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CRITICAL BUCKLING LOAD OF MULTI-STORY R/C BUILDINGS

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SUMMARY

As regards the check of influence of P-Δ effects on the structures, the recommendations of the modern seismic Codes present serious problems of applicability. In the present paper, formulae are given to determine the critical buckling load in plane multistory reinforced concrete frames. By use of the critical buckling load of a frame, the importance of the influence of such effects can be directly estimated, in any case. For the purposes of the present work extended parametric analyses of plane multistory frames have been carried out in order to find their critical buckling load. Based on the estimation of the results of these parametric analyses, appropriate formulae are proposed which give, with a sufficient in practice accuracy, the critical buckling load of frames directly obtained from their elastic and geometric characteristics without the requirement for an additional analysis to be previously performed.

INTRODUCTION

The phenomenon of buckling in buildings, in spite of the research efforts already made, continues to attract strongly the interest of the investigators. The critical buckling load means the initiation of instability in the loaded structure. It was Euler who first determined the critical buckling load by closed mathematical formulae in case of simple beams with several support conditions at their ends. However, the extension of these closed equations in multistory frames is impossible, thus approximate computational methods are applied. Because of the difficulty of determination of the critical buckling load of the structures, it is usual to check the so-called "higher order effects" by use of simpler procedures (Neuss & Maisson [1], Penelis [2], Rutenberdg [3]). The recommendations of the modern seismic Codes aim to reduce the second order effects in the seismic response of multistory buildings, such as the influence of the P-Δ effects and the curving of their vertical structural elements. The P-Δ effects create additional horizontal forces at the levels of the stories, because of the 2nd order moments developed by the axial gravity loads on the vertical stiffness elements of the structure. Also, the curving of the vertical structural elements always affects unfavorably the results of the response, because the equilibrium equations for the P-Δ effects are written with respect to the deformed axis of the structural elements. As a result of the influence of the P-Δ effects, as well as of the curving of the vertical structural elements, the relative displacements of their ends are increased. The modern seismic Codes (FEMA 356/2000 (sect.3.2.5.1.1), UBC-97 (sect.1910.11.4.2), Eurocode No8-2003 (4.4.2.2[2]) and Greek Seismic Code-2003 (EAK-2003 (4.1.2.2[1]))), try to confront the influence of such effects by the use of the index θ of

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relative displacement ability. However, this index is not suitable for the majority of the structures and specifically for all the frames except of the shear building frames (Anastassiadis et all [4]).

In order to avoid the phenomenon of buckling of the structures, by accounting for the 2^{nd} order effects too, the use of the stability index θ_e is proposed which is defined by the ratio P/P_{cr}, where P the total vertical load of the structure and P_{cr} its critical buckling load. However, for the determination of the stability index θ_e, a necessary prerequirement is to find the critical buckling load P_{cr} of the structure. In the present work, formulae are given for the determination of the critical buckling load for plane multistory frames and relevant results are presented after an extended parametric analysis. These frames are divided to three groups: (a) multistory frames with a prominent shear deformation in elevation, (b) multistory frames with a prominent bending deformation in elevation and (c) multistory frames with a mixed (bending-shear) deformation in elevation. For the two first groups suitable approximate formulae are given for the determination of the critical buckling load P_{cr} of the frames directly from their elastic and geometric characteristics, while for the third group the results are satisfactory but a more systematic investigation is needed in order to reach final conclusions.

METHODOLOGY

For the purposes of the present work, the computer finite element program SAP2000v8.00 was used. The compressive axial force reduces the stiffness of an element of a structure, because of the presence of a geometric nonlinearity. By the incremental increase of the total vertical load, a value of the load is reached for which in some diagonal term of the stiffness matrix a negative number or zero appears, having as consequence the interruption of the analysis because the stiffness matrix will be no more positive definite. This value of the load is the critical buckling load P_{cr} of the structure. If the zero term appears in a local stiffness matrix of a structural element then it is mentioned as a local buckling of limited extent, while if it appears in the global stiffness matrix of the structure, it is mentioned as a global buckling of the building. In the present work, the results always refer to the smallest of the buckling loads.

The various parameters affecting the critical buckling load P_{cr} of the plane multistory frames were isolated and separately examined and they are the following:

(a) the number of stories N,

(b) the ratio a/h_c where a is the section side of the column which belongs to its bending plane and h_c is the height of the story,

(c) the ratio $\rho = \dfrac{\sum E \cdot I_b/\ell_b}{\sum E \cdot I_c/h_c}$ which is used in the plane frames with a prominent shear deformation (I_b, I_c, ℓ_b, h_c as defined in fig.1,2),

(d) the static eccentricity e of the structure, that is the horizontal distance of the stiffness center of the structure from the resultant of the vertical loads,

(e) the amount of the structural walls in the flexural frames and mixed frames too.

All the above parameters were appropriately connected to each other in the following proposed formulas for the calculation of the critical buckling load of the structures and each one was directly determined from the elastic and geometric data of the structures, without the prerequirement of another structural analysis.

TYPES OF 2D-MULTISTORY R/C FRAMES

The plane multistory frames are divided, depending on the type of deformation over their height, to frames with deformation of bending, shear and mixed type. Consequently, in every multistory frame under consideration, an estimation of the coupling between its bending and shear behavior must be done first. For this purpose, various parameters of different degrees of accuracy can be used, which the following two are mentioned:

A. The parameter of the amount of structural walls in R/C frames. This is a simple and empirical parameter, which represents the ratio of section area of the structural walls with respect to the area of all above stories (in a similar way to that prescribed by the first Greek Seismic Code of 1959). After relevant investigation in a sufficient number of frames it was obtained that if the total section area A_w of the structural walls in every story is greater than the 1/1500 of the sum $\sum A_{f,i}$ of areas of all above stories, then in the frame the bending deformation predominates.

In more detail it was obtained that:

- if $A_w \geq \frac{\sum A_{f,i}}{150}$ then a clear bending deformation exists,
- if $A_w \geq \frac{\sum A_{f,i}}{1500}$ then mainly bending deformation exists,
- if $\frac{\sum A_{f,i}}{1500} > A_w \geq \frac{\sum A_{f,i}}{8000}$ then mixed deformation with a strong coupling of bending and shear behavior appears,
- if $\frac{\sum A_{f,i}}{8000} > A_w \geq \frac{\sum A_{f,i}}{18000}$ then mixed deformation appears with predominant shear deformation,
- if $A_w \approx 0$ and $I_b >> I_c$ then a clear shear deformation exists.

B. The parameter $\lambda \cdot H$. This is considered as the most accurate parameter in order to recognize the type of a frame. This quantity is not empirical and results from the study of continuous systems, where H is the total height of the frame and $\lambda = \sqrt{G \cdot A_s / E \cdot I}$, ($G \cdot A_s$ is the shear stiffness of the section of the shear subsystem and $E \cdot I$ is the bending stiffness of the section of bending subsystem). When $\lambda \cdot H \leq 1$ the system is characterized as a mainly bending system, when $\lambda \cdot H \geq 15$ the system is characterized as one of shear type and when $1 < \lambda \cdot H < 15$ the multistory system is a mixed one. More specifically, when $1 < \lambda \cdot H < 6$, in the mixed system, the bending subsystem is stronger, while, for $6 < \lambda \cdot H < 15$ in the mixed system, the shear subsystem is stronger. In the present work an investigation was carried out to approximately correlate the quantity $\lambda \cdot H$ with the "divisor" of the sum $\sum A_{f,i}$ of the areas of all above stories, so that to obtain the total area A_w of the sections of structural walls in every story. This correlation is shown in the Table 1.

TABLE 1. Correlation of the quantity $\lambda \cdot H$ with the divisor, which yields the A_w

$\lambda \cdot H$	0	0.3	0.5	0.7	1	6	15	20	∞
Divisor	150	500	800	1000	1500	8000	18000	23000	–

At this point it is useful to refer in detail to the determination of the quantity $\lambda \cdot H$. Starting from the formation of the continuous shear subsystem, we replace the plane multistory frame by a shear column. For this purpose we first separate a diaphragm (story level) with its columns up to the middle of the above and the below story (near the zero point of bending moments of the column, see figure 1) by considering at same time that the stiffness of the beams is uniformly distributed over the height ($h_c/2 + h_c/2$) of the story level under consideration. If $Q = \sum_i Q_i$ is the total shear force developed in the columns for a relative horizontal displacement δ of the top of the story with respect to its base, then the slope of column axis is $\gamma = \delta / h_c$. Then, we simulate the story level under consideration by a shear column element whose section has an equivalent shear stiffness $G \cdot A_s$ and exhibits the same, with the frame, slip angle $\gamma = Q / (G \cdot A_s)$. Consequently, the equivalent shear stiffness of shear column section is $G \cdot A_s = (h_c \cdot Q)/\delta$.

For the determination of the equivalent shear stiffness $G \cdot A_s$ first the shear forces Q_i, due to of the forced unit displacement ($\delta = 1$) and then the resultant $Q = \sum_i Q_i$ of the shear forces are calculated (figure 2). Finally, from the formula $G \cdot A_s = (h_c \cdot Q)/\delta$ the required quantity is directly determined. Alternatively to the above procedure, for the determination of the $G \cdot A_s$, we can use other approximate formulas (Anastassiadis [5]).

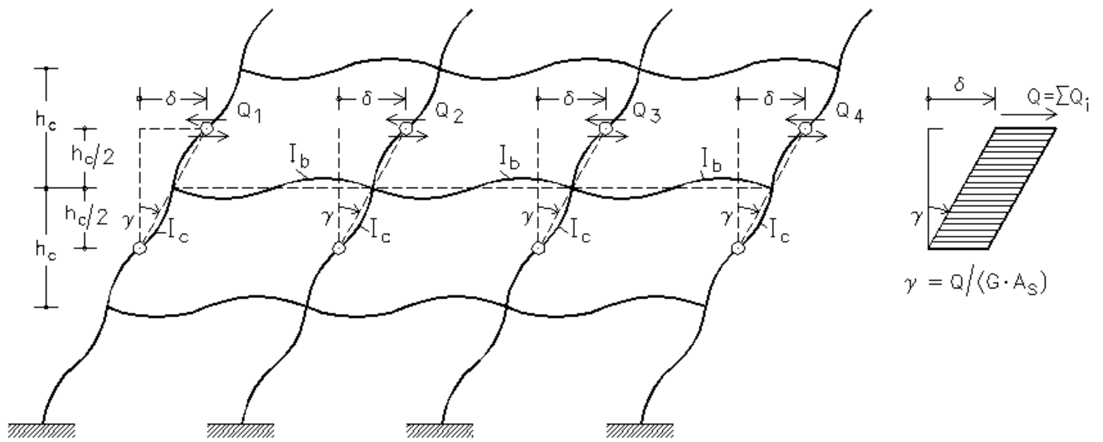

Figure 1. Formation of the continuous model of the shear subsystem

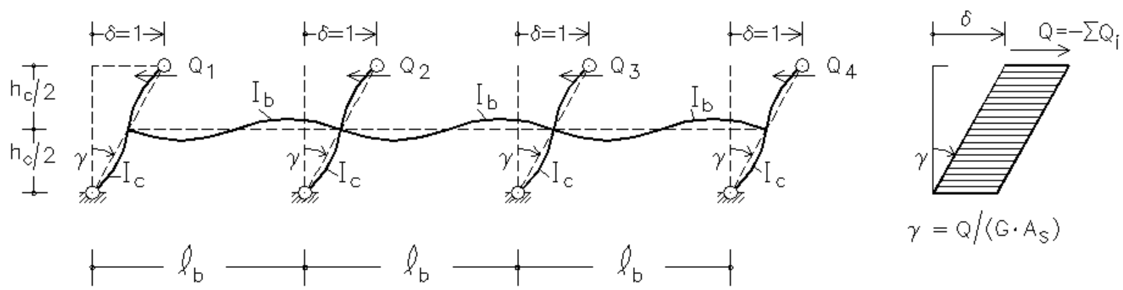

Figure 2. Determination of the equivalent shear stiffness of the frame

CRITERIA FOR THE INFLUENCE OF THE BUCKLING EFFECTS

Stability index θ_e of a frame

As stability index θ_e of a frame is defined the ratio of the total vertical service load P at the base of the structure with respect to the critical buckling load P_{cr}:

$$\theta_e = \frac{P}{P_{cr}} \tag{1}$$

The stability index θ_e of a frame is a characteristic number of the structure, is independent from the horizontal loading (within the validity of the linear stability) and it absolutely depends on the geometric and elastic characteristics of the structure as well as on the distribution of the vertical loading.

In the case that the stability index θ of a frame does not exceed the value 0.10, then the check of the P-Δ effects can be omitted.

In the case that $0.10 < \theta_e \leq 0.20$ then the P-Δ effects can be approximately taken into account by means of an increase by $1 / (1 - \theta)$ of all the 1^{st} order response quantities. Finally, the θ_e must not exceed the value 0.20 in any case.

Index θ of relative displacement ability

The contribution of 2^{nd} order effects on the stability of the structure becomes more significant as the degree of plastic yield of the structure under seismic action increases. Also, this contribution becomes critical just before the appearance of the collapse and while the structure is already subjected to an extended plastic yield. For this reason, the modern seismic codes aim, even in case of linear elastic analysis, to restrict the 2^{nd} order effects to very low levels so that to reduce their action to a significant degree, in the case that an extended plastic yield of the structure appears. Indeed, according to the section 4.1.2.2 of the Greek Seismic Code-2003 (EAK-2003), the index θ of the relative displacement ability is given by the equation (2):

$$\theta = \frac{N_{tot} \cdot \Delta}{V_{tot} \cdot h_c} \qquad (2)$$

where N_{tot}, V_{tot} are respectively the total axial and shear force of the vertical structural elements of the story, h_c is the story height and Δ is the computational relative displacement of story plates (FEMA 356/2000 (sect.3.2.5.1.1), UBC-97 (sect.1910.11.4.2), Eurocode No8-2003 (4.4.2.2[2])).

According to EAK-2003, when in every story the index θ does not exceed the value 0.10 then the check of P-Δ effects can be omitted. This practically means that when $\theta \leq 0.10$, then the influence of P-Δ effects in every story induces a limited increase of the response, which is equivalent to an amount less than 10% of the 1^{st} order response quantities.

In the case that $0.10 < \theta_e \leq 0.20$, it is allowed to approximately take into account the P-Δ effects by means of an increase by $1 / (1 - \theta)$ of all the 1^{st} order response quantities. It is not allowed for the θ to exceed the value 0.20 in any case according to the EAK-2003, whereas according to the previous edition of the Greek Seismic Code-1992, the corresponding limit was 0.30. However, the equation (2), which gives the above-mentioned index θ, is valid only for a one-story cantilever column. So, the extension of its application on multistory plane frames of shear type is abusive, because even from a single-story one-bay frame (frame with two columns), the index θ begins to exhibit divergences. Also, a significant disadvantage of equation (2) is the fact that before its application a static analysis is required to determine the V_{tot} and Δ.

The above equation (2) is not valid to mixed systems (with a bending-shear deformation in elevation) because of the bending behavior of the structural walls. Also, it is not valid to space asymmetric systems, because, in this case, the story displacement under consideration is not uniquely defined (relative displacement in the gravity center of the story as proposed by the Greek Seismic Code of 1992 or the relative displacement in a perimeter frame of the same story as proposed by EAK-2003 or the relative displacement at another point). Finally, the above equation (2) is not valid in the case of use of response spectrum analysis, because the term V_{tot} has not a physical meaning and consequently it cannot be determined. That is, it is indirectly accepted by EAK-2003 that in the case of an irregular frame, in which the use of response spectrum analysis is necessary, we must, for the check of the 2^{nd} order effects, to perform a static analysis so that the determination of the index θ of relative displacement ability become possible.

Criterion of non-displacement of frames

According to the Greek Reinforced Concrete Code of 2000, when, in the structures, the following relations (3) and (4) are satisfied, then the frames under consideration are assumed as "non-displaced", that is, it is allowed to use the 1st order analysis by ignoring the 2nd order effects. However, it is noted that the isolated check, in 2nd order effects, of separated columns must be performed in any case:

$$H \cdot \sqrt{N_{tot} / (E \cdot I_c)} \leq 0.20 + 0.10 \cdot n \qquad \text{for} \quad n \leq 3 \qquad (3)$$

$$H \cdot \sqrt{N_{tot} / (E \cdot I_c)} \leq 0.60 \qquad \text{for} \quad n \geq 4 \qquad (4)$$

where n is the number of stories, H the total height of the structure, $E \cdot I_c$ is the total bending stiffness of the sections of all vertical structural elements (structural walls and columns non-interrupted over the height of the building) and N_{tot} is the sum of all the vertical service loads at the base of the building.

Criterion of non-displacement of frames based on the parameter λ·H

Within the present work, an investigation was also performed aiming to estimate the size of the parameter λ·H which must be possessed by a multistory structure, so that it will be safely characterized as "non-displaced". By using as criterion that "the influence of P-Δ effects in every story must not load more than 10% the response of 1st order analysis", it resulted that in order, for the structure, to be "non-displaceable", $\lambda \cdot H \leq 0.70$ must hold. In a different case, the frame is considered as "displaceable", according to Greek Reinforced Concrete Code of 2000. In last case, the 2nd order effects are significant and more investigation is needed to confront this problem.

Taking into account the correlation of the quantity λ·H and the "divisor" shown in Table 1, we can approximately use the "divisor" to characterize or not a frame as "non-displaceable". After a relevant investigation, it resulted that when the total section area of structural walls in every story is greater than 1/1000 of the sum of areas of all the above stories, then the frame is considered as "non-displaceable". In a different case the frame is considered as "displaceable", the 2nd order effects begin to become significant and more investigation is needed to confront the problem.

CRITICAL BUCKLING LOAD OF THE STRUCTURES

Critical buckling load of isolated columns according to Euler formula

The critical buckling load of a column according to Euler formula refers to an ideal bar, for which the differential equation of the displacements of its deformed configuration has been written and its solution resulted, for various support conditions:

- Cantilever beam $\qquad P_{cr} = \dfrac{\pi^2 \cdot E \cdot I}{(2 \cdot H)^2}$
- Two-hinged ends column $\qquad P_{cr} = \dfrac{\pi^2 \cdot E \cdot I}{(H)^2}$
- One fixed end-column $\qquad P_{cr} = \dfrac{\pi^2 \cdot E \cdot I}{(0.7 \cdot H)^2}$

- Two-fixed ends column $$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{(0.5 \cdot H)^2}$$

where E·I is the bending stiffness of the section, and H the column length. A characteristic mark of the above relations is that the coefficient of the total length H of the column is variable, which is the buckling length of the bar is variable.

Critical buckling load of frames

The plane multistory shear frame can be approximately considered as a vertical cantilever beam with a constant section over its height. Thus, in order to approximately determine the critical buckling lead of the frame, and after a substitution of the real structure by an equivalent cantilever column, we can use one of the above Euler formulas, by appropriately modifying the buckling length. Similar research efforts with different, however, methodologies have been made in the past (MacLeod & Zalka [6], Zalka & MacLeod [7]). So, the critical buckling load of multistory plane frames could be given from the general relation (5):

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{(v \cdot H)^2} \tag{5}$$

where v the coefficient yielding the buckling length of the ideal substitute cantilever column.

The ideal column substituting the multistory frame of total height H, is considered as having a height H equal to the total height of the initial multistory frame, the same modulus elasticity (with that of the initial frame) and a constant section over its height with inertia moment I_{tot} around a horizontal axis, vertical to the bending plane of the frame. We accept that the inertia moment I_{tot} is equal to the sum of inertia moments of all columns of a story level, usually that of the base of the frame. So, we consider that all the parameters affecting the problem of buckling of a frame are taken into account by means of the coefficient v. For the determination of the coefficient v an extended parametric analysis was performed by separating the various parameters affecting it, as already mentioned in previous section. Also, it resulted that this coefficient can be obtained by the following formula:

$$v = \lambda_1 \cdot v_o = \lambda_1 \cdot (v_{o1} + v_{o2}) \tag{6}$$

By the quantity λ_1 the relative stiffness of beams with respect to columns is taken into account in a story level of the frame. By the quantity v_{o1} the slenderness of columns of stories is taken into account depending on the ratio a / h_c, where a is the section side of the column belonging to its bending level (of the structural element) and h_c is the story height. Finally, the quantity v_{o2} takes into account the influence of number N of the stories of frame. From the evaluation of the above parametric analysis the following methodology resulted depending on the type of the frame and the number of its stories.

Critical buckling load of frames with predominant shear deformation

In the shear type frames, first the coefficient $\rho = \frac{\sum E \cdot I_b / \ell_b}{\sum E \cdot I_c / h_c}$ is calculated where E is the elasticity modulus of the structural material, $\sum E \cdot I_b / \ell_b$ the sum of all ratios $E \cdot I_b / \ell_b$ of all beams of a story (I_b , ℓ_b inertia moment of section and length, respectively, of a beam b, see figures 1, 2), $\sum E \cdot I_c / h_c$ the sum of all ratios $E \cdot I_c / h_c$ of all columns of a story (I_c, h_c inertia moment of section and length, respectively, of a column c, see figures 1, 2).

In figure 3, the variation of coefficient v with the increase of number of stories and of the ratio a/h_c is presented. From the results of the above investigation, is shown that in shear type plane frames, the influence of static eccentricity e has not a particular significance, in contrast with the bending type frames.

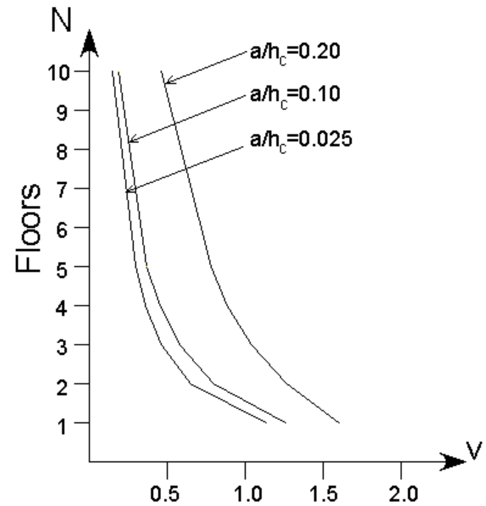

Figure 3. Variation of coefficient v versus the increase of number of stories and of ratio a/h_c

Case A: Number of stories N ≤5

The quantity λ_1 is given by the following equations of the curves of figure 3:

$$\lambda_1 = 0.30 + 2.58 \cdot \rho - 1.11 \cdot \rho^2 \qquad \text{for} \quad \rho \leq 1.40 \,,$$

$$\lambda_1 = 0.001 \cdot \rho + 1.70 \qquad \text{for} \quad \rho > 1.40 \,.$$

If in any case results $\lambda_1 > 2.50$, then we set $\lambda_1 = 2.50$. The quantity v_{o1} is given by the following equation of the curves of figure 3:

$$v_{o1} = 0.306 - 0.48 \cdot \left(\frac{a}{h_c}\right) + 11.20 \cdot \left(\frac{a}{h_c}\right)^2 \qquad \text{for} \quad 0.025 \leq \frac{a}{h_c} \leq 0.20$$

Values with ratios $a/h_c > 0.20$ show frames not belonging to the group of shear systems. The quantity v_{o2} is given by the following equation of the curves of figure 3:

$$v_{o2} = 0.056 \cdot N^2 - 0.562 \cdot N + 1.41$$

Case B: Number of stories N ≥6

$$\lambda_1 = 0.02 + 3.56 \cdot \rho - 1.40 \cdot \rho^2 \qquad \text{for} \quad \rho \leq 1.40 \,,$$

$$\lambda_1 = 0.001 \cdot \rho + 2.24 \qquad \text{for} \quad \rho > 1.40 \,.$$

If in any case results $\lambda_1 > 3.50$, then we set $\lambda_1 = 3.50$.

$$v_{o1} = 0.306 - 0.48 \cdot \left(\frac{a}{h_c}\right) + 11.20 \cdot \left(\frac{a}{h_c}\right)^2 \qquad \text{for} \quad 0.025 \leq \frac{a}{h_c} < 0.30$$

$$v_{o2} = 0.03 \cdot (5 - N)$$

If in any case results $v_{o2} \leq -0.27$ then we set $v_{o2} = -0.27$.

It is noted that in the case $N \geq 6$, the coefficient v_{o2} results always negative.

Critical buckling load of frames with predominant bending deformation

In the figure 4 is presented the variation of coefficient v versus the increase of stories number and of the ratio a/h_c. Here, we observe that in the bending type plane frames, the influence of static eccentricity e is significant for a story number up to 5. In figure 4 the influence of static eccentricity is shown (as a part of the total length L of the frame).

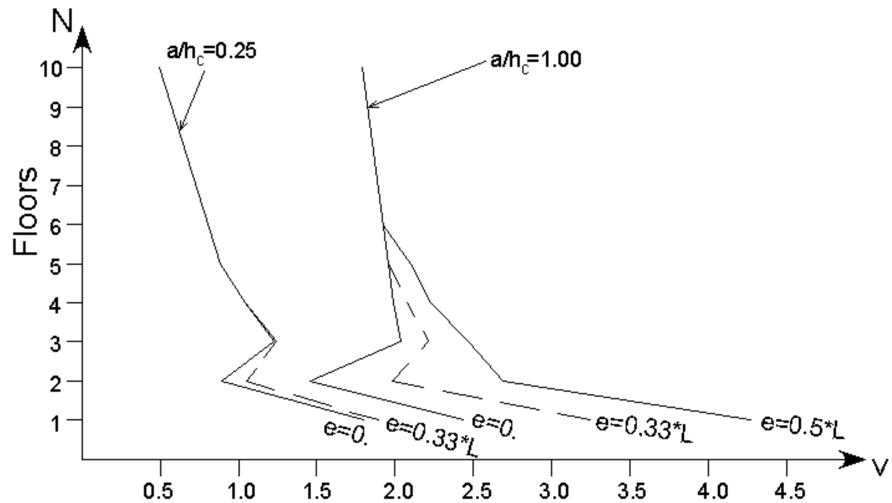

Figure 4: Variation of coefficient v versus the increase of number of stories and of the ratio a/h_c.

Case A: Number of stories $N \leq 5$

The quantity v_{o1} is given by the following equation of the curves of figure 4:

$$v_{o1} = 1.161 + 0.902 \cdot \left(\frac{a}{h_c} \right)$$

The quantity v_{o2} is given by the following equation of the curves of figure 4:

$$v_{o2} = 5.68 - 1.58 \cdot N + 0.17 \cdot N^2$$

The static eccentricity e affects the bending type frames up to five stories. The above formulae describe the envelope of the curves of figure 4 for the maximum eccentricity e = 0.50*L and so, the static eccentricity is eliminated from the formulas under consideration.

Case B: Number of stories $N \geq 6$

The quantity v_{o1} is given by the following equation of the curves of figure 4:

$$v_{o1} = 1.161 + 0.902 \cdot \left(\frac{a}{h_c} \right)$$

The quantity v_{o2} is given by the following equation of the curves of figure 4:

$$v_{o2} = 0.05 \cdot (5 - N)$$

Table 2: Values of the quantity λ_1

	10-story	5-story	Single-story
Structural walls coupled by a diaphragm plate	0.60	0.85	1.00
Structural walls coupled by a usual plate-beam	2.50	1.75	1.00
Structural walls coupled by means of infinite inertia moment	35.00	23.00	1.50

If in any case results $v_{o2} \leq -0.70$ then we set $v_{o2} = -0.70$.

It is noted that in the case $N \geq 6$, the coefficient v_{o2} results always negative. Finally, in the bending frames, the Table 2 approximately gives the quantity λ_1. For cases with a different number of stories, a linear interpolation is sufficient.

Critical buckling load of frames with mixed deformation

In this case, we first determine the critical buckling load of the two subsystems and then, by an appropriate interpolation and depending on the value of parameter $\lambda \cdot H$ (or even on the amount of structural walls), the critical buckling load is estimated. This procedure showed that, in a limited number of frames, which were studied, the results can be considered as satisfactory, however a more systematic investigation is needed. For the moment, the only reliable solution is the use of a special purpose computer program to determine the critical buckling load P_{cr} of the structure, and then the stability index θ_e is calculated from the formula (1).

CONCLUSIONS

In the present work, formulae were given to determine the critical buckling load of plane multistory frames directly from their elastic and geometric characteristics without the requirement for another previous analysis. The critical buckling load is calculated in any case from the equations (5) and (6), while as a criterion to avoid buckling effects the stability index θ_e of the frame is proposed (eq. 1). The various parameters affecting the critical buckling load were separated and examined, such as the number N of stories, the ratio a/h_c, the ratio ρ for plane frames with predominant shear deformation, the static eccentricity e of the structure and the amount of structural walls in the bending type frames. Finally, the frames were divided to groups depending on the type of their deformation, by use of the amount of structural walls or the dimensionless quantity $\lambda \cdot H$.

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