

A CONFINEMENT MODEL OF HIGH STRENGTH CONCRETE

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SUMMARY

An empirical formulation of the lateral confinement effect on high strength concrete is proposed based on a series of uniaxial loading tests on concrete cylinders. An emphasis is placed to develop unloading and reloading paths as well as the envelop of stress vs. strain relations. The proposed formula can be used to evaluate stress vs. strain relations of the laterally confined concrete with a wide range of strength between 30-90 *MPa*.

INTRODUCTION

The lateral confinement effect on concrete has been studied by many researchers to evaluate the ductility capacity of reinforced concrete columns [for example, 1, 2]. However the past confinement models were developed on the normal strength concrete and their application to high strength concrete was not verified. This paper presents a new confinement model on high strength concrete with the target cylinder strength of as high as 90 *MPa*. A series of uniaxial compression tests was conducted on 36 concrete cylinders. An emphasis was placed to develop an empirical model for unloading and reloading paths with various degrees of unloadings and reloadings.

TEST SPECIMENS AND EXPERIMENTAL SET-UP

Thirty six concrete cylinders as shown in Fig. 1 were constructed for the test. They were 600 mm tall with a diameter of 200 mm. Target concrete strength σ_{c0} was 30, 60 and 90 MPa. We used deformed bars with a 6 mm diameter and the yield strength and the strength of 321 MPa and 492 MPa, respectively (SD295) as both the longitudinal and ties bars. Four longitudinal bars were provided in all cylinders. Tie bars were set at every 50, 100 and 150 mm so that the volumetric tie reinforcement ratio ρ_s is 1.36, 0.68 and 0.45 %. Only the top and bottom of the cylinders were confined by ties with smaller spacing. To measure compression strains of a cylinder, an acrylic bar with strain gauges at every 50 mm was embedded at the center [3]. This avoided the error involved in the strain measurement due to imperfect contacts or deformation of plaster between a concrete cylinder and a loading cell. By accumulating the

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Fig. 2 Loading Hysteresis

strains along the height, an averaged cylinder strain was estimated by dividing the total compression of the cylinder by the entire height. The cylinders were loaded under the displacement control by a 5 *MN* loading machine.

Four types of loading were used in the test as shown in Fig. 2: (1) monotonic loading, (2) full unloadings and full reloadings, (3) partial unloadings and full reloadings, and (4) full unloadings and partial reloadings [4]. Unloading and reloading were repeated 1-5 times at specific strains.

STRESS VS. STRAIN RELATIONS

Failure Modes

As the intensity of loading increased, failure of the concrete progressed gradually in the cylinders with the target strength σ_{c0} of 30 and 60 *MPa*, while the cylinders with target strength of 90 *MPa* failed in a brittle manner. However the cylinders with target strength of 90 *MPa* did not fail in the brittle manner when unloadings and reloadings were repeated several times just before the concrete stress reaches the strength σ_{cc} . The post-yield behavior of the cylinders described hereinafter was evaluated under this condition.

Stress vs. Strain Relations

Fig. 3 shows an example of stress σ_c vs. strain ε_c relation of concrete with the target strength σ_{c0} of 30 and 90 *MPa*. Although the lateral confinement effect is not significant prior to the peak strength, it is significant in the post-yield range. It is important however to note that such a confinement effect becomes



Fig. 3 Stress vs. Strain Relation of Concrete with a Target Strength of 90 MPa



Fig. 4 Strength and Strain at the Strength of Unconfined Concrete

less significant as the concrete strength increases.

Fig. 4 shows how the strain at the strength of unconfined concrete ε_{c0} depends on the strength σ_{c0} . It is important to note that ε_{c0} increases as σ_{c0} increases. Although there are many studies which reported that ε_{c0} was independent of σ_{c0} , it seems that this assumption was derived from tests on the confined concrete with a narrow band strength. From Fig. 4, one obtains

$$\varepsilon_{c0} = 1.54 \times 10^{-5} \cdot \sigma_{c0} + 1.04 \times 10^{-3} \tag{1}$$

The predicted value by Eq. (1) well correlates the experimental relation as shown in Fig. 4.

Fig. 5 shows the strength σ_{cc} vs. strain at the strength ε_{cc} relation of the confined concrete. Both σ_{cc} and ε_{cc} increase as the lateral confinement $\rho_s \sigma_{sy}$ increases. They may be expressed as

$$\frac{\sigma_{cc}}{\sigma_{c0}} = \begin{cases} 1 & 0 \le \frac{\rho_s \sigma_{sy}}{\sigma_{c0}} \le 0.0022 \\ 1.825 \cdot \frac{\rho_s \sigma_{sy}}{\sigma_{c0}} + 0.996 & 0.0022 \le \frac{\rho_s \sigma_{sy}}{\sigma_{c0}} \end{cases}$$
(2)
$$\frac{\varepsilon_{cc}}{\varepsilon_{c0}} = \begin{cases} 1 & 0 \le \frac{\rho_s \sigma_{sy}}{\sigma_{c0}} \le 0.0123 \\ 7.494 \cdot \frac{\rho_s \sigma_{sy}}{\sigma_{c0}} + 0.909 & 0.0123 \le \frac{\rho_s \sigma_{sy}}{\sigma_{c0}} \end{cases}$$
(3)



Formulation of Stress vs. Strain Relation of the Envelops

As shown in Fig. 3, the concrete stress σ_c approaches to a certain level of stress σ_u (residual stress) at high strain after reaching the peak strength. The envelop of stress σ_c vs. strain ε_c relation must satisfy the following requirements.

$$\sigma_{c} = 0 \text{ at } \varepsilon_{c} = 0$$

$$d\sigma_{c}/d\varepsilon_{c} = E_{c} \text{ at } \varepsilon_{c} = 0$$

$$\sigma_{c} = \sigma_{cc} \text{ at } \varepsilon_{c} = \varepsilon_{cc}$$

$$d\sigma_{c}/d\varepsilon_{c} = 0 \text{ at } \varepsilon_{c} = \varepsilon_{cc}$$

$$\sigma_{c} = \sigma_{u} \text{ at } \varepsilon_{c} = \infty$$
(4)

in which, E_c is the elastic modulus. As a function which satisfies Eq. (4), the following stress σ_c vs. strain ε_c relation is proposed.

$$\sigma_{c} = \sigma_{u} + \frac{-\sigma_{u} + E_{c} \cdot \varepsilon_{c}}{1 + \frac{E_{c} \cdot \varepsilon_{cc} - \sigma_{cc}}{\sigma_{cc} - \sigma_{u}} \cdot \left(\frac{\varepsilon_{c}}{\varepsilon_{cc}}\right)^{n}}$$
(5)

where,

$$n = \frac{E_c \cdot \varepsilon_{cc}}{\sigma_{cc} - \sigma_u} \tag{6}$$

In this analysis, σ_u is defined as the stress at ε_c =0.02. Based on the test results, the residual stress σ_u is estimated as

$$\frac{\sigma_u}{\sigma_{cc}} = 0.315 \cdot \rho_s^{0.4} \tag{7}$$

Fig. 6 compares the predicted and experimental stress σ_c vs. strain ε_c relations of the confined concrete with the target strength of 30 and 90 *MPa*. Eq. (5) provides a good approximation for the σ_c vs. ε_c relations of the confined concrete with a wide range of strengths between 30 and 90 *MPa*.

FULL UNLOADING AND FULL RELOADING PATHS

Normalized Stress vs. Normalized Strain Relations

If a full unloading occurs from a point at a strain ε_{ul} and stress $\sigma_{ul,1}$ on a skeleton curve, the strain reaches $\varepsilon_{pl,1}$ at zero stress as shown in Fig. 2 (b). The ε_{ul} , $\sigma_{ul,1}$ and $\varepsilon_{pl,1}$ are called unloading strain, 1st unloading stress, and 1st plastic strain, respectively. If the concrete is reloaded from $\varepsilon_{pl,1}$ to ε_{ul} , the stress at ε_{ul} ($\sigma_{ul,2}$) may be smaller than $\sigma_{ul,1}$. If we repeat a set of full unloading and full reloading *n* times, the *n*-th unloading stress and the plastic strain become $\sigma_{ul,n}$ and $\varepsilon_{pl,n}$, respectively.

A normalized stress $\tilde{\sigma}$ and a normalized strain $\tilde{\varepsilon}$ are defined here as

$$\widetilde{\sigma} = \frac{\sigma_c}{\sigma_{ul.n}}; \ \widetilde{\varepsilon} = \frac{\varepsilon_c - \varepsilon_{pl.n}}{\varepsilon_{ul} - \varepsilon_{pl.n}}$$
(8)

and deterioration rates of stress and strain, β_n and γ_n respectively, are defined as

$$\beta_n = \frac{\sigma_{ul.n+1}}{\sigma_{ul.n}} \quad ; \qquad \gamma_n = \frac{\varepsilon_{ul} - \varepsilon_{pl.n}}{\varepsilon_{ul} - \varepsilon_{pl.n-1}} \tag{9}$$

Fig. 7 shows how the full unloading and full reloading paths depend on the unloading strain ε_{ul} . Since the similar results are obtained in other cases, only the case for $\sigma_{c0} = 101.6 \ MPa$ and $\rho_s = 1.36\%$ is presented here. It is obvious that the $\tilde{\sigma}$ vs. $\tilde{\varepsilon}$ relation is almost linearly proportional at small $\varepsilon_{ul}/\varepsilon_{cc}$ (0.111-0.613), while a certain nonlinearity occurs at $\varepsilon_{ul}/\varepsilon_{cc} > 1.0$. As shown in Fig. 8, the dependence of $\tilde{\sigma}$ vs. $\tilde{\varepsilon}$ relation on ρ_s is less significant in the range of $0.45\% \le \rho_s \le 1.36\%$. Figs. 9 and 10 show the dependence of $\tilde{\sigma}$ vs. $\tilde{\varepsilon}$ relations on the concrete strength σ_{c0} and the number loading *n*, respectively. Both the concrete strength σ_{c0} and the number of loading *n* are less sensitive to the $\tilde{\sigma}$ vs. $\tilde{\varepsilon}$ relations.



Fig. 6 Proposed and Experimental Stress and Strain Relation of Confined Concrete

Based on the above clarification on the experimental results, a full unloading path of the concrete stress σ_c vs. strain ε_c relation may be represented by disregarding ρ_s , σ_{c0} and *n* dependence as

$$\widetilde{\sigma} = \widetilde{\varepsilon}^{n_{ul}} \tag{10}$$

where n_{ul} is a parameter which represents the degree of nonlinearity of stress σ_c vs. strain ε_c relation, and this may be expressed from the test results as



$$n_{ul} = 0.541 \left[\tan^{-1} \left\{ 2.2 \left(\frac{\varepsilon_{ul}}{\varepsilon_{cc}} - 1.44 \right) \right\} + \tan^{-1} 3.17 \right] + 1$$
(11)

In a similar way, a full reloading path is represented as,





$$\sigma_{c} = \begin{cases} \left(\frac{\varepsilon_{c} - \varepsilon_{pl,n}}{\varepsilon_{ul} - \varepsilon_{pl,n}}\right)^{n_{rl}} \cdot \sigma_{ul,n+1} & 0 \le \tilde{\varepsilon} < 0.5 \\ E_{rl}(\varepsilon_{c} - \varepsilon_{ul}) + \sigma_{ul,n+1} & 0.5 \le \tilde{\varepsilon} \le 1 \end{cases}$$
(12)

where,

$$E_{rl} = \frac{\sigma_{ul.n+1} - 0.5^{nul} \cdot \sigma_{ul.n+1}}{0.5 \cdot (\varepsilon_{ul} - \varepsilon_{pl.n})}$$
(13)

$$n_{rl} = \begin{cases} 1 & 0 \le \varepsilon_{ul} / \varepsilon_{cc} \le 1 \\ 1 + 0.584 \cdot \left(\frac{\varepsilon_{ul}}{\varepsilon_{cc}} - 1\right) & 1 \le \varepsilon_{ul} / \varepsilon_{cc} \le 1.25 \\ 1.146 & 1.25 \le \varepsilon_{ul} / \varepsilon_{cc} \end{cases}$$
(14)

Deterioration Coefficients β_n and γ_n

The stress deterioration ratio β_n by Eq. (9) represents the degree of deterioration of $\sigma_{ul.i}$ by repeating a set of full unloading and full reloading. Fig. 11 shows how β_n depends on the unloading strain ε_{ul} . The stress deterioration is significant when unloadings occur in the post-yield range. This may be written as



Fig. 11 Dependence of Stress Deterioration Ratio β_n on Unloading Strain ε_{ul}

<u>n=1</u>

<u>n≥2</u>

$$\beta_{\rm I} = \begin{cases} 1 - 0.05 \cdot \left(\frac{\varepsilon_{ul}}{\varepsilon_{cc}}\right) & 0 \le \varepsilon_{ul} / \varepsilon_{cc} \le 1 \\ 1.174 - 0.224 \cdot \left(\frac{\varepsilon_{ul}}{\varepsilon_{cc}}\right) & 1 \le \varepsilon_{ul} / \varepsilon_{cc} \le 1.25 \\ 0.894 & 1.25 \le \varepsilon_{ul} / \varepsilon_{cc} \end{cases}$$
(15)
$$\beta_n = \begin{cases} 1 & 0 \le \varepsilon_{ul} / \varepsilon_{cc} \le 1 \\ 1 + 4(0.027n - 0.133) \cdot \left(\frac{\varepsilon_{ul}}{\varepsilon_{cc}} - 1\right) \le 1.0 \\ 1 \le \varepsilon_{ul} / \varepsilon_{cc} \le 1.25 \\ 0.027n + 0.867 \le 1.0 & 1.25 \le \varepsilon_{ul} / \varepsilon_{cc} \end{cases}$$
(16)

The values by Eqs. (15) and (16) well correlate the experimental results as shown in Fig. 11.

On the other hand, Figs. 12 and 13 show the dependence of plastic strain $\varepsilon_{pl,1}$ and the plastic strain ratios γ_n on the unloading strains ε_{ul} . From the results, $\varepsilon_{pl,1}$ and γ_n may be expressed as



Fig. 12 Dependence of Unloading Strain ε_{ul} on the Plastic Strain $\varepsilon_{pl.1}$



Fig. 13 Dependence of Plastic Strain Ratio γ_n on Unloading Strain ε_{ul}

$$\frac{\varepsilon_{pl.n}}{\varepsilon_{cc}} = \begin{cases} 0.062 \cdot \left(\frac{\varepsilon_{ul}}{\varepsilon_{cc}}\right)^2 & 0 \le \frac{\varepsilon_{ul}}{\varepsilon_{cc}} \le 1\\ 0.889 \cdot \left(\frac{\varepsilon_{ul}}{\varepsilon_{cc}}\right) - 0.827 & 1 \le \frac{\varepsilon_{ul}}{\varepsilon_{cc}} \end{cases}$$
(17)

<u>n=2</u>

$$\gamma_{2} = \begin{cases} 1 & 0 \leq \varepsilon_{ul} / \varepsilon_{cc} \leq 1 \\ 1 - 0.304 \cdot \left(\frac{\varepsilon_{ul}}{\varepsilon_{cc}} - 1\right) & 1 \leq \varepsilon_{ul} / \varepsilon_{cc} \leq 1.25 \\ 0.924 & 1.25 \leq \varepsilon_{ul} / \varepsilon_{cc} \end{cases}$$
(18)

<u>n≥3</u>

$$\gamma_{n} = \begin{cases} 1 & 0 \le \varepsilon_{\rm ul} / \varepsilon_{cc} \le 1 \\ 1 + 4(0.033n - 0.123) \left(\frac{\varepsilon_{ul}}{\varepsilon_{cc}} - 1 \right) \\ & 1 \le \varepsilon_{\rm ul} / \varepsilon_{cc} \le 1.25 \\ 0.003n + 0.967 & 1.25 \le \varepsilon_{\rm ul} / \varepsilon_{cc} \end{cases}$$
(19)



(a) Partial Unloading and Full Reloading Paths (b) Normalized Reloading Paths

Fig. 14 Full Reloading Paths when Reloaded at $\beta_{ul} = 0.5$

PARTIAL UNLOADING AND FULL RELOADING PATHS

Since a partial unloading is a part of a full unloading path until a reloading occurs, the partial unloading path may be represented by a full unloading path by Eq. (10). Consequently, it is needed to formulate a full reloading path from a reloading point at a stress of σ_{rl} on an unloading path. To represent where a reloading occurs after unloaded, a parameter β_{ul} (partial unloading ratio) is defined as

$$\beta_{ul} = \frac{\sigma_{ul.1} - \sigma_{rl}}{\sigma_{ul.1}} \tag{20}$$

Fig. 14 (a) shows an example of the partial unloading and full reloading paths with $\beta_{ul} = 0.5$. Using the normalized stress and strain, $\tilde{\sigma}$ and $\tilde{\epsilon}$, by Eq. (8), a full reloading path from a point corresponding to $\beta_{ul} = 0.5$ becomes as show in Fig. 14 (b). Since the effect of repeating partial unloadings and full reloadings is less significant, a full reloading path may be idealized as

$$\sigma_c = E_{rl}(\varepsilon_c - \varepsilon_{rl}) + \sigma_{rl} \tag{21}$$

where

$$E_{rl} = \frac{\sigma_{ul.n+1} - \sigma_{rl}}{\varepsilon_{ul} - \varepsilon_{rl}}$$
(22)

Predicted full reloading paths by Eq. (21) agree well with the experimental results (refer to Fig. 14(b)).

FULL UNLOADING AND PARTIAL RELOADING PATHS

If a confined concrete is subjected to a full unloading, it reaches the plastic strain $\varepsilon_{pl,1}$ at zero stress. When it is subjected to a reloading from this plastic strain $\varepsilon_{pl,1}$, it may follow a full reloading path by Eq. (12). If a full unloading occurs at a strain $\varepsilon_{ul,in}$ ($\varepsilon_{ul,in} \le \varepsilon_{ul}$) on this path before reaching the unloading strain ε_{ul} , the unloading path has to be determined. Since the unloading above is the 2nd unloading if we count the first unloading from the unloading strain ε_{ul} , it is called here as the 2nd internal unloading. The stress where the 2nd unloading occurs is called the 2nd internal unloading stress $\sigma_{in,2}$. After the 2nd internal unloading, the concrete strain ε_c reaches the 2nd plastic strain $\varepsilon_{pl,2}$. Similarly, the stress at an internal unloading strain $\varepsilon_{ul,in}$ and the plastic strain after a set of *n* times full unloading and internal



Fig. 15 Partial Unloading Paths when Unloaded at Internal Unloading Point with $\sigma_{in.n}$ and $\varepsilon_{in.n}$ (γ_{rl} =0.5)

partial reloading are defined $\sigma_{in.n+1}$ and $\varepsilon_{pl.n+1}$, respectively. To represent a point until where an internal reloading continues, a parameter γ_{rl} (partial reloading ratio) is defined as

$$\gamma_{RL} = \frac{\varepsilon_{ul.in} - \varepsilon_{pl.1}}{\varepsilon_{ul} - \varepsilon_{pl.1}}$$
(23)

If a set of full unloading and partial reloading are repeated five times after fully unloaded, the stress σ_c and strain ε_c hysteresis become as shown in Fig. 15 (a) in the confined concrete with the target strength σ_{c0} of 90 *MPa* and $\rho_s = 0.68\%$. Other combinations of σ_{c0} and ρ_s have the similar relations. The plastic strain $\varepsilon_{pl,n}$ and the internal unloading stress $\sigma_{in,n}$ do not significantly deteriorate during the repeated loadings. This is different to the cyclic full unloadings and full reloadings, in which the plastic strain $\varepsilon_{pl,n}$ and the unloading stress $\sigma_{ul,n}$ deteriorate due to load reversals. This means that the deterioration of plastic strain $\varepsilon_{pl,n}$ and unloading stress $\sigma_{ul,n}$ is limited if unloadings occur before reaching the unloading strain ε_{ul} .

By defining a normalized stress $\tilde{\sigma}_{in}$ and strain $\tilde{\varepsilon}_{in}$ as

$$\tilde{\sigma}_{in} = \frac{\sigma_c}{\sigma_{in,n}}; \ \tilde{\varepsilon}_{in} = \frac{\varepsilon_c - \varepsilon_{pl,n}}{\varepsilon_{ul,in} - \varepsilon_{pl,n}}$$
(24)

the unloading paths presented in Fig. 15 (a) becomes as shown in Fig. 15 (b). The first unloading path from an unloading strain ε_{ul} is presented here as the "1st cycle" for comparison. The normalized unloading paths are very close to the normalized unloading path from the unloading strain ε_{ul} . Consequently, by replacing σ_{ul} and ε_{ul} in Eq. (12) by σ_{in} and $\varepsilon_{ul.in}$, respectively, one obtains

$$\sigma_{c} = \begin{cases} \left(\frac{\varepsilon_{c} - \varepsilon_{pl,n}}{\varepsilon_{ul,in} - \varepsilon_{pl,n}}\right)^{n_{rl}} \cdot \sigma_{in,n+1} & 0 \le \tilde{\varepsilon}_{in} < 0.5 \\ E_{rl}(\varepsilon_{c} - \varepsilon_{ul,in}) + \sigma_{in,n+1} & 0.5 \le \tilde{\varepsilon}_{in} \le 1 \end{cases}$$
(25)



Fig. 16 Application of Proposed Model to Full Unloadings and Full Reloadings ($\rho_s = 0.68\%$)



Fig. 17 Application of Proposed Model to Full Unloadings and Partial Reloadings

Fig. 18 Application of Proposed Model to Partial Unloadings and Full Reloadings

APPLICATION OF THE LOADING AND UNLOADING MODEL

Fig. 16 shows an application of the proposed model to a series of full unloadings and full reloadings of the concrete with the target concrete strength of 90 and 30 *MPa*, and $\rho = 0.68$ %. The proposed model provides a good estimate of the lateral confinement effect on the concrete with a wide range of strength between 30 and 90 *MPa*.

Figs. 17 and 18 show applications of the proposed model to a series of full unloadings and partial reloadings and a series of partial unloadings and full reloadings, respectively, of the concrete with σ_{c0} =90 MPa. The models show a good agreement with the test results.

CONCLUSIONS

An empirical constitutive model for the lateral confinement of concrete cylinders with a wide range of strength between 30 and 90 *MPa* was developed based on a series of loading test. Based on the results presented herein, the following conclusions may be deduced:

- 1. The strain at the peak strength ε_{c0} of the unconfined concrete depends on the peak strength σ_{c0} , and this dependence of ε_{c0} and σ_{c0} can be represented by Eq. (1).
- 2. The peak strength σ_{cc} and the strain at the peak strength ε_{cc} of the confined concrete can be represented by Eqs. (2) and (3), respectively.
- 3. The envelops of the stress vs. strain relation of the confined concrete can be represented by Eq. (5).
- 4. Unloading and reloading paths for a combination of full unloadings and full reloadings, full unloadings and partial reloadings, and partial unloadings and full reloadings can be represented by Eqs. (10), (12), (21), and (25).

REFERENCES

- 1. Mander, J.B., Priestley, M.J.N. and Park, R. "Theoretical stress-strain model for confined concrete." Journal of Structural Division, ASCE, 1988: 114(8), 1804-1826.
- Hoshikuma, J., Kawashima, K., Nagaya, K. and Taylor, A. "Stress-strain model for confined reinforced concrete in bridge piers." Journal of Structural Engineering, ASCE, 1997: 123(5), 624-633.
- Lertsrisakulrat, T., Watanabe, K., Matsuo, M. and Niwa, J. "Experimental study on parameters in localization of concrete subjected to compression." Proc. JSCE, 2001: 669(50), 309-321.
- 4. Sakai, J. and Kawashima, K. "An unloading and reloading stress-strain model for concrete confined by tie reinforcements." 12 WCEE, 2000: Paper No. 1432 (CD-ROM), New Zealand.