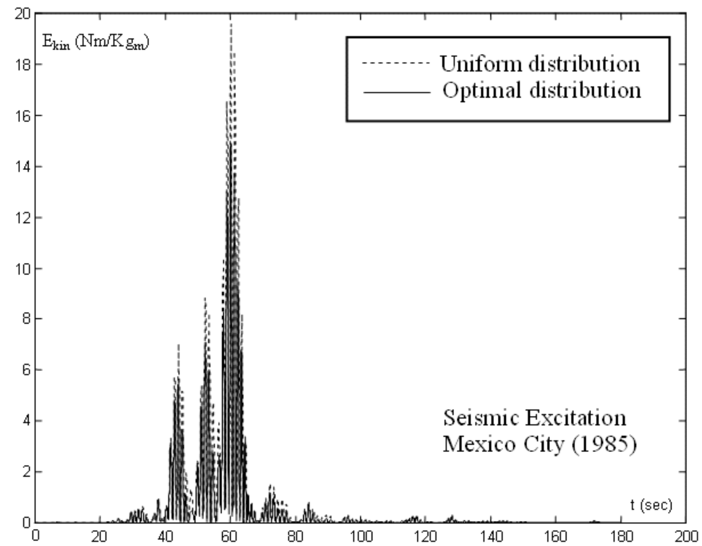

Figure 6: Kinetic energy: comparison between uniform and optimal distribution of extra-structural dissipation (Structure B)

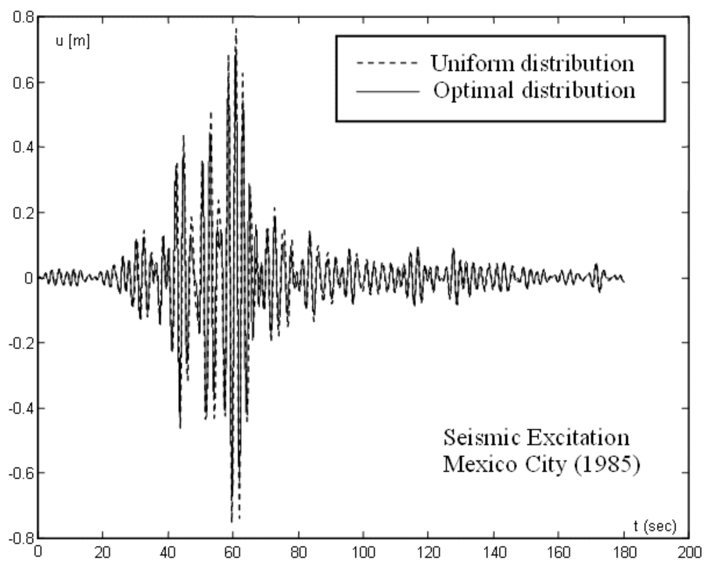

Figure 7: Roof displacement: comparison between uniform and optimal distribution of extra-structural dissipation (Structure B)

In particular, figures respectively show the comparison among the elastic energy, the kinetic energy, the roof displacement and the overall input and dissipated energy.

For the Mexico City event, optimal damping device placement leads to a 33% reduction in elastic energy, 24% in kinetic energy and 10% in roof displacement with respect to the uniform distribution. As regards dissipated energy behavior, optimal design increases the overall input of energy and decreases the structure's dissipated energy as well as the damage.

For all the seismic events under consideration, tables 6-9 show the maximum response ratios in the case of optimal design and uniform device placement for elastic energy and kinetic energy.

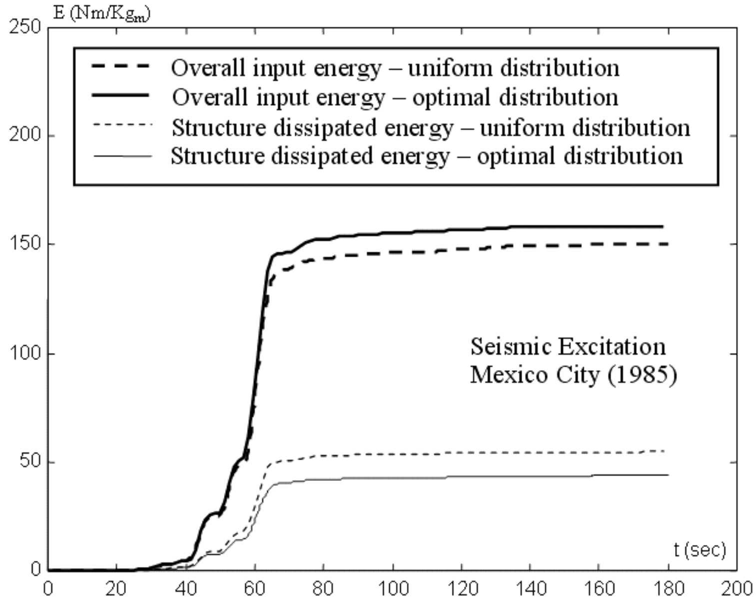

Fig. 8: Overall dissipated energy: comparison between uniform and optimal distribution of extra-structural dissipation (Structure B)

Table 6: Elastic energy ratio between optimal and uniform distribution – Structure A

	C_{tot} = 11 KNsec/m		*C_{tot} = 36.3 KNsec/m*	
modal forms	n=1		n=1	
spectrum	**A**	**C**	**A**	**C**
Imperial Valley (1940)	0,9102	0,9127	0,8284	0,8303
Kern County (1952)	0,8938	0,8936	0,8631	0,8629
Loma Prieta (1989)	0,9526	0,9526	0,9194	0,9239
Mexico City (1995)	0,9300	0,9318	0,9823	0,9813
Petrovac (1979)	0,8543	0,8568	0,8110	0,8142
Pacoima (1971)	0,8877	0,8866	0,9462	0,9390
Parkfield (1966)	0,9048	0,9046	0,8511	0,8473

Results show a general improvement in seismic response for the optimally designed structures. In the case of structures characterized by a main vibration period in the high energy range of the seismic demand spectrum, maximum gains are achieved. Generally, the higher modal forms contribution in the proposed design methodology lead to better seismic performance against the aleatory character of the input excitements.

Table 7: Elastic energy ratio between optimal and uniform distribution – Structure B

	C_{tot} = 610 KNsec/m		*C_{tot} = 2205 KNsec/m*	
modal forms	n=1		n=1	
spectrum	**Class A**	**MC**	**Class A**	**MC**
Imperial Valley (1940)	0,8768	0,8805	0,7745	0,7787
Kern County (1952)	0,8587	0,8599	0,8665	0,8604
Loma Prieta (1989)	0,9107	0,9108	0,9107	0,9216
Mexico City (1995)	0,7533	0,7502	0,6813	0,6673
Petrovac (1979)	0,9121	0,9125	0,8456	0,8522
Pacoima (1971)	0,9374	0,9368	0,9101	0,9148
Parkfield (1966)	0,8422	0,8424	0,9081	0,9101

MC = Mexico City seismic excitation spectrum

Table 8: Kinetic energy ratio between optimal and uniform distribution – Structure A

	C_{tot} = 11 KNsec/m		*C_{tot} = 36.3 KNsec/m*	
modal forms	n=1		n=1	
spectrum	**Class A**	**C**	**Class A**	**C**
Imperial Valley (1940)	0,9603	0,9612	0,9432	0,9440
Kern County (1952)	0,9328	0,9315	0,9533	0,9514
Loma Prieta (1989)	0,9684	0,9695	0,9355	0,9379
Mexico City (1995)	0,9973	0,9976	0,9967	0,9970
Petrovac (1979)	0,9463	0,9425	0,9453	0,9441
Pacoima (1971)	0,9779	0,9772	0,9810	0,9821
Parkfield (1966)	0,9277	0,9189	0,9602	0,9608

Table 9: Kinetic energy ratio between optimal and uniform distribution – Structure B

	C_{tot} = 610 KNsec/m		*C_{tot} = 2205 KNsec/m*	
modal forms	n=1		n=1	
spectrum	**Class A**	**MC**	**Class A**	**MC**
Imperial Valley (1940)	0,9445	0,9448	0,9238	0,9234
Kern County (1952)	0,9543	0,9550	0,9721	0,9819
Loma Prieta (1989)	0,9573	0,9554	0,9885	0,9912
Mexico City (1995)	0,7550	0,7545	0,7753	0,7590
Petrovac (1979)	0,9731	0,9758	0,9441	0,9465
Pacoima (1971)	0,9464	0,9495	0,9878	0,9793
Parkfield (1966)	0,8736	0,8755	0,9174	0,9271

MC = Mexico City seismic excitation spectrum

CONCLUSION

A passive control strategy for framed structures, based on the optimal placement of extra-structural dissipation devices, is proposed. The design methodology is based on maximization of the instant

maximum power dissipated by the overall system in consideration of an assigned response spectrum. In particular, this maximization is pursued through modal analysis in the state space.
The proposed design methodology has been tested by analyzing the seismic response of two structural steel-framed typologies which were subjected to different recorded seismic excitations. The results showed the effectiveness of the proposed optimization method. Particularly, reductions greater than 30% are achieved in seismic response in the case of optimal distribution for extra-structural dissipation in comparison with uniform distribution.

REFERENCES

1. Aiken I.D., Kelly J.M. (1990). Earthquake simulator Testing and Analytical Studies of Two Energy Absorbing Systems for Multistory Structures, *Repost No UCB/EERC-90/03, Earthquake Engineering Research Center, University of California, Berkeley, CA.*
2. Chopra A.K. (1995). Dynamics of structures. Theory and Applications to Earthquake Engineering. Prentice-Hall: Englewood Cliffs, NJ.
3. Ciampi V., Paolone A. And De Angelis M. (1992). On the seismic design of dissipative bracings. *Proc. of 10th World Conference on Earthquake Engineering, Madrid (Spagna), 7, pp 4133-4138.*
4. Ciampi V. And Samuelli Ferretti (1990). Energy dissipation in buildings using special bracing system. *Proc. of 9th European Conference on Earthquake Engineering, Mosca, 3, pp 9-18.*
5. Colajanni P. and Papia M. (1996). Energy design approach for braced frames with dissipative devices. *Proc. 11WCEE, Paper N° 354, Acapulco (Mexico).*
6. Constantinou M.C. and Symans M.D. (1993). Experimental study of seismic response of buildings with supplemental fluid dampers, *The Structural Design of Tall Buildings, 2, pp 93-132.*
7. Dolce M. (1994) Passive control of structure. *Proc. of 10th European Conference on Earthquake Engineering, Vienna (Austria).*
8. Dolce M., Marnetto R. (1999). Dispositivi di controllo passivo basati sulle leghe a memoria di forma. *Atti del Convegno ANIDIS '99 Torino (Italia).*
9. Fu Y., Cherry S. (1999). Frame retrofit using friction dampers. *Proc. of 8th Canadian Conference on Earthquake Engineering, Vancouver (Canada).*
10. Gluck N., Reinhorn A.M., Gluck J., Levy R. (1996). Design of supplemental dampers for control of structures. *Journal of structural engineering, ASCE, 122(12), 1394-1399.*
11. Paolacci F., Ciampi V. And De Angelis M. (1998). Optimal design of viscoelastic bracing system for earthquake protection of buildings. *Proc. of 11th ECEE, Parigi (Francia).*
12. Palazzo B., Petti L., De Iuliis M. (2001). Posizionamento ottimale di elementi dissipativi extrastrutturali per controventi metallici. *Convegno CTA: Giornate italiane della costruzioni in acciaio – Venezia 26-28 settembre 2001.*
13. Pall A.S. and Marsh C. (1982). Response of friction damped braced frames. *Journal of Structural Division ASCE, 108:1313-1323.*
14. Perko L. (1993) Differential Equations and Dynamical Systems. *Springer – Verlag – New York, Berlin, Heidelberg.*
15. Ribakov Y.(1999). Optimal design of ADAS damped MDOF structures. *Earthquake spectra 15-2, 317-330.*
16. Singh M.P. And Moreschi L.M. (2001). Optimal seismic response control with dampers. *Earthquake engineering and structural dynamics 30, 553-572.*
17. Soong T.T. (1990). Active Structural Control: theory and practice. *John Wiley & Sons, Inc., New York.*
18. Soong T.T. and Constantinou M.C. (1994) Passive and Active Structural Vibration Control in Civil Engineering, *Springer-Verlang, Wien.*

19. Soong T.T. and Dargush G.F. (1997). Passive Energy dissipation system in structural engineering. *Wiley & Sons, Chichester, England.*
20. Spencer B.F., Christenson R.E., Dyke S.J. (1998) "Next generation Benchmark Control Problem for Seismically Excited Buildings".
21. Takewaki I., Yoshitomi S., Uetani K. And Masaaki Y. (1999). Non-monotonic optimal damper placement via steepest direction search. *Earthquake engineering and structural dynamics 28, 196-202*.
22. Takewaki I. (1997). Optimal damper placement for minimum transfer functions. *Earthquake engineering and structural dynamics 26, 1113-1124.*
23. Vulcano A. (1994). Design of damped steel bracing system for seismic control of framed structures. *Proc. of 10th European Conference on Earthquake Engineering. Vienna, Balkema, Rotterdam, 4, 1567-1572.*
24. Whittaker A.S., Bertero V.V., Alonso J., Thompson C.L. (1989). Earthquake Simulator Testing of Steel Plate Added Damping and Stiffness Elements, *Report UCB/EERC-89/02, Earthquake Engineering Research Center, University of California, Berkeley, CA.*
24. Wilson E.L., Der Kiureghian A. and Bayo E.P. (1981). A Replacement for SRSS Method in Seismic Analysis, *Earthquake Eng. Struct. Dyn., 9, 187-194.*
25. Zhang R.H., Soong T.T. (1992). Seismic design of viscoelastic dampers for structural applications. *Journal of structural engineering 18, 1375-1392.*
26. Zhang R.H., Soong T.T. And Mahmoodi P. (1989). Seismic response of steel frame structures with added viscoelastic dampers. *Earthquake engineering and structural dynamics 18, 389-396.*