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NON-LINEAR INTERACTION OF NORMAL AND TANGENTIAL INTERNAL FORCES ON 3D RC BEAM-COLUMN STRUCTURAL SYSTEMS

J. M. BAIRAN GARCIA[1], A. R. MARI BERNAT[2]

SUMMARY

The accurate behaviour assessment of complete structural systems is essential in developing Performance Based Engineering methodologies. In this aspect, structural models based on beam-column elements are still a competitive option compared to solid-FEM models; mainly because of the practical data generation, results interpretation and the direct use of internal forces in design. Nevertheless, BC-elements are developed under plane sections hypothesis and under null or simple imposed shear deformation along cross sections allowing for reasonable non-linear evaluation of normal internal forces (axial force and biaxial bending moment) but not for tangential internal forces (biaxial shear and torsion) whose influence on the behaviour of structural systems with irregular distribution of stiffnesses, masses or resistances has proved to be determinant.

A NL-fibrewise sectional model for concrete structures capable of simulating the total interaction between all six beam internal forces and deformations with arbitrary shaped cross section and longitudinal reinforcement and stirrups arrangement has been developed. Crack concrete is simulated as a 3D orthotropic material which in combination with steel reinforcements yields, in general, a full constitutive matrix coupling all stress and strain components. The model enables an explicit expression of the sectional stiffness matrix considering the interaction of the complete set of internal sectional forces on a cracked section.

A reinforced concrete cross section is analysed for increasing levels of combined bending and shear loads. Noticeable dependencies on the concomitant load are shown on the overall performance of both moment-curvature and shearing force-deformation diagrams.

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[1] Ph.D. Candidate. Department of Construction Engineering. Technical University of Catalonia.Campus Nord, Jordi Girona St., Mod.C-1.Barcelona 08034, Spain. Fax: +(34) 93.401.36.80. email: jesus.miguel.bairan@upc.es

[2] Professor of Concrete Structures. Department of Construction Engineering. Technical University of Catalonia, Spain

INTRODUCTION

Accurate understanding of non-linear behaviour of complete structural systems under a broad range of loading, including ultimate loads and different levels of service loads, is an essential issue in the development of Performance Based Engineering (PBE) methodologies. This holds not only for the case of Performance Based Earthquake Engineering, but also for every design philosophy where the designer should assure a certain structural behaviour and damage states for several limit states.

In spite of the great advance that structural analysis using solid and 2D finite elements has experienced, structural models based on frame elements are, still today, the type of numerical model more employed in engineering practice, for both linear and non-linear analysis of complete structural systems, see Marí [1] and Marí [2]. Now a day, this is mainly because of the more practical and direct generation of data and interpretation of the analysis results rather than to computational limitations. Although this last aspect continues being important.

Nevertheless, traditional frame elements are only capable to represent appropriately, at non-linear level, the structural behaviour under axial stresses. This is due to the absence of a theory that allows incorporating the influence of shear stresses in this type of elements with similar success as the Navier-Bernoulli plane section hypothesis does for axial forces and bending moments. Shear forces and torsion moments effects are then considered, in general, in a very simplified and decoupled fashion.

The behaviour of many structures can be considered dominated by axial stresses; being, in such cases a traditional frame model acceptable. But very often, frame structures are submitted to a combination of forces in which both axial and shear stresses are of similar importance, thus the performance – in terms of stiffness, yield load and ultimate load - evaluated from a decoupled frame model shall differ from the real behaviour. Structures presenting this behaviour are not so infrequent. Usually they are characterized by an irregular stiffness configuration, Calvi and Pinto [3]; this is the case of most viaducts. Also, curve and skew bridges may exhibit this strong interaction since torsion and flexure effects are cinematically coupled.

In this paper, the non-linear coupled behaviour of reinforced concrete (RC) sections is studied by means of a previously developed numerical model, Bairan [4] which takes into consideration the 3D state of stresses on each concrete fibre, internal equilibrium and compatibility in the cross section's domain and material anisotropy induced by generated cracks in concrete. In the development of the sectional model, sufficient generality has been considered in order to allow any arbitrary shape of the cross section as well as different materials and reinforcement arrangements.

Two main peculiarities exhibit this model compared to others coupled models, for instance Petrangeli et al [5]. First, it allows the full three dimensional interaction of axial force, skew shear forces, skew bending and torsion moments regardless of the shape of the cross section. Second, an explicit sectional stiffness matrix can be written showing the terms that actually couples all internal forces on the section. In the case of a cracked section, this matrix has shown to be full.

A beam specimen tested under shear forces by Kani [6] has been analysed modelling only one cross section in order to show the validity of the model. Further, moment-curvature and shear-deformation curves of the same cross section are obtained under increasing simultaneous shear forces and bending moments respectively. Results show that when the simultaneous actions are strong enough, influence is exhibited on these characteristic curves, not only in terms of ultimate and yield loads, but also in terms of their overall shape and pre-yield stiffness.

DESCRIPTION OF THE SECTIONAL MODEL

A numerical sectional model, developed by Bairan [4], for the analysis of cracked concrete cross sections under general 3D coupled axial and shear forces will be briefly described here. The cross section is considered as a 2D domain submitted to the traditional sectional deformations, i.e. axial elongation, skew shear deformation, skew bending curvatures and a twisting curvature. From this domain and imposed deformation, a 3D stress and strains field is to be obtained.

In order to be able to reproduce a three-dimensional strain field, the sectional displacements are expressed as a combination of the traditional Navier-Bernoulli plane section hypothesis ($\mathbf{u}^{\mathbf{ps}}$), allowing a kinematical relation between the axial strain parallel to the directrix of the bar, and a warp ($\mathbf{u}^{\mathbf{w}}$) considered a three dimensional vector field with three components, so changes in the section's shape are considered allowing elongation on the transversal reinforcements, and lateral deformation on concrete. Warp vector field is discretized on nodal values ($\mathbf{d}_{\mathbf{F}}$) along the cross section and then interpolated by means of an interpolation matrix ($\mathbf{N}_{\mathbf{F}}$), see equation (2).

$$\mathbf{u} = \mathbf{u}^{\mathbf{ps}} + \mathbf{u}^{\mathbf{w}} \tag{1}$$

$$\mathbf{u}^{\mathbf{w}} \approx \mathbf{N}_{\mathbf{F}} \mathbf{d}_{\mathbf{F}} \tag{2}$$

In addition, the warp field is considered a function of the axial elongation of the bar's axis, twist curvature, the two bending curvatures and their corresponding derivatives respect to direction of the bar's axis; each of those quantities define the components of vector $\boldsymbol{\xi}^*$. Further, the nodal values of the warp displacement are written as a multiplication of matrix **A**, as shown in the next equation, times vector $\boldsymbol{\xi}^*$.

$$\begin{aligned} &\mathbf{d}_{\mathbf{F}} = \mathbf{A}\boldsymbol{\xi}^* \\ &\boldsymbol{\xi}^* = \left[\varepsilon_0, \phi_x, \phi_y, \phi_z, \varepsilon'_0, \phi'_x, \phi'_y, \phi'_z\right]^T \end{aligned} \tag{3}$$

Matrix **A** in equation (3) has dimensions of [n X 8], and relates each component of the cross section warp to the four quantities described above and their derivatives. The expression for the complete three dimensional strain field can be written as shows equation (4). Vector $\boldsymbol{\varepsilon}_{\mathbf{s}}$ contains the components of deformation of the beam. $\mathbf{N}^{\mathbf{PS}}$ is the typical operator containing Navier-Bernoulli relationships for a plane section hypothesis. Matrix $\mathbf{B}_{\mathbf{F}}$ contains derivatives of $\mathbf{N}_{\mathbf{F}}$ and is used as an interpolation matrix for strain components due to warp.

$$\begin{aligned} &\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{\mathbf{PS}} + \boldsymbol{\varepsilon}^{\mathbf{w}} \\ &\boldsymbol{\varepsilon}^{\mathbf{PS}} = \mathbf{N}^{\mathbf{PS}} \boldsymbol{\varepsilon}_{\mathbf{s}} \\ &\boldsymbol{\varepsilon}^{\mathbf{w}} \approx \mathbf{B}_{\mathbf{F}} \mathbf{d}_{\mathbf{F}} = \mathbf{B}_{\mathbf{F}} \mathbf{A} \boldsymbol{\xi}^* \\ &\boldsymbol{\varepsilon}_{\mathbf{s}} = [\varepsilon_0, \gamma_y, \gamma_z, \phi_x, \phi_y, \phi_z]^T \end{aligned} \tag{4}$$

Provided a suitable three dimensional non-linear constitutive equation, expressed in a general fashion as in equation (5), matrix **A** is then evaluated as described in Bairan [4]. The procedure followed takes into account the three dimensional internal equilibrium equations at each material point and compatibility conditions in order to obtain a warp field that produces shear and in plane axial stresses that are compatible with the applied normal axial stresses and the shape of the cross section. The resulting matrix **A** result dependent on the shape of the cross section and on the actual non-linear state on each material point.

$$\begin{aligned} \boldsymbol{\sigma} &= \mathbf{D}\boldsymbol{\varepsilon} = \mathbf{D}\left(\boldsymbol{\varepsilon}^{\mathbf{PS}} + \boldsymbol{\varepsilon}^{\mathbf{w}}\right) = \boldsymbol{\sigma}^{\mathbf{PS}} + \boldsymbol{\sigma}^{\mathbf{W}} \\ \boldsymbol{\sigma}^{\mathbf{PS}} &= \mathbf{D}\boldsymbol{\varepsilon}^{\mathbf{PS}} \\ \boldsymbol{\sigma}^{\mathbf{W}} &= \mathbf{D}\boldsymbol{\varepsilon}^{\mathbf{W}} \end{aligned} \quad (5)$$

In order to be consistent in energy and compatibility terms, shear deformations of the beam's axis shall be obtained from the shear strain field over the cross section, which in turn has been computed from the warp field. The shear deformation of the beam element evaluated this way assures a correct computation of the energy absorbed by shear and torsion effects a more accurate estimation of the frame's deformation.

Nevertheless, in general when a skew shear is applied to an arbitrary shaped cross section built from different materials, the resulting shear deformation does not have the same skew angle as the applied shear force at each increment of forces. This represents a problem in a non-linear three dimensional analysis because two shear components are to be obtained from only one equation, i.e. the balance of energy equation. In reference [4] this is discussed and a way to handle this situation is deduced.

Finally, by equating the work performed by the stress field and the internal forces on the cross section, the following explicit expression for a total interaction stiffness matrix is obtained.

$$\begin{aligned} \boldsymbol{\sigma}_{\mathbf{s}} &= \mathbf{K}_{\mathbf{s}}\, \boldsymbol{\varepsilon}_{\mathbf{s}} \\ \mathbf{K}_{\mathbf{s}} &= \boldsymbol{\Xi}^{\mathbf{T}} \boldsymbol{\Omega}^{\mathbf{T}} \iint \mathbf{B}^{*^{\mathbf{T}}}\, \mathbf{D}\mathbf{B}^{*} dA\, \boldsymbol{\Omega}\boldsymbol{\Xi} \end{aligned} \quad (6)$$

Where $\mathbf{B}^*$ is an operator matrix for evaluating the full three dimensional strain vector from the sectional deformation vector. $\mathbf{B}^*$ depends on both matrixes $\mathbf{N}^{\mathbf{PS}}$ and **A**. Matrix **Ω** stands for the consistent evaluation of beams shear deformation as commented above, and matrix Ξ condensates the derivatives of axial elongation and torsion curvature (ε'_0 and ϕ'_x) taking into account the actual distributed axial load and torsion moment.

It can be seen that sectional stiffness matrix presented above is symmetric. More over, it satisfies internal equilibrium and compatibility conditions over the cross section. This matrix may be used on any shaped cross section, composed of different materials with different constitutive equations, even considering constitutive anisotropy

CONSTITUTIVE EQUATIONS USED IN THIS STUDY

For this study, a rotational smeared crack approach has been considered for plain concrete in order to take into account crack-induced anisotropy and its influence on shear resistance mechanism of reinforced concrete sections. An orthotropic equivalent uniaxial model has been used for represent plain concrete. Stress and strain principal directions are considered coincident and a uniaxial stress-strain equation is stated for each thee directions. In compression, each stress-strain curve maintains the same shape, however they are a function of maximum lateral strain according to the next equation from Vecchio and Collins [7]. Where ε_i is a principal compression strain, ε_{max} is the maximum lateral strain and f_c is the strength of the concrete. Equation (7) was originally formulated for analysis in plane-stress state; however it has been applied for three dimensional analyses and has shown reasonable results, Vecchio and Selby [8].

$$\sigma_i = f_{c\max}\left[2\left(\frac{\varepsilon_i}{\varepsilon_0}\right)-\left(\frac{\varepsilon_i}{\varepsilon_0}\right)^2\right]$$
$$f_{c\max} = \frac{f_c}{0.8-\left(0.34\,\varepsilon_{\max}/\varepsilon_0\right)} \le f_c \tag{7}$$

For concrete in tension, a linear elastic behaviour is considered previous cracking. After tensile strength is reached, a non-linear softening curve is considered (Cervenka [9]) in order to take into account tension-stiffening effects. In this equation, k_2 is a parameter specifying the shape of the softening branch, c is a strain parameter specifying the strain for null tensile stress on concrete. For the present study, values k_2=0.5 and c=0.004 has been considered.

$$\sigma_i = f_t\left[1-\left(\frac{\varepsilon_i}{c}\right)^{k_2}\right] \ge 0 \tag{8}$$

Shear stiffness on the principal direction of concrete is specified in order to maintain the coincidence of the stress and strain principal directions for each load increment. The following expression for the shear stiffness assures coaxiality of the numerical solution, Jirásek and Bazant [10], Bazant [11], Zhu et al [12].

$$D_{ij} = \frac{1}{2}\frac{\sigma_i-\sigma_j}{\varepsilon_i-\varepsilon_j} \tag{9}$$

Constitutive matrix **D** is assembled on a coordinate system coincident with the principal directions. After rotating this matrix to the original coordinate system is a full matrix results.

In reinforcing steel, the usual elastoplastic uniaxial stress-strain relationship, with yield stress f_y, is considered.

NUMERICAL IMPLEMENTATION

The numerical model has been implemented considering with the possibilities of some numerical facilities such as load and displacement control and line search, Crisfield [13]. Non-linear solution is reached by means of a Newton-Raphson algorithm that allows using a tangential or a secant stiffness matrix.

NUMERICAL STUDY

The critical cross section of a beam specimen tested under shear loads by Kani [6] was studied using the previously developed non-linear sectional model. The dimensions of the cross section studied is shown on figure 1 and the experimental set-up is described in figure 2. The concrete strength was 28.2 MPa, strain at peak compression stress was 0.0022 and the tensile strength was 1.75 MPa. Longitudinal reinforcement was eight bars of 25 mm diameter and yield stress of 442 MPa. Stirrups were of 10 mm diameter spaced at 100 mm and 400 MPa yield stress.

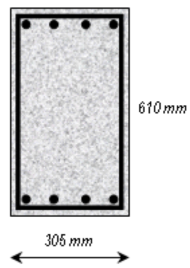

Figure 1. Cross section studied

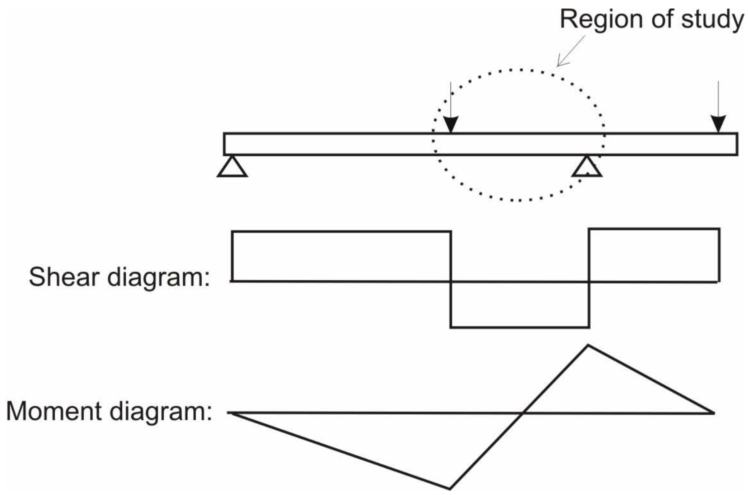

Figure 2. Test set-up used by Kani [6]

A comparison of the numerical solution of shear force versus sectional shear stress and the experimental results is presented in figure 3. Good agreement between both curves is observed.

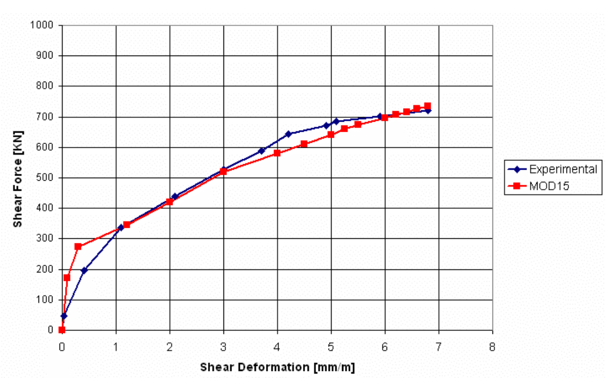

Figure 3. Sectional model compared to an experimental beam specimen under shear.

The influence of the shear-bending interaction on the performance of force–deformation diagrams of the cross section is studied by calculating shear–deformation diagrams under different levels of constant bending moments, figure 4. Under low level bending moment, shear force-displacement curves present little variation both on ultimate resistance and overall stiffness during the entire loading process. Only a reduction on the shear force producing initial crack is appreciable. When the concomitant moment increases, a significant reduce on both ultimate shear force and stiffness during the non-linear loading is observed.

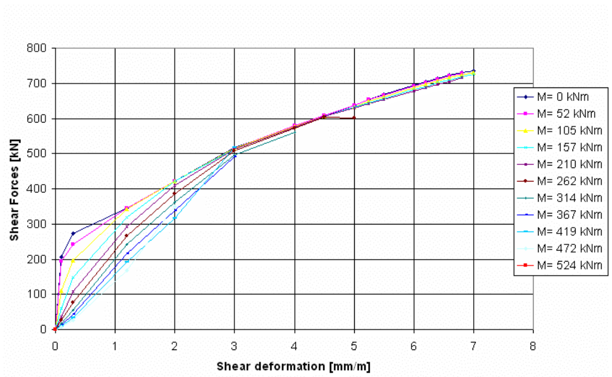

Figure 4. Influence of concomitant moment on shearing force-deformation curves.

The influence of concomitant shear forces on moment-curvature diagrams is also studied on the next figure. It is seen that for increasing shearing forces, not only the ultimate bending moment is reduced but also overall shape of the curve changes resulting on lower flexure stiffness for the cross section.

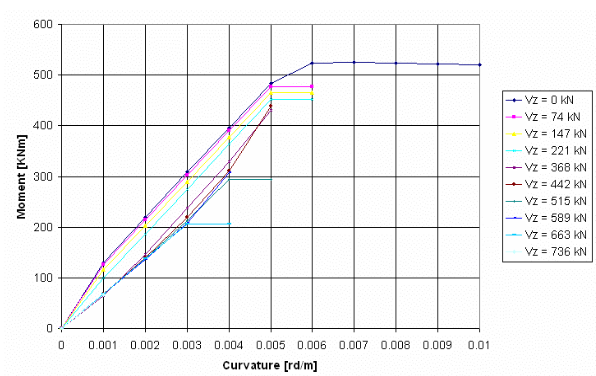

Figure 5. Influence of concomitant shearing force on moment-curvature curves.

CONCLUSIONS

A non-linear model for the analysis of reinforced concrete cross sections under combined three dimensional loading has been developed. The sectional model is general enough to allow a correct simulation of any shaped cross section built of different materials. Particularly, non-linear behaviour, and crack induced anisotropy has been taken into consideration.

An equivalent uniaxial model with rotating smeared cracks has been used as a constitutive model for concrete in this study. The influence of shear-flexure combination on the performance of a particular cross section has been investigated.

A shear test on a beam element having the same cross section has been simulated and good agreement with the experimental results have been found by analysing a single cross section under pure shear. Performance of both moment-curvature and shearing force-deformation on the cross section has shown to be very dependent on the concomitant action when these present for moderate to high value. Interaction effects were neglectable for low concomitant actions.

Influences of the axial-tangential forces interaction were present on both the ultimate and stiffness of the pre-yield branch predicted by the sectional model. It is to be expected that effects of normal and tangential interaction influence the overall performance of structural elements under general loading and complete constructions, determining the ultimate load and non-linear redistribution capacity when strong combinations take place.

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