

# A SIMPLIFIED DAMAGE MODEL FOR SHEAR DOMINATED REINFORCED CONCRETE WALLS UNDER LATERAL FORCES

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# SUMMARY

In this paper, a new model for simulating the damage of squat RC shear walls under lateral loads is proposed which takes into account the reduction in stiffness and strength due to diagonal cracking and permanent deformations that occur due to yield of reinforcement and shear slipping across cracks. The model is based on the principles of continuum damage mechanics and fracture mechanics and can be classified in the group of lumped plasticity models.

In a first section the expressions are developed for monotonic loading. A yield function is proposed and the experimental identification of a crack resistance function is explained. This crack resistance function is based on the use of the Griffith criterion of fracture mechanics. A numerical simulation is presented of a shear wall tested under monotonic loading.

In a later section the necessary expressions are developed for hysteretic type loadings and several numerical simulations of experimental tests are presented that validate the applicability of the proposed model.

# INTRODUCTION

Many types of models that allow the simulation of nonlinear behavior of shear walls have been published. These models can be classified into three large groups: a) lumped plasticity models; b) distributed plasticity models; and c) multi-layer models.

Lumped plasticity models are the simplest and easiest to implement, because they consider all material nonlinearities concentrated in nonlinear springs or plastic hinges of zero length. The nonlinear behavior of these hinges is described by more or less complicated rules. Several of the most commonly models used are those reported by Mahin [1], Bazant [2], Ma [3] and Rheinhorn [4]. The weakness of these models results from the difficulty in the choice of appropriate model parameters. These models usually represent real behavior when applied to experimental results, but when they are used to simulate the behavior of real structures there are many uncertainties in the correct choice of adequate parameters.

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Distributed plasticity models are slightly more complicated, as they take into account the distribution of inelastic effects along a finite length as described by Kunnath [5]. They are less popular than the lumped plasticity models, because they have the same shortcomings of these models with an added uncertainty in the estimation of the length of the zone where inelastic effects are distributed.

Multi-layer models, which are based mainly on the finite element method, use a representation of the member by a certain number of discrete elements. Material behavior is represented by constitutive relations that are usually well known. Results obtained with these models are very good in general; however, the computational cost and the time consumed in the preparation of the necessary input data render these models of limited use when large shear wall structures need to be modeled. Vulcano [6] analyzes several models which fall into this category comparing analytical simulations with experimental results. He concludes that those models based on a macroscopic approach are more effective than those based on a microscopic approach. Other authors as Colotti [7] and Ghobarah [8] report multi-component models similar to those analyzed by Vulcano [6], which include some refinements that allow a better representation of the nonlinear behavior, but there is basically no improvement in computational cost and ease of modeling. More recently, Mazars [9] have proposed a model based on the framework method that uses lattice meshes for concrete and reinforcement bars and uniaxial constitutive laws based on continuum damage mechanics and plasticity. The authors indicate that the success of the simulation depends on the value of the angle that the diagonal compressive trusses form with the horizontal ones and discuss some ways to estimate this value.

In this paper, a new model for simulating the damage of squat RC shear walls under lateral loads is proposed which takes into account the reduction in stiffness and strength due to diagonal cracking and permanent deformations that occur due to yield of reinforcement and shear slipping across cracks. The model is based on the principles of continuum damage mechanics and fracture mechanics and can be classified in the group of lumped plasticity models.

In the first section the expressions are developed for monotonic loading. A yield function is proposed and the identification of a crack resistance function is explained. This crack resistance function is based on the use of the Griffith criterion of fracture mechanics. A numerical simulation is presented of a shear wall tested under monotonic loading.

In a later section the necessary expressions are developed for hysteretic type loadings and several numerical simulations of experimental tests are presented that validate the applicability of the proposed model.

## STIFFNESS AND FLEXIBILITY MATRICES OF A DAMAGED SHEAR WALL.

Let us consider a planar frame made of a number of structural members linked together with rigid joints. In figure 1, a member of a frame is isolated. In order to characterize the behavior of the frame member, the same notation described by Flórez-López [10] will be used. The generalized stresses and deformations are given by the matrices  $\{M\}^t = (M_i, M_j, N)$  and  $\{\Phi\}^t = (\phi_i, \phi_j, \delta)$  respectively, where these terms have the interpretation indicated in Figure 1.(a) and (b).

In an elastic frame member these matrices are related by:

$$\{\Phi\} = [F_0]\{M\}$$
(1)



Figure 1. a) Generalized stresses b) Generalized deformations

Where [F<sub>0</sub>] is the flexibility matrix in local coordinates whose expression is:

$$[F_0] = [F_0^a] + [F_0^f] + [F_0^s]$$
<sup>(2)</sup>

The matrices  $[F_0^a]$ ,  $[F_0^f]$  and  $[F_0^s]$  represent the flexibility due to axial forces, flexure effects and shear respectively. These matrices have the following expressions:

$$[F_0^a] = \frac{L}{EA} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [F_0^f] = \frac{L}{3EI} \begin{bmatrix} 1 & -1/2 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [F_0^s] = \frac{1}{GA_v L} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3)$$

where E is the elastic modulus, A is the area,  $A_v$  is the effective shear area, I is the moment of inertia, G the shear modulus and L the length of the member. It can be seen that for large values of L, the shear term becomes small while the flexure term increases. This is the case of slender members where shear deflections can be neglected.

The modeling of damage in slender RC members based on damage and fracture mechanics has been presented by Flórez-López [10]. The goal of this paper is the analysis of structural frame members where the latter term in expression (2) is more important than the former ones. For the sake of simplicity, it will be assumed that the member does not undergo flexural related damage or plastic hinge rotations. In other words, all damage and plastic deformations are due to shear. This assumption is reasonable in the case of squat shear walls.

Following the concepts of damage mechanics used also by Flórez-López, [10], the shear damage variable  $d_s$  is introduced so that the shear flexibility of a damaged wall can be expressed as:

$$[F^{s}(d_{s})] = \frac{1}{GA_{v}L(1-d_{s})} \begin{bmatrix} 1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 0 \end{bmatrix}$$
(4)

This damage variable can take values between zero and one. A value of zero represents a non-damaged wall whose flexibility is given by the elastic flexibility matrix as shown in equation (2). A value of one characterizes a totally damaged wall with infinite flexibility. It is assumed that damage evolves continuously from zero to one as a function of the loads on the wall and after the damage law that is described in the next section. Physically, the damage variable measures the degree of concrete cracking in

the wall, i.e.  $d_s = 0$  indicates that there is no concrete cracking,  $d_s = 1$  represents a wall so cracked that it has no shear stiffness at all.

The flexibility and stiffness matrices of a damageable shear wall have the following expressions:

$$[F(d_s)] = [F_0^a] + [F_0^f] + [F^s(d_s)] \quad \text{and} \quad [S(d_s)] = [F(d_s)]^{-1}$$
(5)

Where,

$$S_{11} = S_{22} = \frac{4EI[(1-d_s)GA_vL^2 + 3EI]}{L[(1-d_s)GA_vL^2 + 12EI]}; \qquad S_{33} = \frac{EA}{L}$$
(6)

$$S_{12} = S_{21} = \frac{4EI[(1-d_s)GA_vL^2 - 6EI]}{L[(1-d_s)GA_vL^2 + 12EI]}; \qquad S_{13} = S_{23} = S_{31} = S_{32} = 0$$
(7)

It can be noticed that if the shear coefficient  $(1-d_s)GA_vL^2$  is much larger than EI, the elements of the stiffness matrix tend to the familiar terms 4EI/L and 2EI/L.

The expression of the stiffness and flexibility matrices in global coordinates (the 6x6 matrices) can be obtained from (5) and (6) by conventional methods.

#### SIMPLIFIED GRIFFITH CRITERION OF A SHEAR WALL

The complementary strain energy of a damaged frame member, W<sup>\*</sup>, can be written as:

$$W^* = 1/2\{M\}^t [F(d_s)]\{M\}$$
(8)

Therefore, the energy release rate of a damaged shear wall can be defined as:

$$G_{s} = -\frac{\partial W^{*}}{\partial d_{s}} = \frac{L(M_{i} + M_{j})^{2}}{2GA_{v}L^{2}(1 - d_{s})^{2}} = \frac{LV^{2}}{2GA_{v}(1 - d_{s})^{2}}$$
(9)

Where,  $V = (M_i + M_j)/L$  is the shear force on the member.

The Griffith criterion, which is the basis of Fracture Mechanics, states that there may be crack propagation only if the energy release rate equals the crack resistance of the wall:

$$d_s > 0$$
 only if  $G_s = R(d_s)$  (10)

Where,  $R = R(d_s)$  is the crack resistance of the wall that is assumed to be a function of the damage state of the wall. As in Fracture Mechanics, the crack resistance function has to be identified with the help of experimental results, as described in a following section.

It can be noticed that the damage in the wall indeed depends on the level of shear forces. For instance, a member subjected to pure flexure ( $M_i = -M_j \neq 0$ ) would not develop shear damage since the resulting energy release rate would be zero.

#### YIELDING FUNCTION OF A DAMAGED SHEAR WALL

Equations (1) to (3) show that deformations in a member can be separated into three terms. The first term is related to axial forces and generates no rotations, only elongation of the chord. The second term is related to flexural effects and the last term is due to shear effects. When actions on the member exceed some critical value, permanent or plastic deformations appear in the member. As mentioned earlier, it is assumed that there is no permanent elongation of the chord and there are no plastic hinge rotations. That is, there are no plastic deformations that can be related to the first and second class of deformations. Physically, this assumption means that there is no yielding of the longitudinal reinforcement.

However there may be plastic deformations due to the yielding of the transverse reinforcement. These plastic rotations are related to shear effects and will be taken into account in the model described in this paper. One particularity of the shear-related rotations is that they have the same value and sign. Therefore, the state law of a member with shear deformations, damage and plastic rotations is:

$$\{\Phi - \Phi^P\} = [F(d_s)]\{M\}$$
<sup>(11)</sup>

Where the plastic deformation matrix  $\{\Phi^{p}\}$  has the following general form:

$$\{\Phi^{p}\} = \phi_{s}^{p}(1,1,0) \tag{12}$$

The yielding function  $f_y$  that allows the computation of the shear plastic rotation  $\phi_s^p$  of a damaged shear wall can be derived from the same general principles described by Flórez-López [10]. The only difference is that the yielding function in the present case depends on the shear force V:

$$f_{y} = \left| \frac{V}{1 - d_{s}} - c_{s} \phi_{s}^{p} \right| - V_{y}$$
<sup>(13)</sup>

Where,  $c_s$  and  $V_y$  are member dependent properties. There may be plastic rotation evolution only if the yielding function is equal to zero:

$$\phi_s^p > 0 \qquad \text{only if} \qquad f_y = 0 \tag{14}$$

#### **IDENTIFICATION OF THE CRACK RESISTANCE FUNCTION**

The model that describes the behavior of a shear wall is therefore composed by the state law (11), the yielding function (13), the plastic rotation evolution law (14) and the Griffith criterion (9) and (10). It can be noticed that only the crack resistance term needs experimental identification. In order to carry out this identification, a shear wall (specimen MC-01) was subjected to a lateral loading such as indicated in Figure 2. Shear wall MC-01 was designed so that its shear strength was smaller than its flexural strength, thereby forcing a failure dominated by shear, which is in accordance with the assumptions used in the development of the present model.



Figure 2. Shear wall specimen geometry and loading

The state law (11) and the boundary conditions of the problem under consideration lead to the following relationship between force and displacement:

$$P = Z(d_{s})(t - t_{p}) \qquad Z(d_{s}) = \frac{1}{\frac{H}{3EI} + \frac{1}{GA_{v}H(1 - d_{s})}}$$
(15)

It can be noticed that the term Z represents the slope of the elastic unloading. As this slope can be measured, it is possible to compute the damage at each elastic unloading from the following equation.

$$d_{s} = 1 - \frac{1}{GA_{v}L} \left( \frac{1}{\frac{1}{Z} - \frac{H}{3EI}} \right)$$
(16)

It can be noticed that this is simply a variation of the stiffness variation method well known in continuum damage mechanics (Lemaitre [11]).

Figure 3 shows a plot of the damage due to shear as a function of the energy release rate. From these results, it is possible to formulate an expression of the crack resistance function  $R(d_s)$ :

$$R(d_s) = G_{crs} + q_s \frac{\ln(1-d_s)}{(1-d_s)}$$

$$\tag{17}$$



Figure 3. Energy release rate vs damage variable for shear wall MC-01

Two member dependent parameters are necessary to define this function:  $G_{crs}$  and  $q_s$ . A plot of the crack resistance function with appropriate values of the two parameters can also be seen in. It is evident that the general trend of the damage evolution is represented by this function.

A simulation of the test of specimen MC-01 was carried out to verify the model. The results of this simulation are shown in Figure 4. As can be seen, the proposed model represents adequately the evolution of the damage due to shear and the accumulation of plastic deformations in the wall.



Figure 4. Test of shear wall MC-01 and simulation

The proposed model has four parameters:  $G_{crs}$ ,  $q_s$ ,  $V_y$ ,  $c_s$ , which depend on the geometry of the wall and the horizontal and vertical reinforcement. Instead of estimating these constants directly, it is preferable to compute them by the numerical resolution of the following nonlinear system of equations.

In the case of monotonic loading,

$V = V_{cr}$	implies	$d_s = 0$	(18a	l)
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 $V = V_p$  implies  $\phi^p = 0$  (18b)

$$V = V_u \text{ implies } dV = 0 \tag{18c}$$

 $\mathbf{V} = \mathbf{V}_{\mathrm{u}} \quad \text{implies} \quad \boldsymbol{\phi}^{p} = \boldsymbol{\phi}_{u}^{p} \tag{18d}$ 

Where,  $V_{cr}$  is the shear that produces first diagonal crack,  $V_p$  is the shear that makes the horizontal reinforcement yield,  $V_u$  is the ultimate shear resisted by the wall and  $\phi_u^p$  is the ultimate plastic rotation resisted by the wall. It is evident that all model parameters can be calculated based on reasonably well known characteristics from conventional reinforced concrete theory.

#### HYSTERETIC MODELING

The complementary strain energy of a damaged member undergoing hysteretic actions can be written as:

$$W^* = \frac{1}{2} \left\{ \left\langle M \right\rangle_+ + \left\langle M \right\rangle_- \right\} \left\{ \left[ F \left( d_s^+ \right) \right] \left\langle M \right\rangle_+ + \left[ F \left( d_s^- \right) \right] \left\langle M \right\rangle_- \right\} \right\}$$
(19)

Where,  $\langle M \rangle_{+}$  = positive part of M and  $\langle M \rangle_{-}$  = negative part of M, ie.

$$\langle M \rangle_{+} = M \quad if \quad M > 0 \quad and \quad \langle M \rangle_{+} = 0 \quad otherwise$$
 (20a)

$$\langle M \rangle_{-} = M \quad if \quad M < 0 \quad and \quad \langle M \rangle_{-} = 0 \quad otherwise$$
 (20b)

There are now two damage variables for shear:  $d_s^+$  and  $d_s^-$ , which characterize the state of damage due to, respectively, positive and negative actions (a representation of the meaning of these damage variables can be seen in Figure 5). The use of two different damage variables allows the description of "unilateral" behavior. The term "unilateral" is associated, in conventional damage mechanics, to the assumption that the damage originated by positive actions has no influence on the behavior under negative actions and vice versa. This hypothesis must be considered as an idealization of the real behavior and not as an experimental observation.



Figure 5. Representation of positive and negative shear damage

The flexibility matrices have the same basic form of equation (5) substituting  $d_s$  for  $d_s^+$  and  $d_s^-$ , respectively for  $\left[F\left(d_s^+\right)\right]$  and  $\left[F\left(d_s^-\right)\right]$ .

Now there exist energy release rates for positive actions and negative actions:

$$G_{s}^{+} = -\frac{\partial W^{*}}{\partial d_{s}^{+}} = \frac{1}{2GA_{v}L(1-d_{s}^{+})^{2}} \left\{ \left( \left\langle M_{i} \right\rangle_{+} + \left\langle M_{j} \right\rangle_{+} \right)^{2} + \left\langle M_{i} \right\rangle_{+} \left\langle M_{j} \right\rangle_{-} + \left\langle M_{i} \right\rangle_{-} \left\langle M_{j} \right\rangle_{+} \right\}$$
(21a)

$$G_{s}^{-} = -\frac{\partial W^{*}}{\partial d_{s}^{-}} = \frac{1}{2GA_{v}L(1-d_{s}^{-})^{2}} \left\{ \left( \left\langle M_{i} \right\rangle_{-} + \left\langle M_{j} \right\rangle_{-} \right)^{2} + \left\langle M_{i} \right\rangle_{+} \left\langle M_{j} \right\rangle_{-} + \left\langle M_{i} \right\rangle_{-} \left\langle M_{j} \right\rangle_{+} \right\}$$
(21b)

In this case it can be seen that the energy release does not depend exclusively on the shear (V). However, in the case of a shear wall where there are no lateral loads applied to the member (only at nodes), the terms  $\langle M_i \rangle_+ \langle M_j \rangle_-$  and  $\langle M_i \rangle_- \langle M_j \rangle_+$  are zero because M<sub>i</sub> and M<sub>j</sub> have the same sign. Therefore, for a shear wall, the energy release rate can be written in a manner similar to the monotonic case:

$$G_{s}^{+} = \frac{V^{2}L}{2GA_{v}(1-d_{s}^{+})^{2}} \qquad and \qquad G_{s}^{-} = \frac{V^{2}L}{2GA_{v}(1-d_{s}^{-})^{2}}$$
(22)

The state law for hysteretic actions now becomes:

$$\{\Phi - \Phi^{P}\} = [F(d_{s}^{+})]\{\langle M \rangle_{+}\} + [F(d_{s}^{-})]\{\langle M \rangle_{-}\}$$

$$\tag{23}$$

The evolution of shear damage is described according to the Griffith criterion:

$$\dot{d}_s^+ > 0$$
 only if  $G_s^+ > R(d_s^+)$  and  $\dot{d}_s^- > 0$  only if  $G_s^- > R(d_s^-)$  (24)

The plastic evolution law is similar to the monotonic model, but the yield function now has two expressions, one for positive actions and another for negative actions.

$$if \left[\frac{V}{\left(1-d_{s}^{+}\right)\left(1-d_{s}^{-}\right)}-\alpha.c_{s}.\phi_{s}^{p}\right] \geq 0 \quad then \quad f_{y} = \frac{V}{\left(1-d_{s}^{+}\right)}-X-R$$

$$else \qquad f_{y} = -\frac{V}{\left(1-d_{s}^{-}\right)}+X-R$$

$$(25)$$

Where X is a kinematic hardening term, and R is an isotropic hardening term, which are defined as follows:

$$X = \alpha c_s \phi_s^p \tag{26}$$

$$R = (1 - \alpha) \cdot c_s \cdot p_s - V_y \tag{27}$$

The variable  $p_s$  is the maximum plastic rotation due to shear at any given time of the entire plastic deformation history. The parameter  $\alpha$  is a constant that takes values between zero and one. This parameter can be interpreted as the percentage of plastic hardening that corresponds to a kinematic hardening. In the numerical simulations carried out a value of  $\alpha = 0.6$  was used, therefore, this value is suggested as appropriate for this type of shear walls.

Additionally, on observation of experimental results, "pinching" effects in the hysteretic loading curves are noticeable. The evidence is that this phenomenon is due to sliding shear across the diagonal cracks as

they close after the load changes sign. In the following section the basis for the modeling of this phenomenon is explained.

#### PINCHING EFFECTS IN SHEAR WALLS

Consider an interface between two different continua as shown in Figure 6(a). Let  $\sigma$  and  $\tau$  be the normal and shear stresses on the interface. If the surface is characterized by a Coulomb friction criterion, the relative horizontal displacement h between the blocks obeys the following law:

 $\begin{cases} \dot{h} > 0 & \text{if } |\tau| - \tau_s(\sigma) = 0\\ \dot{h} = 0 & \text{if } |\tau| - \tau_s(\sigma) < 0 \end{cases}$ (28)



Figure 6. a) Interface between two media, b) Non-slip domain

Where, the term  $\tau_s$  is the slip resistance that depends on the normal stress. The non-slip domain, assuming an arbitrary resistance, is represented in Figure 6(b). It can be noticed that slip occurs when the shear stress reaches the value of the slip resistance. The latter value is not constant but depends on the normal stress. For higher values of the compressive normal stress, higher values of the slip resistance are obtained. A general presentation of interface behavior can be seen in plasticity textbooks (see for instance Salençon [12]).

# SLIP FUNCTION OF A SHEAR CRACK

The process of slip across a shear crack can also be explained in terms of Coulomb friction plasticity. Consider a shear crack in a shear wall which has formed under positive load. As the load is reduced to zero, the crack remains open. Once the load starts to be applied in the negative direction, friction across the crack is small, but as the crack begins to close, friction increases gradually, which can be seen as a gradual increase in the normal stress and consequentially in the slip resistance. Additionally, if yield of the reinforcement has occurred as the crack opens, it is evident that in order to close the crack completely, the reinforcement must be yielded in compression. Therefore, there is an interaction between two phenomena: slip across shear cracks and yield of the reinforcement. Both phenomena generate plastic rotations in the wall.

In order to model slip across shear cracks, the lumped dissipation hypothesis will be used again. Thus, it is assumed that plastic rotations generated by slip across shear cracks can be lumped into the internal variable:  $\Phi_{e}^{p}$ .

A generalization of the concept of Coulomb friction plasticity can be used to describe the behavior of an inelastic shear wall with slip. Thus, the following "slip function" is introduced:

$$f_s = |V| - k_s \tag{29}$$

Expression (29) must be interpreted as follows: there will be increments of the plastic rotations due to slip across shear cracks if the shear force reaches the critical value  $k_s$ , otherwise these increments are nil.

In the case of Coulomb friction plasticity, it is accepted that the slip critical value depends on the normal stresses on the interface. For slip across shear cracks it will be assumed that the critical value  $k_s$  corresponds to a hardening function. The analytical determination of the hardening function is a very complex problem, therefore the following phenomenological expression is proposed:

$$k_s = V_o.e^{sign(V).\gamma.\phi_s^{p}}$$
(30)

An exponential function of the plastic rotation has been chosen so that the typical pinched curves are obtained when slip is present in the wall. The term  $V_o$  will be called "slip resistance" and is a concept similar to the yield shear force in plasticity, i.e.,  $V_o$  is the shear force that produces slip when no plastic rotations have occurred yet. The computation of the parameters  $V_o$  and  $\gamma$  will be discussed in a following section.

To model sliding shear together with damage due to cracking, a slip function due to sliding shear is proposed, similar to that proposed by Picon [13],[14], for a similar phenomenon observed in beams with bond failure. This slip function ( $f_s$ ) is defined as follows:

$$if \quad \left[\frac{V}{\left(1-d_{s}^{+}\right)\left(1-d_{s}^{-}\right)}-\alpha.c_{s}.\phi_{s}^{p}\right] \geq 0 \quad then \quad f_{s} = \left|\frac{V}{\left(1-d_{s}^{+}\right)}\right|-V_{o}.e^{sign(V).\gamma.\phi_{s}^{p}}$$

$$else \qquad f_{s} = \left|\frac{V}{\left(1-d_{s}^{-}\right)}\right|-V_{o}.e^{sign(V).\gamma.\phi_{s}^{p}}$$

$$(31)$$

Now, there are two yielding functions which interact, one due to actual yielding of horizontal reinforcement and the other due to sliding shear. The function which controls the evolution of plastic deformations will be the one with the largest value at any given time as is illustrated in Figure 7. This takes into account the fact that on closure of the shear cracks, there are two effects which oppose closure: friction between crack faces and the presence of horizontal reinforcement subject to compression forces. This phenomenon is similar to the "crack closure effect" described by Ladeveze [15].



Figure 7. Interaction between yield and slip functions

Low cycle fatigue effects cannot be represented with the model proposed here, however, these effects can be included as shown by Puglisi [16] and Thomson [17].

## CALCULATION OF SLIDING SHEAR PARAMETERS

In expression (31), two new parameters are introduced:  $V_o$  and  $\gamma$ .  $V_o$  represents the value of shear force which produces slip across a crack for zero plastic rotation and  $\gamma$  is a parameter which can be calculated by solving the following equations:

when 
$$f_y = f_s = 0$$
 then  $G_s^+ = R(d_s^+)$  for positive actions  
or (32)  
when  $f_y = f_s = 0$  then  $G_s^- = R(d_s^-)$  for negative actions

As a result, the following expression is obtained for positive actions:

$$\gamma = \frac{\left(1 - d_{s}^{+}\right)}{2} \frac{\ln\left(\frac{2GA_{v}R\left(d_{s}^{+}\right)}{LV_{o}^{2}}\right)}{\sqrt{\frac{2GA_{v}\left(1 - d_{s}^{+}\right)^{2}R\left(d_{s}^{+}\right)}{L}} - \left(1 - d_{s}^{+}\right)V_{y} - \left(1 - \alpha\right)\left(1 - d_{s}^{+}\right)c_{s}p_{s}}$$
(33)

The expression for negative actions is obtained in a similar manner substituting  $d_s^-$  for  $d_s^+$ . The effect of the  $\gamma$  parameter on the hysteretic curves can be seen in Figure 8.



**Plastic Rotation** Figure 8. Effect of γ parameter

#### NUMERICAL SIMULATIONS OF EXPERIMENTAL TESTS

The model proposed in this paper has been included in the user element library of a commercial finite element program called Abaqus [18]. Several numerical simulations were carried out to validate the applicability of the model. The first simulation can be seen in Figure 9, of shear wall specimen MC-04 tested under hysteretic loads at the Universidad de Los Andes by the first author.



Figure 9. Simulation of test for specimen MC-04

Several other numerical simulations of tests reported in the literature were also carried out. The test of a specimen identified as Wall 1 in a paper by Paulay [19] and the corresponding simulation are shown in Figure 10. Note that low-cycle fatigue effects are not accounted for by the model, therefore the simulation of second cycles to a previous maximum displacement (10 mm) does not show any reduction in force. The test of a three level shear wall identified as Specimen 6, reported by Vulcano [20] and the simulation are shown in Figure 11.



Figure 10. Test of Wall 1, Paulay [19] and simulation



Figure 11. Test of Specimen 6, Vulcano [20] and simulation

## CONCLUSIONS

A simplified model for the simulation of damage to squat RC shear walls under monotonic and hysteretic lateral loads has been proposed. It is based on the notions and methods of continuum damage mechanics and fracture mechanics. It allows, at least in a qualitative manner, the representation of the following effects:

- Stiffness and strength degradation due mainly to diagonal cracking of concrete.
- Plastic deformations due to yield of the horizontal reinforcement.
- Sliding shear across diagonal cracks ("pinching effect").
- Unilateral behavior.

This model is relatively simple and the parameters that need to be defined are related to wall strength and geometry through variables that can be determined from conventional reinforced concrete theory. The model does not include low cycle fatigue effects but these could easily be included. Also, the model in its present state does not account for combined damage due to shear and flexure, as in taller shear walls, where cracking due to flexure may be more significant than cracking due to shear.

This model can be included in the library of standard finite element programs which allow for nonlinear analysis.

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