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AN ANALYSIS METHOD FOR NONLINEAR BEHAVIOR OF RIGID PLATE ON SOIL UNDER CYCLIC LOAD

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SUMMARY

In this paper, a model with a rigid plate connected by discrete nonlinear springs to linearly elastic soil body is used to simulate the nonlinear load-settlement behavior of steel plate under cyclic loads in loading test. The nonlinearity of actual soil is considered in the springs of which the internal forces are proportional to the square roots of their displacements and the hysteretic characteristics fulfill Masing Law. At the same time, the internal forces of springs are limited by the bearing capacity due to soil shear failure. While, the soil in this model is a linearly elastic body, which is expressed by Boussinesq solution. Parameters of elastic modulus, bearing capacity and nonlinear coefficient of spring are back-calculated from test data. This hybrid model is used to predict the behavior of steel plate under cyclic vertical loads. Analyses are carried out for both circular and square plates on cohesive and cohesionless soils. Comparisons show that the analyses are in good agreement with test data.

INTRODUCTION

In the field of earthquake engineering, prediction of soil behavior under cyclic load is one of the most important research topics. And it is typically the first step in the evaluation of structural performance in the soil-foundation-structure interaction problem. Since soil is a highly nonlinear material, any proper analysis should reflect these characteristics.

For spread foundation design, loading test is often performed to estimate elastic modulus and bearing capacity of soil. Authors are trying to propose a new method for evaluating vertical performance of building foundation through plate loading tests. The load-settlement relationship of test data usually shows nonlinear behavior of the loaded plate due to nonlinearity of soil. To simulate this nonlinear behavior, authors [1][2] have established a hybrid model consisting of rigid plate, discrete nonlinear springs and elastic soil body. In this model, following assumptions are made: response of soil is expressed by Boussinesq solution; non-linearity of actual soil is simulated by nonlinear springs; internal forces of springs are proportional to the square roots of their displacements and limited by the bearing capacity due

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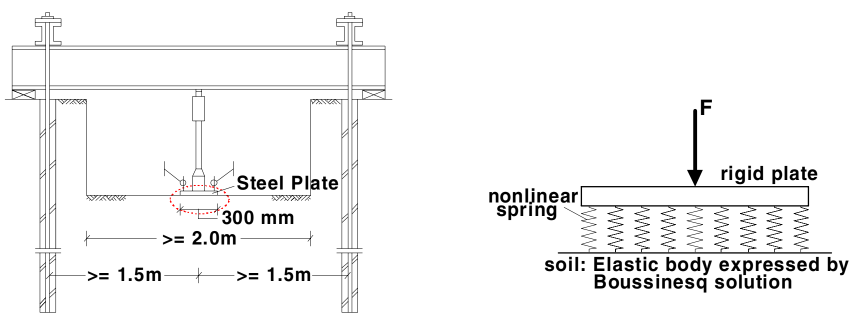
to shear failure of soil. The analysis of plate behavior under vertical load has been carried out and the results were found to be in good agreement with test results.

The previous studies were focused on the behavior of plate under monotonic loading. In earthquake engineering, however, cyclic loadings are the most important to be considered in foundation analysis. So, the study of steel plate loading test should accordingly be carried out not only for monotonic loading but also for cyclic loadings. In this study, an analytical method considering hysteretic characteristics of nonlinear springs is proposed. It is assumed that Masing law is fulfilled. Based on the parameters back-calculated from test data, the proposed hybrid model is used to simulate the plate behavior under cyclic loadings.

ANALYSIS MODEL

Numerical Analysis Model

Loading test performed on steel plate (see Fig. 1(a)) provides the complete load-settlement data of soil response developing from linear elasticity to general failure, not only the information of the two special characteristics of soil - elastic modulus for elasticity and bearing capacity for general failure. The load-settlement test data indicate that the steel plate under working loads behaves nonlinearly. To study this non-linearity, a spring-soil model in Fig. 1(b) is proposed to analyze the steel plate behavior.



a) Loading Test on Steel Plate **b) Spring-Soil Hybrid Model**

Fig.1 Loading Test Analysis Model

The internal force of nonlinear spring N is assumed to be expressed as:

$$N = kx_{sp}^{1/2} \qquad \cdots (1)$$

where, k is the nonlinear coefficient of spring and x_{sp} denotes the displacement of spring.
The response of linearly elastic soil is assumed to be expressed by Boussinesq solution as:

$$x_{so} = \frac{P(1-\nu^2)}{\pi E_s \zeta} \qquad \cdots (2)$$

where x_{so} represents the deformation of soil at the evaluation point on surface, E_s is elastic modulus of soil and ν is Poisson's ratio, P is vertical load acting on soil surface, and ζ is the distance from external loading point to the evaluation point.

If the displacement of the rigid plate under external force F is S, and letting $y_i = x_{spi}^{1/2}$, (i=1, ・・・,n), then the analytical equations for this spring-soil system can be expressed as follow:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \\ 1 & 1 & \cdots & 1 \end{bmatrix} k \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix} = \begin{Bmatrix} S - y_1^2 \\ S - y_2^2 \\ \vdots \\ S - y_n^2 \\ F \end{Bmatrix} \qquad \cdots (3)$$

where i (i=1, ・・・,n) is the ith section of the soil that is divided into n sections in accordance with discrete springs. And the partial matrix of Eq.(3) is called the linear characteristics matrix of soil and can be expressed by Boussinesq solution as follow:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

where

$$a_{ij} = \frac{1-\nu^2}{\pi E_s \zeta_{ij}}, \; (i = 1, \cdots, n; j = 1, \cdots, n) \qquad \cdots (4)$$

Hysteresis Loop of Spring

The hysteretic characteristics of actual soil under cyclic loadings are represented by the nonlinear spring satisfying the Masing law with a coefficient of 2, as shown in Fig.2 [3]. The N-x_{sp} curves can be expressed by a virgin curve or skeleton curve (O-A or O-C in Fig.2), an ascending and a descending curves or

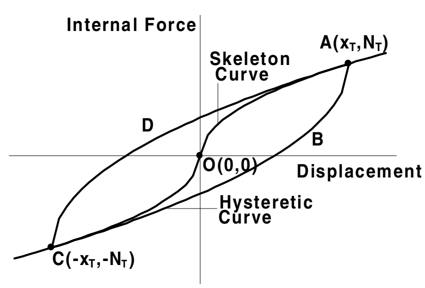


Fig.2 Hysteretic Curve of Spring

hysteresis curves (C-D-A or A-B-C in Fig.2.). The skeleton curve is expressed by Eq.(1). And the analytical representation of the hysteresis curves can be obtained from Eq.(1), satisfying Masing law. Thus,

$$\frac{|N - N_T|}{2} = k \left| \frac{x_{sp} - x_{spT}}{2} \right|^{1/2} \qquad \cdots (5)$$

in which the point (N_T, x_{spT}) is the most recent turning point.

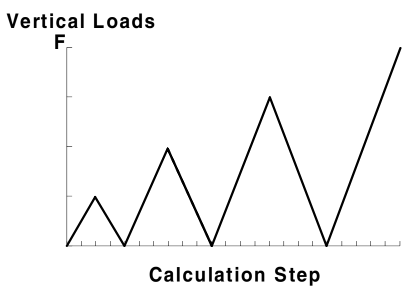
Letting $y_i=[(x_{spTi} - x_{spi})/2]^{1/2}$, ($i=1, \cdots, n$), the analysis equation of the proposed spring-soil system of unloading can be expressed as Eq.(3) changed according to Eq.(5) as:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{Bmatrix} N_{T1} - 2ky_1 \\ N_{T2} - 2ky_2 \\ \vdots \\ N_{Tn} - 2ky_n \end{Bmatrix} = \begin{Bmatrix} S - (x_{spT1} - 2y_1^2) \\ S - (x_{spT2} - 2y_2^2) \\ \vdots \\ S - (x_{spTn} - 2y_n^2) \\ F \end{Bmatrix} \qquad \cdots (6)$$

Letting $y_i=[(x_{spi} - x_{spTi})/2]^{1/2}$, ($i=1, \cdots, n$), the analysis equation of the proposed spring-soil system of reloading can be expressed as Eq.(3) changed according to Eq.(5) as:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{Bmatrix} N_{T1} + 2ky_1 \\ N_{T2} + 2ky_2 \\ \vdots \\ N_{Tn} + 2ky_n \end{Bmatrix} = \begin{Bmatrix} S - (x_{spT1} + 2y_1^2) \\ S - (x_{spT2} + 2y_2^2) \\ \vdots \\ S - (x_{spTn} + 2y_n^2) \\ F \end{Bmatrix} \qquad \cdots (7)$$

The load-settlement behavior of steel plate or *F*-*S* relationship in cyclic loading test is obtained by solving Eqs.(3), (6) and (7) using Newton-Raphson's iterative method for initial loading, unloading and reloading states, respectively.



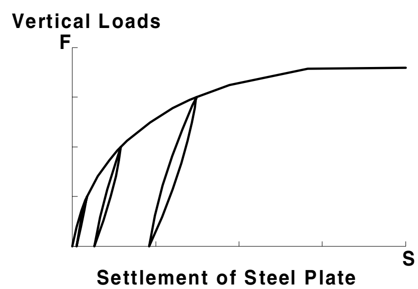


Fig.3 Plate Behavior under Simple Pattern of Load

Behavior under Ideal Cyclic Loads

Fig. 3 illustrates an example of *F*-*S* curve computed using the above analysis model. Vertical load acting on the plate is as indicated in the left figure. The parameters used here will be stated in next section.

PARAMETER DETERMINATION

Elastic Modulus and Ultimate Stress Distribution

Following hyperbolic function is used to approximate the nonlinear load-settlement curve obtained in loading test [4]:

$$F = \frac{S}{(aS+b)} \qquad \cdots (8)$$

where a and b are constants, with a denoting the reciprocal of the asymptotic value of this function and b indicating the reciprocal of the slope of initial tangential line. The elastic modulus of soil is retrieved from the initial tangential line together with Boussinesq equation:

$$E_s = \frac{1-\nu^2}{Bb} I_s \qquad \cdots (9)$$

B is the width of steel plate, I_s is the settlement coefficient and I_s=1.0 for circular plate and I_s=0.88 for square plate

The ultimate resistance F_u of soil can be calculated from following equation:

$$F_u = 0.90 * F_a \qquad \cdots (10)$$

F_a is the asymptotic value of above hyperbola, $F_a = 1/a$. It is considered that the ultimate resistance of soil is composed of two parts. One is provided by cohesion and the other is provided by friction. It is assumed that the cohesion part is evenly distributed along the contact surface while the friction part is distributed in the form of a parabola with zero at the edges and maximum at the center. And further assumption is made that the ultimate stress Q_u is evenly distributed along each concentric circle for circular plate and each concentric square for square plate. Thus, Q_u can be expressed as follow:

$$Q_u(r) = cN_c + \frac{2F_u - cN_c A}{A}\left[1 - \left(\frac{2r}{B}\right)^2\right] \qquad \cdots (11)$$

Here, A denotes the area of steel plate. r is the distance from evaluation point to the center of plate. c is cohesion of soil and N_c is bearing capacity factor, N_c=5.70 is used in this study [5].

When the stress of spring reaches the ultimate stress, Q_u, it is considered that the spring has failed and the internal force of spring is maintained at the level of ultimate stress. General failure of actual soil happens when all springs are failed.

Nonlinear Coefficient of Spring

After elastic modulus and ultimate stress are determined, the proposed analysis model is then used to derive the nonlinear coefficient of spring from the test data for each kind of soil. Eq.(3) is solved for k with known data of F and S. k-values obtained are slightly different for each loading step of F-S data and the average is taken as the nonlinear coefficient for that kind of soil.

COMPARISONS WITH TEST RESULTS

The proposed method, which includes the analytical hybrid model and the ways to determine the elastic modulus, the ultimate stress and the nonlinear coefficient of springs, is applied in following tests.

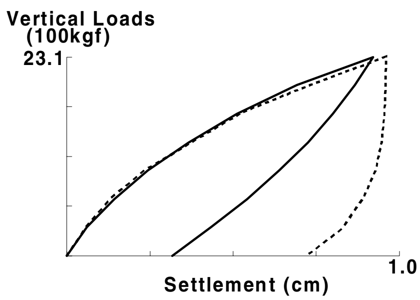
Four cases are taken under consideration. Test 48 and T-24 are performed on circular plate of Ø300mm×25mm on loam and sand, respectively. Test 270 and 118 are performed on square plate of

300mm×300mm×25mm on sand and gravel, respectively. The outlines of these four tests are listed in Table 1.

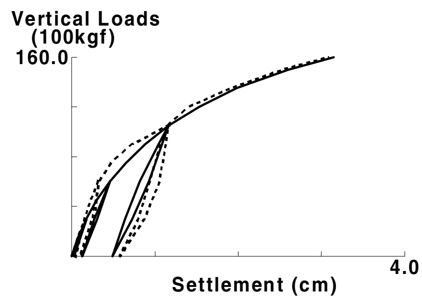
The test data of load-settlement relationships are plotted with dotted curves in Fig.4. Characteristics of soil, elastic modulus, ultimate stress and nonlinear coefficient, are derived from these data by the way described above and listed in Table 1. Assumptions are made that the Poisson's ratio is 0.50 for loam and 0.30 for sand and gravel [6].

Table 1 Outline and Characteristics of Two Cases

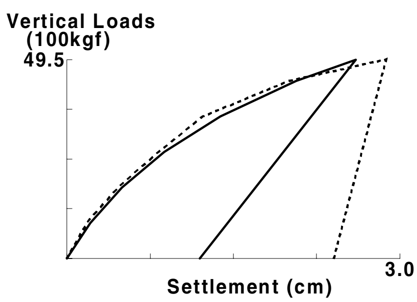
Parameters	Test-48	Test-270	Test T-24	Test-118
Embedded depth (m)	0.40	11.77	1.75	9.50
SPT N-Value	-	18	7	50
Unconfined ultimate strength q_u (kgf/cm^2)	8.90	-	-	-
Elastic Modulus E_s (kgf/cm^2)	150.25	556.11	156.25	532.80
Ultimate resistance (kgf)	3409.10	18072.30	6751.70	7377.00
Maximum vertical load (kgf)	2310.00	16000.00	4948.00	8000.00
Maximum measured settlement (cm)	0.96	3.09	2.87	3.86
Nonlinear coefficient of spring k (kgf/cm$^{1/2}$)	4.07	15.24	5.63	7.94



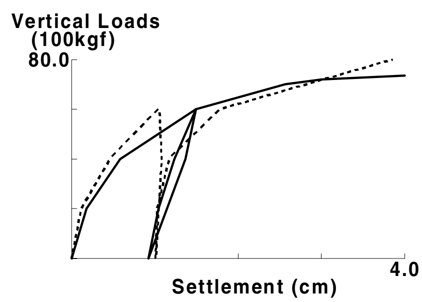
Test 48 (Loam)



Test 270 (Sand)



Test T-24 (Sand)



Test 118 (Gravel)

Fig.4 Load-Settlement Curves for Examples

Predicted behavior (load-settlement relationship) of steel plate under vertical cyclic loads is also plotted with solid curves in Fig.4. From these examples it is found that the proposed analytical method is applicable to predict the behavior of steel plate under cyclic loads.

From these four examples, it is clear that in the cases of Test-270 and Test-118, the analytical results are in good agreement with the test data, while in the cases of Test-48 and Test -24, the calculated residual settlements of steel plate are much less than the measured ones in the tests. For this difference, we can resort for the parameters of soil. Because the embedded depth of Test-48 and Test-24 are 0.4m and 1.75m, respectively, which are very shallow compared with Test-270 of 11.77m and Test-118 of 9.5m. Therefore, the soils in Test-48 and Test-24 are very loose due to the lower confining stresses and a large amount of settlement of steel plate happened when subjected to vertical loadings, among which a relative larger residual settlement remained even when the external load is removed.

CONCLUSIONS

An analytical method is proposed to predict steel plate behavior under vertical cyclic loadings with a new hybrid analysis model consisting of rigid plate, nonlinear spring and linearly elastic soil. Nonlinearity and hysteresis of actual soil are considered with discrete nonlinear springs, with internal force proportional to square root of displacement and hysteretic characteristics fulfilling Masing law. Examples of prediction of steel plate behavior and comparisons with test results show that the proposed method is effective, especially for dense soil.

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