13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 799

INTERACTION OF TORSION AND P-DELTA EFFECTS IN TALL BUILDINGS

A.S. MOGHADAM[1] and A. AZIMINEJAD[2]

SUMMARY

In this paper the importance of asymmetry of building on the P-Delta effects in elastic and inelastic ranges of behavior are evaluated. The contribution of lateral load resisting system, number of stories, degree of asymmetry, and sensitivity to ground motion characteristics are assessed. Four buildings with 7, 14, 20 and 30 story are designed based on typical design procedures, and then their elastic and inelastic static and dynamic behavior, with and without considering P-Delta effects, are investigated. Each building is considered for 0%, 10%, 20% and 30% eccentricity levels. The results indicate that the type of lateral load resisting system plays an important role in degree that torsion modifies the P-Delta effects. It is also shown that although in the elastic static analyses, torsion always magnifies the P-Delta effects, but the same is not always true for dynamic analyses. The results of dynamic analyses also show high level of sensitivity to ground motion characteristics.

INTRODUCTION

In the traditional first order analysis of structures, the effects of change in the structure actions due to structure deformations are neglected. However, when a structure deforms, the applied loads may cause additional actions in the structure that are called second order or P-Delta effects.

The P-Delta effect is dependent on the applied load and building characteristics. In addition to parameters such as height and stiffness of a building, the degree of its asymmetry may also be of importance. The building asymmetry is often due to unbalanced distribution of its mass, stiffness or strength. The induced torsional deformations usually cause uneven displacements among lateral load resisting elements and therefore concentration of damage in some of them. Therefore, torsionally unbalanced buildings are normally more susceptible to earthquake damages. The deformations caused by torsion can affect the P-Delta consequences. As a result, it is expected that torsion and P-Delta have interaction in the seismic behavior of some buildings. A long list of parameters is likely to be effective in this interaction. Lateral and torsional stiffness of building, the level of its eccentricity, mass moment of inertia, height, the properties of loading and ground motions are some of these parameters. To include the effect of P-Delta in

[1] Assistant Professor, International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran. Email: moghadam@iiees.ac.ir
[2] PhD Candidate, Structural Engineering Research Center, International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran. Email: Armin@iiees.ac.ir

the analysis of asymmetric buildings some procedures have been proposed in the literature such as the ones by Rutenberg [1] or Wilson [2]. Wynhoven [3] studied the effect of torsion on the inelastic lateral stability of frame-shear wall building systems. To consider the influence of torsion on the load carrying capacity of the structure, two asymmetric models were constructed by moving the shear-wall locations in the plan of building models. They noticed a reduction in the ultimate lateral load carrying capacity due to induced torsion between 50% to 60 %.

Buildings designed based on traditional building codes are expected to experience inelastic actions when undergo their design level earthquakes. The interaction of torsion and P-Delta can further be complicated if a study extends to inelastic range of behavior. Inelastic actions can substantially modify both torsion and P-Delta effects.

In the present study, the significance of asymmetry of building on the P-Delta effects in elastic and inelastic ranges of behavior are evaluated. The contribution of lateral and torsional stiffness, number of stories and ground motions are assessed. Four building are designed based on typical design procedures, then their static and dynamic elastic and inelastic behavior with and without considering P-Delta effects are investigated for cases with different eccentricity levels.

BUILDING MODELS

Four three dimensional building models of figure 1 are used as the basic models in this study; the torsionally unbalanced models are then derived from these basic models. The buildings have 7, 14, 20 and 30 stories.

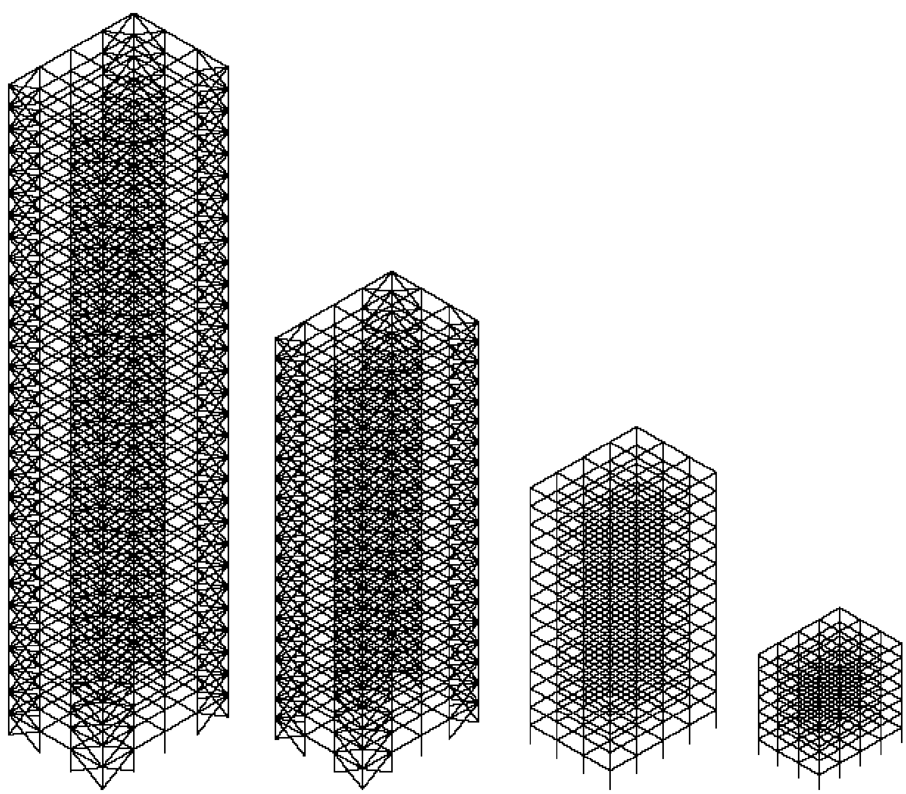
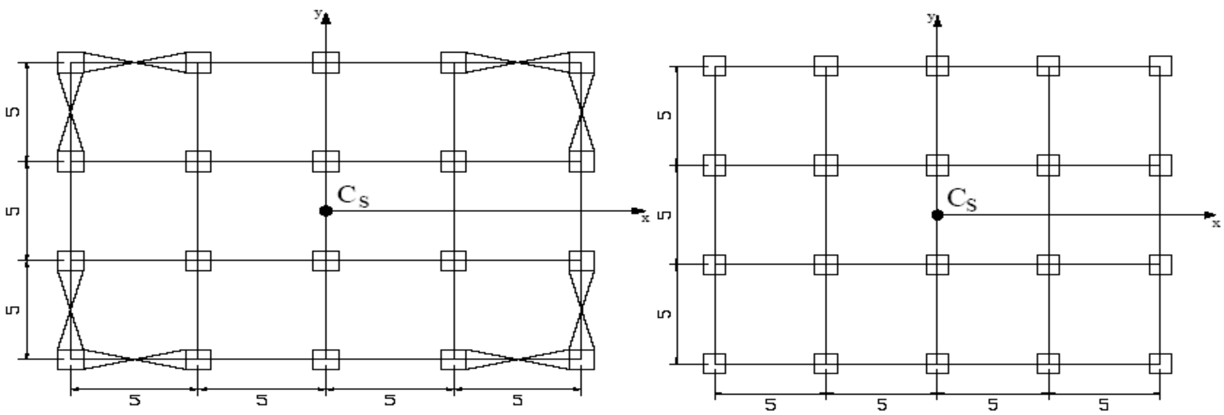


Figure 1. Building models

The lateral load resisting system of 7 and 14 story buildings is consists of steel moment resisting frames, while the 20 and 30 story buildings have a dual moment resisting and braced frame system. Their geometrical and design information are provided in the following sections.

Building characteristics

The plan of all buildings is 15 by 30 meter as shown in figure 2. Bay length of buildings in each direction is 5 and their story height is 3 meters. The floor diaphragms are assumed to be rigid in their plane. The design base shears of the buildings are calculated according to the Iranian Building Code and member designs are done based on AISC-ASD89. The P-Delta and 5% accidental eccentricity are considered in their design. The bracing of 20 and 30 story buildings are in four bays in each direction, that are the first and last bay of the perimeter frames. The mass of the floors are assumed as 240 ton which is equal to dead plus live load of the floor.



(a) Dual system (b) Moment resisting system

Figure 2. Typical floors of building models

Torsionally unbalanced building models

The asymmetry of a building can be due to its unbalanced distribution of stiffness or strength. The asymmetry can also be due to unbalanced distribution of mass. It is shown in the literature that in general, these two types of asymmetry may result in different behavior for the buildings.

In the present study the torsionally unbalanced models are derived from the basic models by change in mass distribution of the floors. To illustrate the mass distribution as assumed in this study, consider the rectangular floor in figure 3. Total mass of the floor m_0 is consists of two mass distribution. The first mass distribution is a uniform distribution with a total sum of m_1. The second mass distribution is a uniform distribution at a 1 meter width band shown in figure 3. The total sum of second mass distribution is βm_1. In figure 3, c_s is the center of rigidity of the floor, c_m is the center of mass of the floor and c'_m is the center of mass of 1 meter band. If $\beta = 0$, then c_m and c_s coincide and the normalized eccentricity e_s (ratio of eccentricity to floor width) equals zero. In this case the floor mass moment of inertia will be:

$$I_o = \frac{m_o}{12}(a^2 + b^2)$$

Values of β and m_1 will depend on the value of normalized eccentricity e_s:

$$\beta = \frac{e_s b}{(\frac{b}{2} - e_s b - 0.5)}$$

$$m_1 = \frac{m_o}{1 + \beta}$$

Then the value of mass moment of inertia with respect to c_m will be:

$$I = \frac{m_1}{12}(a^2 + b^2) + e_s^2 b^2 m_1 + \frac{\beta m_1}{12}(a^2 + 1) + \beta m_1 (\frac{b}{2} - e_s b - 0.5)^2$$

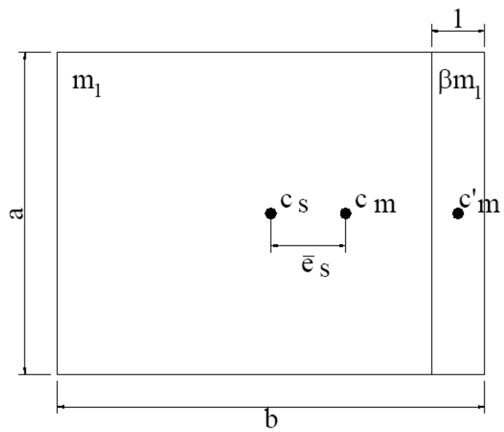


Figure 3. Mass distribution of asymmetric models

The values of floor mass moment of inertia for four level of eccentricity (0%, 10%, 20% and 30%) are calculated from the equation above and shown in table 1.

Table 1. The mass moment of inertia for different eccentricities

Mass moment of Inertia $(Kg.m)\times 10^7$	Eccentricity %
1.25	0
1.442	10
1.442	20
1.25	30

Thus from each of the four basic building, four models are derived with eccentricities from 0% to 30%.

COMPUTER AND P-DELTA MODELING

Software and Element modeling

To include the effects of torsion in analysis, the software has to be able to model the frame components as three dimensional beam-column elements. At the same time, it should have the capability of taking into account the P-Delta effects. In this study two computer programs are employed. A general purpose finite element software is employed for conducting three dimensional inelastic time history analyses with and without P-Delta effects. Another three dimensional structural analysis program is also employed for conducting three dimensional elastic time history analyses with and without P-Delta effects. The outputs of the two computer programs in the elastic range of behavior are in satisfactory agreement.

P-delta modeling

P-Delta effects, in general, include both the instability of the frame elements and global instability of frame itself. There are different procedures for including the P-Delta effects in analysis, such as utilizing the second order stiffness matrix in the analysis. The second order stiffness matrix is derived from applying equilibrium equation to the deformed shape of a beam-column element. The matrix consists of stability functions that can be expanded using Taylor series. To estimate the P-Delta effects with satisfactory precision, often only the first two terms of series are sufficient. In this case, the stiffness matrix of an element can be written as the sum of two matrices. The first matrix is the first order stiffness matrix and the second one is the geometrical stiffness matrix of the element. The geometrical stiffness matrix of beam-column element is [4]:

$$K_P = \frac{P}{\ell}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & \frac{6}{5} & \frac{\ell}{10} & 0 & -\frac{6}{5} & \frac{\ell}{10} \\ & & \frac{2\ell^2}{15} & 0 & -\frac{\ell}{10} & -\frac{\ell^2}{30} \\ & & & 0 & 0 & 0 \\ sym & & & & \frac{6}{5} & -\frac{\ell}{10} \\ & & & & & \frac{2\ell^2}{15} \end{bmatrix}$$

Therefore, the geometrical stiffness matrix is a function of element length (l) and its axial load (P). According to Goto [5], as the value of element axial load is not known prior to analysis there is a need for suitable nonlinear analysis algorithms to deal with the problem.

Another approach for including the P-Delta effects in analysis is the negative stiffness method. The method is proposed by Nixon [6] and modified by Rutenberg [7]. The approach is based on simulating the P-Delta effects by reducing the stiffness of the structure. It can be performed either by directly reducing the stiffness matrix or indirectly by introducing virtual elements in the structure. It is therefore possible to modify the stiffness matrix and include the global P-Delta effects in analysis by conducting a first order analysis. The advantage of the procedure is in completing the analysis in one cycle, it is a great deal of help in performing static and dynamic analyses and in including the P-Delta effects in natural frequencies and mode shapes of the structure. However it does not take into account the effects of (element-wise) instability of the frame elements.

ANALYSES, RESPONSE PARAMETERS AND GROUND MOTIONS

Three types of analyses are performed on the building models. First, elastic static analyses are conducted on the 7, 14, 20 and 30 story buildings, once without P-Delta effects and then with the P-Delta effects. Each case modeled for four levels of eccentricity ratios that are 0%, 10%, 20% and 30%. As the analyses were in elastic range of behavior, the P-Delta magnification factors are employed in the analyses.

The second type of analysis was elastic dynamic one. Five ground motion records were applied to the models. The first two ground motion, meaning Tabas and Naghan are among the most famous ground motions recorded in Iran. They are representative of classical tectonic and geological features of Iranian earthquakes. Three other records, meaning El Centro, San Fernando and Kern County ground motions are among records often used by other researchers; hence they are taken as reference records in the present study. The 7, 14, 20 and 30 story buildings, once without P-Delta effects and then with P-Delta effects were modeled for four levels of eccentricity ratios of 0%, 10%, 20% and 30%. Again, as the analyses were in elastic range of behavior, the P-Delta magnification factors are employed in the analyses.

The third type of analysis was inelastic dynamic one. Four ground motion records, Naghan, El Centro, Kern County, and San Fernando were applied to the models. The 7, 14, and 30 story buildings, once without P-Delta effects and then with the P-Delta effects were modeled for four levels of eccentricity ratios of 0%, 10%, 20% and 30%.

Maximum interstory drift ratio is selected as the response parameter in this study. The drift ratios were only calculated in the flexible side of buildings (the edge near to center of mass). To quantify the effect of P-Delta, the maximum interstory drift ratio of each story in the analyses with P-Delta effect included, is divided to the same parameter in the corresponding case without P-Delta effect. This parameter is called ratio of drifts in this study.

DISCUSSION OF ANALYSES RESULTS

In static analyses, triangle static lateral loading of Iranian earthquake code is used. The shape of this loading is very similar to static lateral loading of UBC 97. The P-Delta effects in these analyses are amplified by a factor equal to 0.4*R, where R is force modification factor (behavior factor). The results of elastic static analyses are shown in figure 4. The ratio of drifts (ratio of maximum interstory drift ratio in analysis with P-Delta effect to maximum interstory drift ratio in analysis without P-Delta effect) is shown for every story of the buildings.

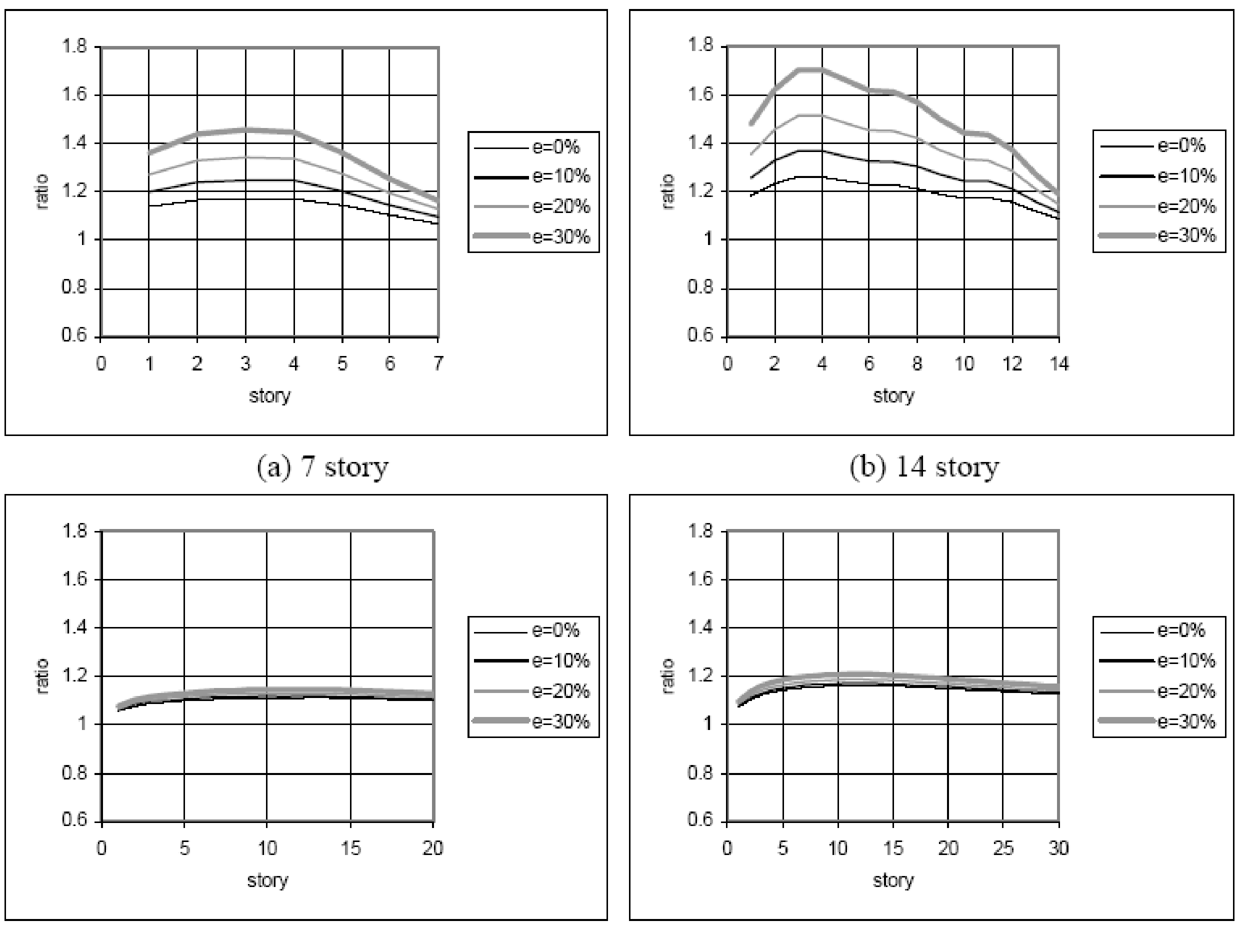
Comparing the results for the four buildings in no torsion cases (e = 0%) shows that in both types of buildings: with moment resisting system (7 and 14 story) and dual system (20 and 30 story), effect of P-Delta increases with increase in number of stories of buildings.

Comparing the results for the four buildings when eccentricity exists, indicates that in all cases with increase in eccentricity the effect of P-Delta has also increased (more than 40% increase in 14 story building). It can also observed that in both types of buildings: with moment resisting system (7 and 14 story) and dual system (20 and 30 story), effect of torsion (increase in eccentricity) on P-Delta increases with increase in number of stories of buildings.

"Importance of torsion on P-Delta effect" mainly depends on the type of lateral load resisting system of building, as in figure 4, the curves in graphs (a) and (b) of the figure are more separate that curves in graphs (c) and (d).

Figure 5 demonstrates the ratio of drifts for the 7 story building subjected to each of the five ground motions. The maximum values of responses are also shown in graph (f) of this figure. Comparing the results for no torsion cases (e = 0%) shows that effect of P-Delta increases the responses in most cases, however there are cases such as Kern County earthquake, in graph (c), that the responses have been reduced due to P-Delta effects. This result may seem strange, as it is usually expected that P-Delta effects have to increase the responses. However, as the implementing P-Delta effects causes change in stiffness matrix of building, the natural periods and other dynamic properties of the building will change. If acceleration response corresponding to the new natural period of building, in response spectrum of the earthquake, is less than acceleration response corresponding to the original natural period, then reduction in building responses for the case with P-Delta can be expected. Comparing the results for no torsion cases (e = 0%) shows further that effect of P-Delta is quite sensitive to ground motion characteristics such as its frequency content.

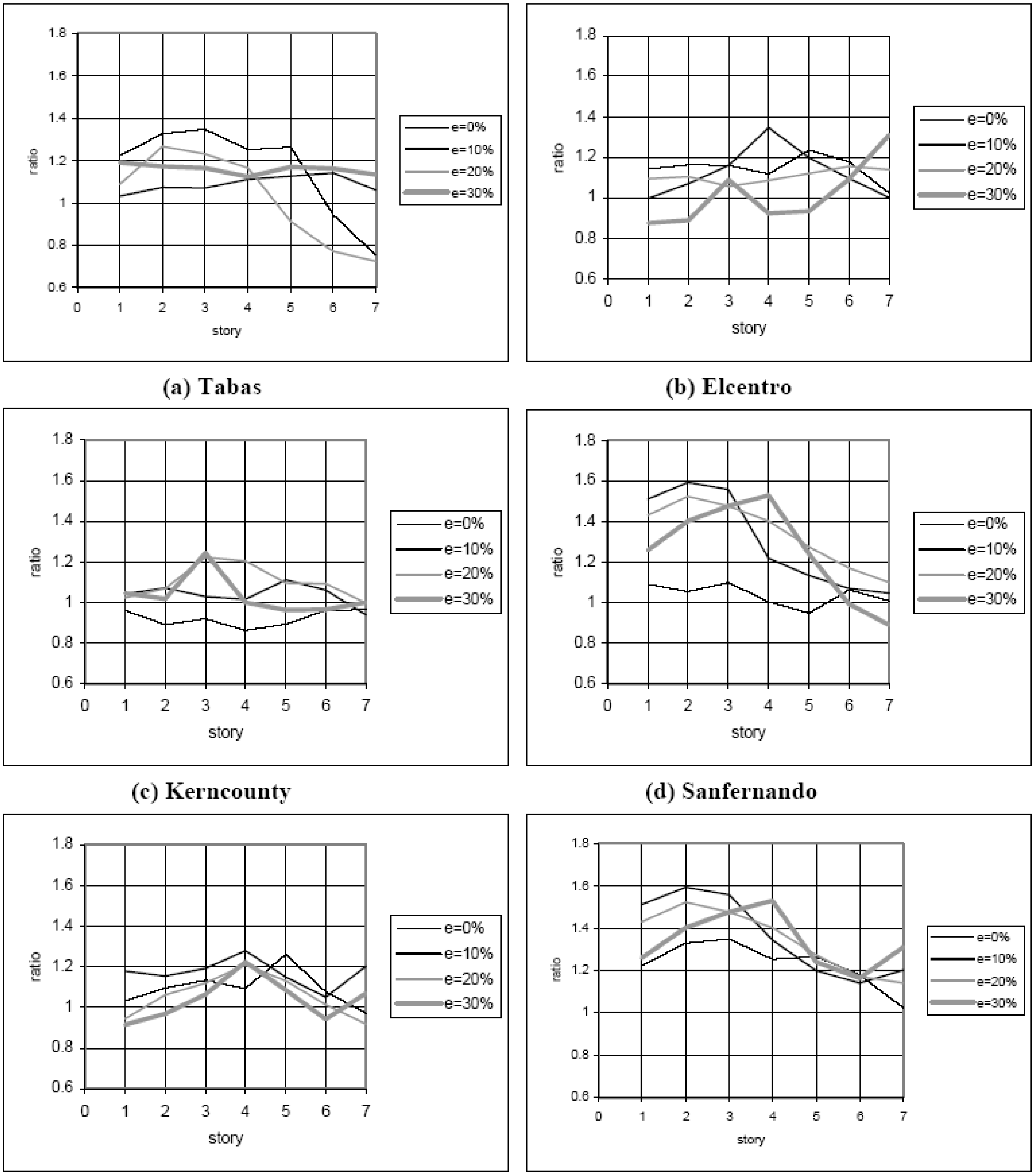
Comparing the results for the 7 story building, when eccentricity exists, indicates that in all cases with increase in eccentricity the effect of P-Delta varies remarkably (up to 60%). However, the variation does not have a constant increasing or decreasing trend. One of the reasons is the fact that with increase in the eccentricity, the mass moment of inertia has not increased in all cases. Table 1 shows that as eccentricity increases from 0% to 30%, the mass moment of inertia increases for eccentricity in the range of 0% to 10%, then it is almost constant in the eccentricity range of 10% to 20%, and finally it decreases in the eccentricity range of 20% to 30%. Furthermore, comparing the results of figure 5 shows that effect of P-Delta is quite sensitive to ground motion characteristics in the cases with eccentricity too.



(d) 30 story

Figure 4. Ratio of drifts for 7, 14, 20 and 30 story models from elastic static analyses

The maximum results of elastic dynamic analyses for 7, 14, 20 and 30 story building models are shown in figure 6. The curves are constructed by calculating the maximum value of ratio of drifts from results of five earthquake ground motion. The ratio of drifts is shown for every story of the buildings. Comparing the results for the four buildings in no torsion cases (e = 0%) shows that in both types of buildings: with moment resisting system (7 and 14 story) and dual system (20 and 30 story), effect of P-Delta generally increases with increase in number of stories of buildings.



(a) Tabas (b) Elcentro

(c) Kerncounty (d) Sanfernando

(e) Naghan (f) max

Figure 5. Ratio of drifts for 7 story models from elastic dynamic analysis

Comparing the results for the four buildings when eccentricity exists, indicates that with increase in eccentricity the effect of P-Delta varies. For example in figure 6(b) P-Delta effect in 14 story building with increase of eccentricity from zero to 20% increases more than 40% and then with increase of eccentricity from 20% to 30% (and decrease of stories moment of inertia) drift ratio decrease more than 20%. It can be also observed that for buildings with moment resisting system (7 and 14 story), effect of torsion (increase in eccentricity) on P-Delta increases with increase in number of stories of buildings but this is not true for the buildings with dual system (20 and 30 story).

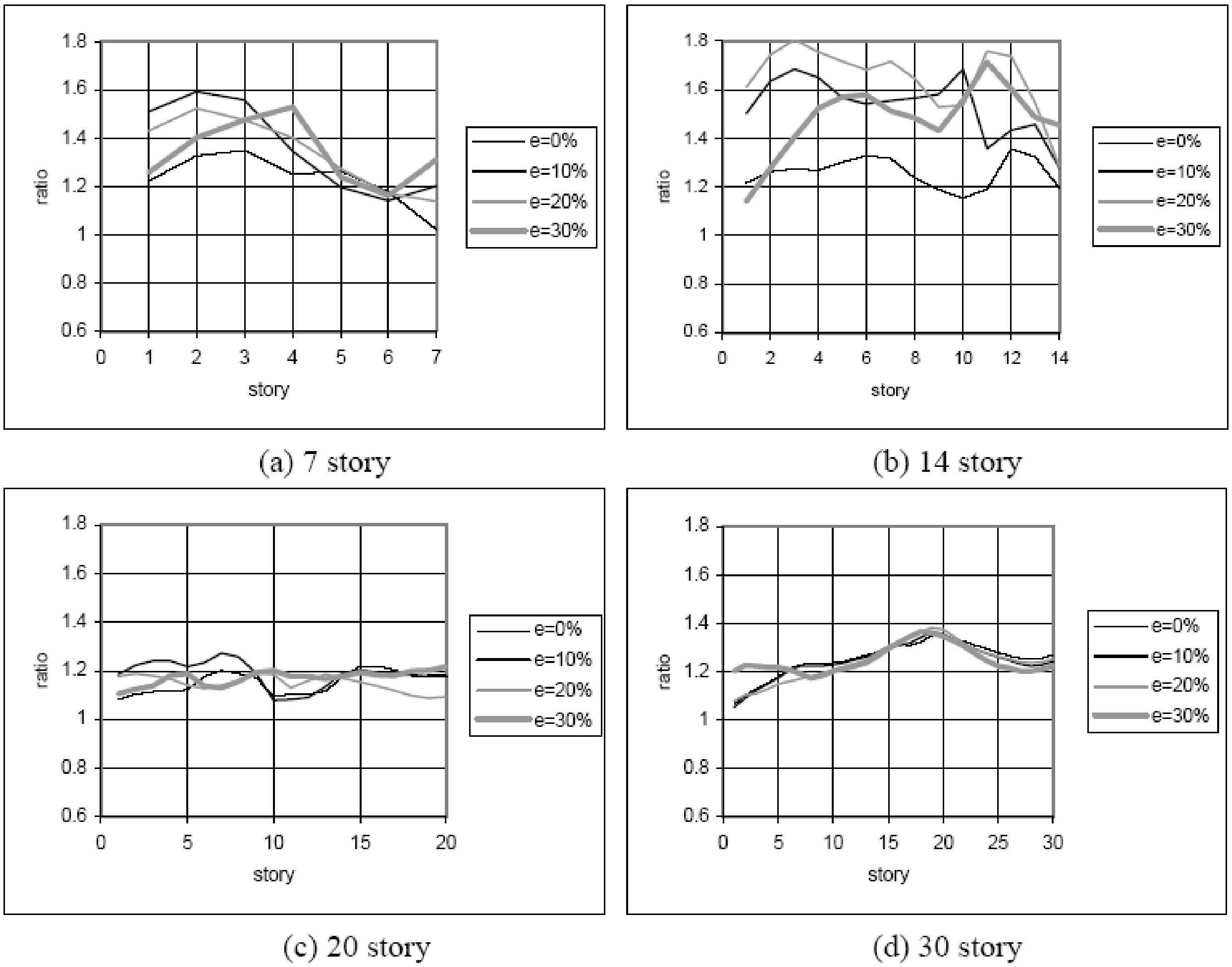


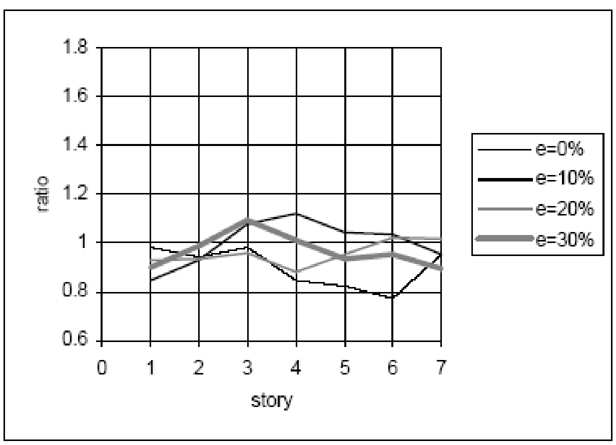
Figure 6. Maximum ratio of drifts for 7, 14, 20 and 30 story models from elastic dynamic analysis

In figure 6, the curves in graphs (a) and (b) of the figure are more separate that curves in graphs (c) and (d). As the first two graphs are for moment resisting frames and the other two are for dual systems, it can be concluded that the “Importance of torsion on P-Delta effect” mainly depends on the type of lateral load resisting system of buildings.

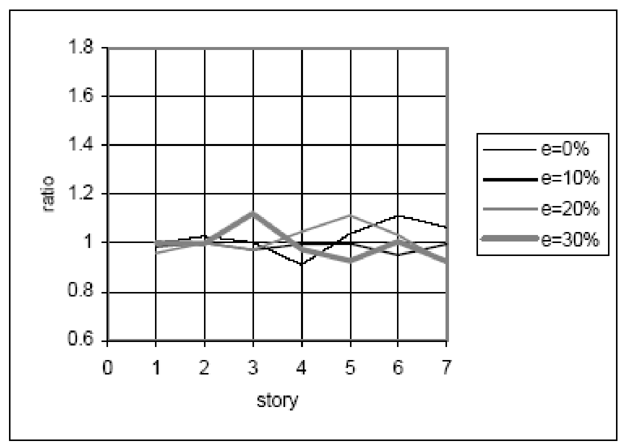
Figure 7 demonstrates the ratio of drifts of inelastic dynamic analyses for the 7 story building subjected to four ground motions. Comparing the results for no torsion cases (e = 0%) shows that effect of P-Delta increases the responses in some cases, however there are also cases that the responses have been reduced due to P-Delta effects. These kinds of results were also observed in the elastic dynamic cases. As explained there, a reason can be due to change in stiffness matrix of building because of P-Delta effects. When stiffness matrix of a building changes, the natural periods and other dynamic properties of the building will also change. If acceleration response corresponding to the new natural period of building, in response spectrum of the earthquake, is less than acceleration response corresponding to the original natural period, then reduction in building responses for the case with P-Delta can be expected, even when inelastic response is assumed. Comparing the results for no torsion cases (e = 0%) shows further that effect of P-Delta is sensitive to ground motion characteristics, however the sensitivity is less than the elastic dynamic cases.

Comparing the inelastic dynamic results for the 7 story building when eccentricity exists, indicates that in all cases, with increase in eccentricity, the effect of P-Delta varies but in a smaller range, compare with elastic dynamic results. Again, the variation does not have a constant increasing or decreasing trend. One of the reasons is the fact that with increase in the eccentricity, the mass moment of inertia has not increased in all cases. Furthermore, comparing the results of figure 7 shows that effect of P-Delta is quite sensitive to ground motion characteristics in the cases with eccentricity. It seems the sensitivity to ground motion increases, as the eccentricity increases. However, the sensitivity to ground motion in inelastic dynamic cases are less than elastic dynamic ones.

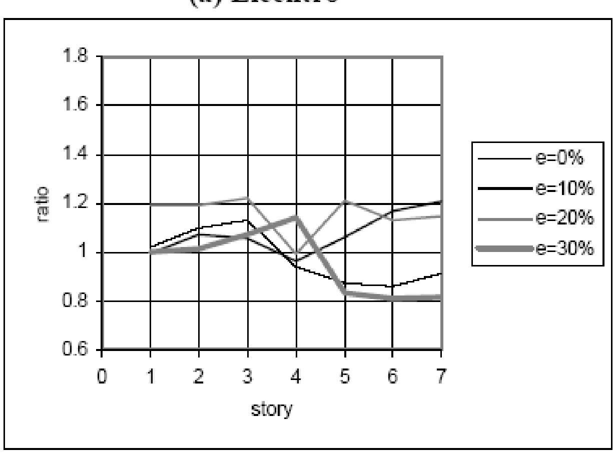
The maximum responses of inelastic dynamic analyses for 7, 14, and 30 story building models are shown in figure 8. The curves are constructed by calculating the maximum value of ratio of drifts from results of four earthquake ground motions. The ratio of drifts is shown for every story of the buildings. Comparing the results for the three buildings in no torsion cases (e = 0%) shows that the effect of P-Delta generally increases with increase in number of stories of buildings.



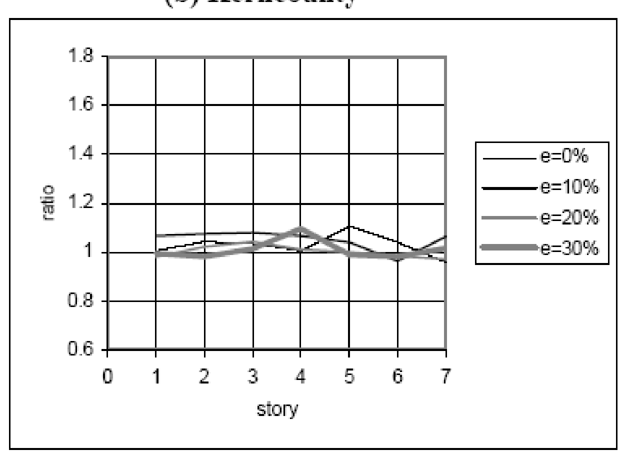
(a) Elcentro



(b) Kerncounty



(c) Sanfernando

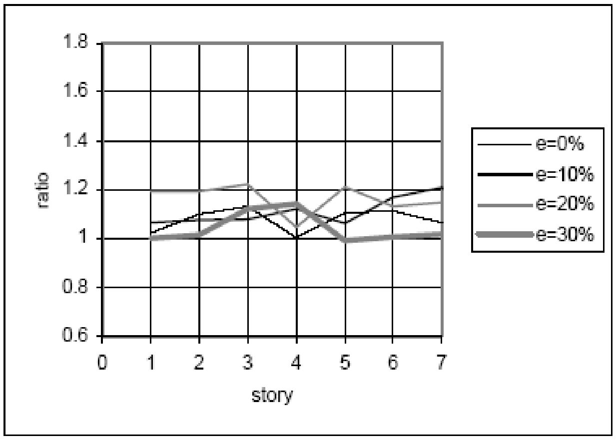


(d) Naghan

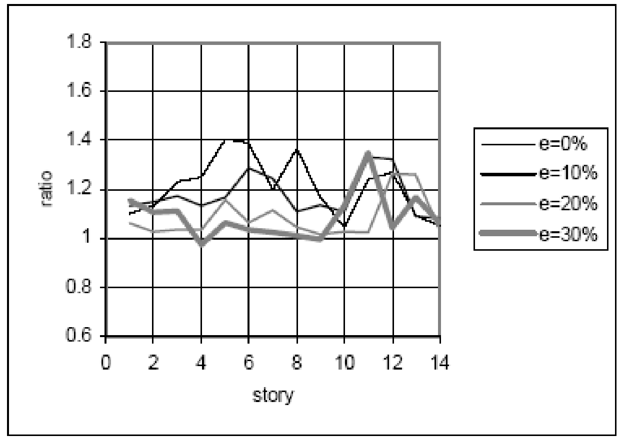
Figure 7. Ratio of drifts for 7 story models from inelastic dynamic analyses

Comparing the results for the three buildings when eccentricity exists, indicates that with increase in eccentricity the effect of P-Delta varies. It can be also observed that for buildings with moment resisting system (7 and 14 story), effect of torsion (increase in eccentricity) on P-Delta increases with increase in number of stories of buildings.

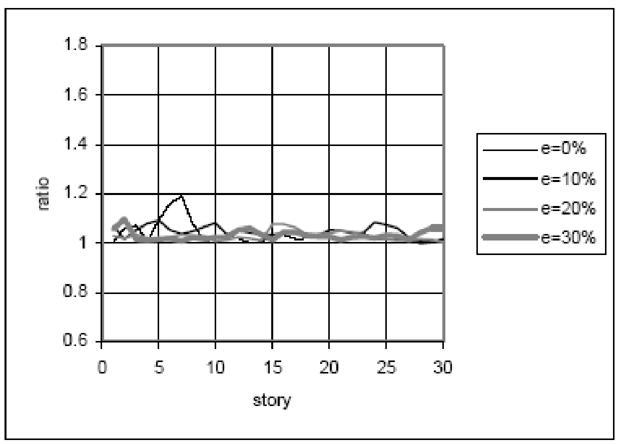
In figure 8, the curves in graphs (a) and (b) of the figure are more separate that curves in graph (c). As the first two graphs are for moment resisting frames and the other one for dual system, it can be concluded that the "Importance of torsion on P-Delta effect" mainly depends on the type of lateral load resisting system of buildings.



(a) 7 story



(b) 14 story



(c) 30 story

Figure 8. Maximum ratio of drifts for 7, 14 and 30 story models from inelastic dynamic analyses

CONCLUSIONS

In this paper the interaction of asymmetry of building on the P-Delta effects in elastic and inelastic ranges of behavior is evaluated. Contributions of lateral load resisting system, number of stories, degree of asymmetry, and sensitivity to ground motion characteristics are assessed. Four buildings with 7, 14, 20 and 30 story are designed based on typical design procedures, and then their elastic and inelastic static and dynamic behavior, with and without considering P-Delta effects, are investigated. Each building is considered for 0%, 10%, 20% and 30% eccentricity levels. The main results of this study are as following:

1. In the elastic static analyses, effect of P-Delta always is increasing, as number of stories of buildings or their eccentricity increases.
2. In the elastic or inelastic dynamic analyses, the effects of P-Delta sometimes increase the responses and sometimes decrease the responses. The reason is that implementing P-Delta effects in analysis causes change in stiffness matrix of building, thus the natural periods and other dynamic properties of the building will change. If acceleration response corresponding to the new natural period of building, in response spectrum of the earthquake, is less than acceleration response corresponding to the original natural period, then reduction in building responses for the case with P-Delta can be expected.
3. "Importance of interaction of torsion and P-Delta effect" mainly depends on the type of lateral load resisting system of building. The results indicate that the type of lateral load resisting system plays an important role in degree that torsion modifies the P-Delta effects. It is concluded that the characteristics of lateral load resisting system has far more importance compare with the number of stories in building.
4. It is seen that the effects of P-Delta is quite sensitive to ground motion characteristics such as the frequency content of earthquake. In inelastic analyses, the sensitivity is still important but less than the elastic dynamic cases. In general, the sensitivity to ground motion increases, as the eccentricity increases.
5. In elastic or inelastic dynamic analyses, increase in eccentricity causes change in the effect of P-Delta. The change is very important in elastic analyses and is somewhat less important in inelastic analyses. However, the variation does not have a constant increasing or decreasing trend. One of the reasons is the fact that with increase in the eccentricity, the mass moment of inertia has not increased in all cases.

ACKNOLEDGEMENTS

The authors wish to thank the International Institute of Earthquake Engineering and Seismology (IIEES) for supporting the work reported in this paper.

REFERENCES

1- A. Rutenberg, "Simplified P-Delta Analysis for Asymmetric Structures", Struct. Div. ASCE, P1993-2013 (1987).
2- Wilson, E.L., Eeri, M. and Habibullah, A. "Static and Dynamic Analysis of Multi Story Building Including P-Delta Effects", Earthquake spectra, 3, 2 (1987).
3- Wynhoven, J. H. and Adams, P. F., "Behavior of Structures Under Loads Causing Torsion", J. Structural Div. ASCE 98, No. ST7, 1361-1376, July 1972.
4- Chen, W.F. and Lui, E.M. "Stability Design of Steel Frame", CRC Press, Boca Raton, FL (1991).
5- Goto, Y. and Chen, W.F. "Second order Analysis for frame design", Journal of Structural Engineering, ASCE, 113, 7 (1987).
6- Nixon, D. and Beaulieu D. "Simplified Second Order Frame Analysis", Canadian Journal of Civil Engineering, 2, 4, (1975).
7- Rutenberg, A. "A Direct P-Delta Analysis Using Standard Plane Frame Computer Programs", Computer and Structures, 14, 1-2 (1987).