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A SIMPLE LOW CYCLE FATIGUE MODEL AND ITS IMPLICATIONS FOR SEISMIC DESIGN

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SUMMARY

A seismic design procedure that does not take into account the maximum and cumulative plastic deformation demands that a structure will likely undergo during severe ground motion could lead to unreliable performance. Seismic design methodologies that account for low cycle fatigue can be formulated using simple damage models. The practical use of one such methodology requires the consideration of the severity of repeated loading through a normalized plastic energy parameter. The inconsistencies inherent to the use of such approach can be corrected through simple empirical rules derived from an understanding of the effect of the history of energy dissipation in the assessment of the level of structural damage.

INTRODUCTION

Current philosophy for seismic design of typical residential or commercial structures accepts the possibility that significant inelastic behavior will occur during severe seismic excitations. The structural properties of a structure deteriorate when deformations reach the range of inelastic behavior. Such deterioration can be important during long and severe ground motions, when several excursions into the inelastic range are expected. A possible consequence of deterioration of the hysteretic behavior of a structure is failure of critical elements at deformation levels that are significantly smaller than its ultimate deformation capacity. In this paper, this failure mode will be termed **low cycle fatigue**.

The concept of target ductility complemented with the use of simple damage indices offers a valuable tool for practical design against low cycle fatigue. A damage model used to assess low cycle fatigue requires the explicit consideration of the severity of cumulative loading, and of at least two structural parameters: One to characterize the ultimate deformation capacity of the structure, and a second one to characterize the stability of its hysteretic behavior.

A simple energy-based low cycle fatigue model is formulated in this paper. This model is then used for seismic design of single-degree-of-freedom systems. The applicability and reliability of the model is assessed through the comparison of design results obtained from the model and other damage indices.

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BASIC CONCEPTS

This paper assumes familiarity of the reader with the concept of low cycle fatigue, and the tools that have been developed to assess its occurrence. For the sake of completeness some basic concepts are introduced.

A parametric approach to performance-based numerical seismic design

Traditionally, earthquake-resistant design has been formulated as a demand-supply problem. First, all relevant seismic demands in the building have to be estimated, and then they must be satisfied with adequate seismic supplies as follows:

$$\begin{array}{ccc} \textit{SEISMIC DEMANDS} & \leq & \textit{SEISMIC SUPPLIES} \\ & \textit{in terms of} & \\ \textit{Stiffness} & \ldots\ldots\ldots\ldots & \textit{Stiffness} \\ \textit{Strength} & \ldots\ldots\ldots\ldots & \textit{Strength} \\ \textit{Maximum and cumulative} & \ldots\ldots & \textit{Maximum and cumulative} \\ \textit{deformation capacity} & & \textit{deformation capacity} \end{array} \tag{1}$$

Although Equation 1 should be formulated explicitly for different design objectives, this paper will focus on structures that undergo severe plastic cycling when subjected to intense ground motion. Within a numerical performance-based methodology, the structural properties included in Equation 1 should be supplied so that the response of the structural and non-structural members is limited within response threshold levels established as a function of the required seismic performance. Teran [1] has observed that recently proposed design methodologies contemplate this check at three different steps: A) *Global predesign*, quick and reasonable estimates of global seismic demands should be established and checked against global threshold levels. The judicious use of response spectra provides information that allows the determination of a set of global structural properties (base shear, period of vibration, and ultimate and cumulative deformation capacities) that can adequately control and accommodate, within technical and cost constraints, the global response of the structure [2]; B) *Preliminary local design*, once the global structural properties have been determined, it is necessary to establish the member properties and detailing at the local level. This step contemplates the analyses of complex analytical models of the structure; C) *Revision of the preliminary design*, dynamic analyses should be carried out to establish and assess the global and local performance of the structure.

In this paper, seismic design will be approached from the *Global Predesign* step. A parametric approach will be used [1]; that is, each one of the relevant structural properties of the structure will be handled during the design process through a structural parameter: base shear (V_b) and fundamental period of translation (T) to define the global lateral strength and stiffness, respectively; and the ultimate displacement ductility (μ_u) and a fourth structural parameter (b) to characterize the global maximum and cumulative deformation capacities, respectively. Although the values of these four parameters are interrelated, the relations between the parameters are complicated and hard to characterize for design purposes in a manner that assessment of low cycle fatigue requires explicit and independent consideration of each one.

Design approaches that use the concept of target ductility

Target ductility is defined as the maximum ductility (μ_{max}) the structure can reach during the design ground motion before the level of structural damage exceeds a preset threshold. Usually, this threshold corresponds to incipient collapse.

In general, it has been agreed that as the plastic energy demand increases, μ_{max} should decrease with respect to the ultimate ductility (μ_u) the structure is able to undergo under monotonically increasing lateral deformation (unidirectional loading). How much smaller μ_{max} should be with respect to μ_u (or how much bigger μ_u with respect to μ_{max}) depends on three variables: the value of the known ductility (either μ_{max} or μ_u), a ground motion parameter that quantifies the severity of the plastic demands, and a structural parameter that characterizes the cycling capacity of the structure.

A key issue during the development of design methodologies to control low cycle fatigue has been the recognition that the lateral strength of a structure plays an instrumental role in controlling the seismic demands that eventually induce this type of failure. Within the context of design against low cycle fatigue, it is important to emphasize that lateral strength is not supplied to enhance the deformation capacity of a structure, but as means to control maximum and cumulative plastic deformation demands, and avoid uncontrolled and excessive degradation of its structural properties. Using the concept of target ductility, two approaches can be considered for the formulation of a performance-based design methodology that accounts for low cycle fatigue:

1) *Approach A*. This approach requires the estimation of a threshold for the maximum plastic response in the structure given that its deformation capacity is known (requires estimating μ_{max} given that μ_u is known). Table 1 summarizes the steps involved in Approach A. First, the value of T is established. Bertero and Bertero [3] and Priestley [4] discuss in detail the determination of the period of a structure within the context of performance-based design. Next, a decision needs to be made about the type of detailing to be used for the structure (ductile vs. non-ductile), and values of μ_u and b should be established according to the selected detailing. Then, the value of μ_{max} is established as a function, among other things, of μ_u and b. Once the value of μ_{max} is known, it is possible to establish the design base shear that will allow the structure to control its maximum plastic demands within this threshold.
2) *Approach B.* This approach requires the estimation of the ultimate deformation capacity given that the maximum allowable plastic deformation demand is known (requires estimating μ_u given that μ_{max} is known). Table 1 summarizes the steps involved in Approach B. Once again, the value of T is established first. Next, a decision needs to be made about the type of detailing to be used for the structure, and a value of b is established according to this decision. Once a preliminary value of μ_{max} is assumed, the values of μ_u and V_b can be established as a function of it.

Table 1 Target ductility-based design approaches

Step	Approach A	Approach B
1	Determine T	Determine T
2	Assume μ_u and $b = f$ (detailing)	Assume $\mu_{max} = f$ (judgement), and $b = f$ (detailing)
3	Estimate $\mu_{max} = f\,(T, \mu_u, b)$	Estimate $\mu_u = f\,(T, \mu_{max}, b)$
4	Estimate $V_b = f\,(T, \mu_{max})$	Estimate $V_b = f\,(T, \mu_{max})$

Normalized Plastic Energy

The total plastic energy dissipated through inelastic hysteretic response by a structure during an earthquake ground motion, denoted herein as $E_{H\mu}$, provides a rough idea of its cumulative plastic deformations. Nevertheless, it is convenient to take into account simultaneously $E_{H\mu}$, and the strength and stiffness of a system, as follows:

$$NE_{H\mu} = \frac{E_{H\mu}}{F_y \delta_y} \tag{2}$$

where $NE_{H\mu}$ is the normalized plastic energy, and F_y and δ_y are the yield strength and yield displacement, respectively. $NE_{H\mu}$ constitutes a direct measure of severity of cycling loading and it allows for the rational estimation of structural damage [5].

GROUND MOTIONS

Four sets of ground motions are considered herein, three of them corresponding to the Los Angeles (*LA*) urban area and one corresponding to the lake zone of Mexico City. The ground motions for *LA*, established as part of the FEMA/SAC Steel Project [6], were grouped in sets of twenty motions as follows: A) Design earthquake for firm soil with 10% exceedance in 50 years (*LA 10in50*); B) Design earthquake for firm soil with 50% exceedance in 50 years (*LA 50in50*); C) Design earthquake for soft soil with 10% exceedance in 50 years (*LA Soft*). The set of Mexican motions (*Mexico Soft*) was formed of seven narrow banded long duration ground motions recorded in the Lake Zone of Mexico City [7]. Although this paper shows results derived from *LA 50in50* and *Mexico Soft*, it should be mentioned that the results obtained from *LA 10in50* and *LA Soft* follow similar tendencies.

Figure 1 shows strength spectra obtained from elasto-perfectly-plastic behavior and 5% of critical damping (S_a stands for pseudoacceleration). The circles identify the corner period (T_s), defined as the period at which the strength spectra decreases after peaking either at a single point or at a plateau. *LA 50in50* has a corner or dominant period around 0.3 sec, while that of *Mexico Soft* is close to 2.0 sec. Figure 1 also shows $NE_{H\mu}$ spectra. There is a distinctive feature in the $NE_{H\mu}$ spectra corresponding to the sets of *LA* motions: starting from very small T, the $NE_{H\mu}$ demand tends to increase until T reaches the value of the corner period, after which it remains fairly constant. For the *Mexico Soft* set, $NE_{H\mu}$ tends to increase until T reaches the value of the corner period. After that, it tends to gradually decrease with a further increase in T. Note that the corner period delimits two distinctive zones in the $NE_{H\mu}$ spectra, and that the maximum $NE_{H\mu}$ demands for *Mexico Soft* are about two to three times larger than those corresponding to the *LA* motions. For constant ductility, the *LA* motions can be considered to have low and moderate energy content, while *Mexico Soft* has very large energy content.

Figure 2 shows the coefficient of variation (*COV*) associated with the mean strength spectra shown in Figure 1. The *COV* is presented for two purposes: A) To provide an idea of the uncertainty and variability involved in establishing mean strength spectra; and B) To provide reference values against which the *COV* associated to the use of the low cycle fatigue model developed here can be assessed.

LOW CYCLE FATIGUE MODELS

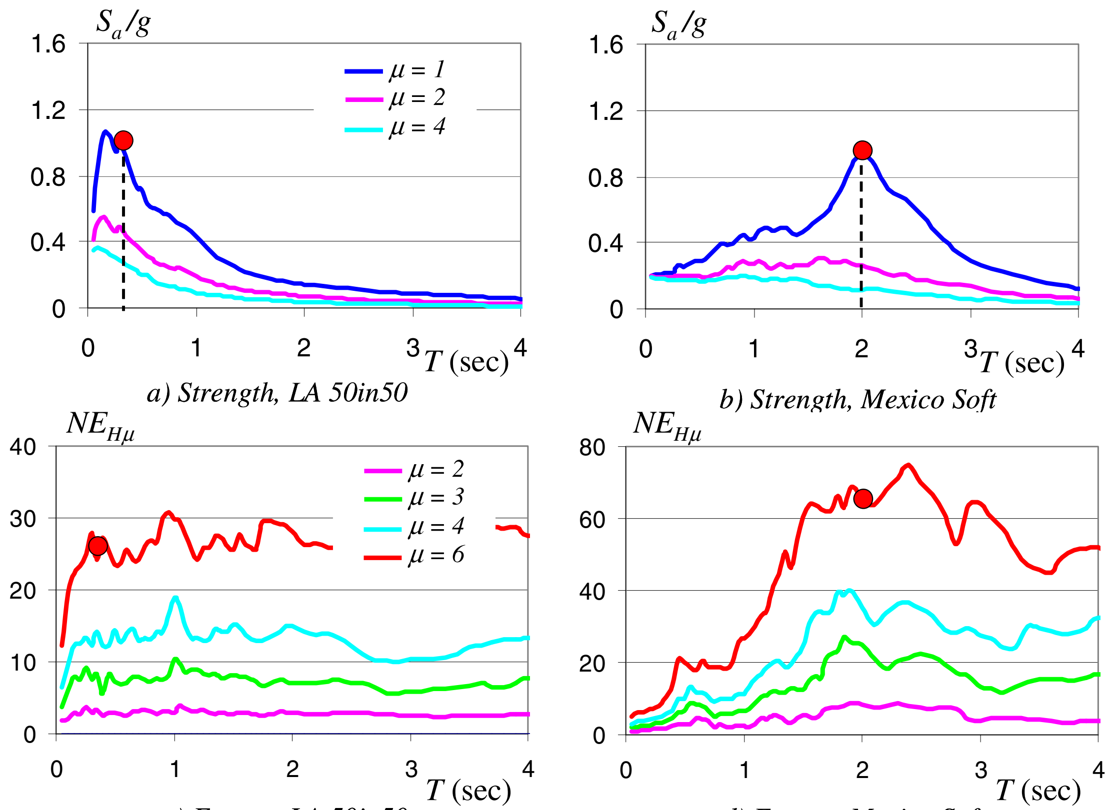
Three low cycle fatigue models are discussed next. Two of these models are well-known and have been used extensively to formulate seismic design methodologies that account for low cycle fatigue [3,8]. The third model is a simple energy-based model that will be introduced in this paper.

Park and Ang damage index

Park and Ang [9] have formulated a damage index to estimate the level of damage in reinforced concrete elements and structures subjected to cyclic loading:

$$DMI_{PA} = \frac{\mu_{max}}{\mu_u} + \beta \frac{NE_{H\mu}}{\mu_u} \tag{3}$$

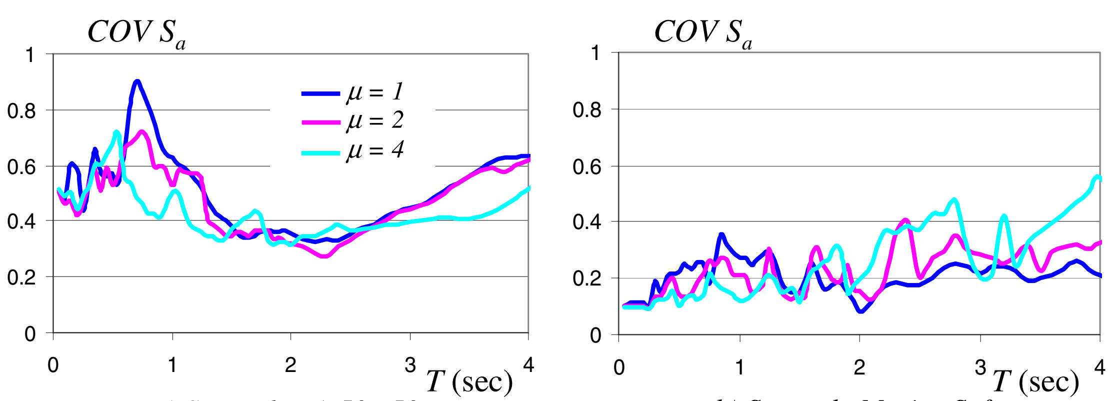
where μ_{max} is the maximum ductility demand, μ_u is the ultimate ductility and β is the structural parameter that characterizes the stability of the hysteretic behavior. In Equation 3, *DMI* denotes damage index; and the subscript *PA*, Park and Ang. The work done by several researchers suggest that β of 0.15 corresponds to systems that exhibit fairly stable hysteretic behavior; while values of β ranging from 0.2 to 0.4 should be used to assess damage in systems exhibiting substantial strength and stiffness deterioration [10,11,12]. Further discussion on the Park and Ang damage index can be found in Chung et al. [13].



a) Strength, LA 50in50　*b) Strength, Mexico Soft*

c) Energy, LA 50in50　*d) Energy, Mexico Soft*

Figure 1. Strength and normalized plastic energy spectra, 5% critical damping



a) Strength, LA 50in50　*b) Strength, Mexico Soft*

Figure 2. COV of strength spectra, 5% critical damping

DMI_{PA} less than 0.4 implies repairable damage; from 0.4 to 1.0, irreparable damage; and greater than 1.0, failure of the element. Under the presence of repeated cyclic loading into the plastic range, 1.0 represents the threshold value at which low cycle fatigue is expected to occur. For incipient occurrence of low cycle fatigue, Equation 3 yields:

$$\mu_u = \mu_{max} + \beta NE_{H\mu} \tag{4}$$

Linear cumulative damage theory

The linear cumulative damage theory (Miner's hypothesis) accounts for the change in the energy dissipating capacity of a structure as a function of its displacement history. Miner's hypothesis considers that damage induced by each plastic excursion is independent of damage produced by any other excursion, in such way that there is a need for a clear convention to define and delimit each excursion. Powell and Allahabadi [14] suggest that for earthquake induced deformations, the Rainflow Counting Method is a good option to achieve this.

Once the displacement history is separated into plastic excursions, the linear cumulative damage theory requires these excursions to be classified into intervals according to their amplitude. *Ndif* will denote the number of different intervals into which all plastic excursions are classified according to their amplitude, and δ_{pi}, the plastic displacement (amplitude) associated to the *ith* interval. For earthquake loading, the linear cumulative damage theory can be formulated as:

$$DMI_{MH} = \sum_{i=1}^{Ndif} \frac{n_i}{N_i} \tag{5}$$

where N_i is the number of plastic excursions the structure can actually undergo before failure when cycled exclusively to excursions with amplitude δ_{pi} (i.e., corresponding to the *ith* interval), and n_i is the number of plastic excursions of amplitude δ_{pi} resulting from the ground motion demands on the structure. In Equation 5, *DMI* denotes damage index; and the subscript *MH*, Miner's Hypothesis. DMI_{MH} equal to one implies incipient failure. Equation 5 can be reformulated as [8]:

$$DMI_{MH} = \sum_{i=1}^{Nexc} \left(\frac{\delta_{pi}}{\delta_{ucp}} \right)^b \tag{6}$$

where *Nexc* is the total number of plastic excursions, δ_{ucp} is the ultimate cyclic plastic displacement, δ_{pi} is now the plastic displacement of the *ith* excursion, and *b* is the structural parameter that characterizes the stability of the hysteretic behavior. After the review of experimental work carried out by several researchers on reinforced concrete and steel elements, Powell and Allahabadi [14] suggest that for low cycle fatigue, typical values of *b* range from 1.6 to 1.8. Furthermore, it has been suggested that a *b* of 1.5 is a reasonably conservative value to be used for seismic design and damage analysis of reinforced concrete and steel ductile structures [8,15].

Because low cycle fatigue implies the presence of multiple plastic excursions, a need arises to define cyclic deformation measures. In this sense, it is important to note that δ_{ucp} and δ_{pi} are cyclic measures of plastic displacement that can be applied to a plastic excursion. The normalization of δ_{ucp} and δ_{pi} by δ_y yields μ_{ucp} (ultimate plastic cyclic ductility) and μ_{pi} (plastic cyclic ductility in the *ith* excursion), respectively. Under the presence of repeated cycling into the plastic range, DMI_{MH} equal to one implies incipient failure due to low cycle fatigue. Under these circumstances, Equation 6 yields:

$$\mu_{ucp}^b = \sum_{i=1}^{Nexc} \mu_{pi}^b \tag{7}$$

A simple model to predict low cycle fatigue

By contrasting the simplicity and range of DMI_{PA} and DMI_{MH}, the following issue arises: Under what circumstances is the knowledge of the number and amplitude of the plastic excursions instrumental in assessing failure due to low cycle fatigue? Consider the case in which n_i and N_i can be related, for all i in Equation 5, through the same proportionality constant α:

$$n_i = \alpha N_i \tag{8}$$

If Equation 8 is substituted into Equation 5, the value of N_i cancels out for each term in the summation. Under the assumption of proportionality, the level of damage in a structure depends exclusively on its $NE_{H\mu}$ demand. In this section, a simple low cycle fatigue model is developed. Basically, this model represents a simplification of the linear cumulative damage theory through the assumption of a fixed shape for the distribution of plastic excursions. If Equation 8 holds, up to failure a structure can dissipate normalized plastic energy equal to:

$$NE_{H\mu} = \sum_{i=1}^{Ndif} n_i \mu_{pi} = \sum_{i=1}^{Ndif} \alpha N_i \mu_{pi} = \sum_{i=1}^{Ndif} \alpha \left(\frac{\mu_{ucp}}{\mu_{pi}} \right)^b \mu_{pi} \tag{9}$$

Equation 9 can be formulated in closed form as follows:

$$NE_{H\mu} = \int_0^{\mu_{ucp}} n \mu_p \, d\mu_p = \int_0^{\mu_{ucp}} \alpha \left(\frac{\mu_{ucp}}{\mu_p} \right)^b \mu_p \, d\mu_p = \alpha \mu_{ucp}^b \frac{\mu_{ucp}^{2-b}}{2-b} = \alpha \frac{\mu_{ucp}^2}{2-b} \tag{10}$$

where n is the number of plastic excursions of amplitude δ_p demanded by the ground motion, and μ_p, equal to δ_p/δ_y, is the plastic cyclic ductility. The value of DMI_{MH} corresponding to Equation 10 can be estimated according to the closed form of Equation 5 as:

$$DMI_{MH} = \int_0^{\mu_{ucp}} \frac{n}{N} d\mu_p = \alpha \int_0^{\mu_{ucp}} d\mu_p = \alpha \mu_{ucp} \tag{11}$$

By considering the right hand sides of Equations 10 and 11, a simplified estimate of DMI_{MH} can be obtained:

$$DMI_{MH}^S = (2-b) \frac{NE_{H\mu}}{\mu_{ucp}} \tag{12}$$

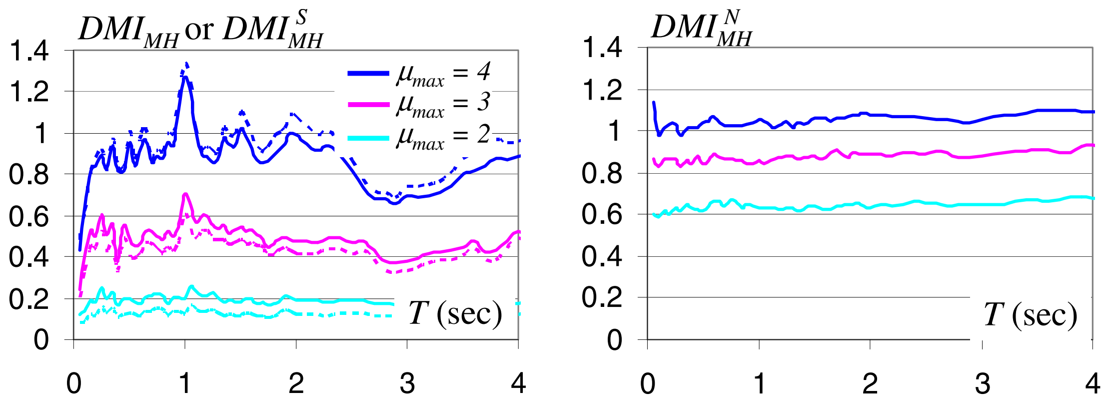
In Equation 12, $NE_{H\mu}$ becomes the ground motion parameter that quantifies the severity of the plastic demands, and μ_{ucp} and b the structural parameters that characterize the ultimate and cumulative deformation capacities of the structure. The analytical upper limit for the value of μ_{ucp} is given by $2(\mu_u - 1)$. In reality, the physical upper limit of μ_{ucp} will be somewhat less than this, because a plastic excursion close to μ_u will damage significantly the capacity of a structure to accommodate plastic deformation in the opposite direction:

$$\mu_{ucp} = 2r(\mu_u - 1) \tag{13}$$

where r is a reduction factor (less than one). For incipient collapse, Equation 12 can be reformulated in terms of μ_u as ($DMI_{MH} = 1$):

$$\mu_u = \frac{(2-b) NE_{H\mu}}{2r} + 1 \tag{14}$$

Figure 3a compares, for *LA 50in50*, damage estimates derived from Equations 6 and 12 ($b = 1.5$ and $\mu_{ucp} = 7.5$). The value of μ_{ucp} was established from Equation 13 by assuming a μ_u of 6 and r equal to 0.75. The discontinuous lines correspond to Equation 6. Equation 12 yields, with respect to Equation 6, higher estimates of damage for μ_{max} of *2*, slightly higher estimates for μ_{max} of *3*, and slightly lower estimates for μ_{max} of *4*.



a) Mean damage *b) Normalized mean damage*

Figure 3. Estimates of damage for *LA 50in50*

The energy dissipating capacity of a structure increases as the amplitude of its plastic excursions decreases. In the case of μ_{max} of *2*, the amplitude of the majority of the plastic excursions is small with respect to μ_u. While Equation 6 accounts for an increased energy dissipation capacity, Equation 12 does not, so that the latter yields higher estimates of damage. As the value of μ_{max} increases, the mean amplitude of the plastic excursions increases with respect to the ultimate deformation capacity. Because the energy dissipating capacity of a system will tend to decrease under these circumstances, Equation 12 yields similar estimates of damage than Equation 6 for μ_{max} of *3* and *4*.

Figure 3b shows the mean ratio of the damage estimates obtained from Equations 6 and 12. The ratio shows a strong dependence on μ_{max} and a weak variation with respect to T. While the results obtained for *LA 10in50* and *LA Soft* are similar to those shown in Figure 3b, the ratios corresponding to *Mexico Soft* are slightly smaller due to its higher energy content.

There are two facts regarding the relation that exists between the mean amplitude of the plastic excursions and the energy content of ground motion: A) For systems undergoing a given μ_{max} during ground motion, the amplitude of the plastic excursions tends to decrease as the energy content of the motion increases; and B) As the plastic energy demand increases, the value of the target ductility should decrease with respect to μ_u, so that an increase in the energy content of the ground motion requires the amplitude of the plastic excursions to be reduced. The results shown in this paper indicate that as the amplitude of the plastic excursions decreases, the estimates of damage derived from Equation 12 tend to increase with respect to those obtained from Equation 6, in such way that Equation 12 yields (with respect to Equation 6): A) Lower damage estimates when applied to structures subjected to motions with low energy content; B) Similar estimates of damage when applied to motions with moderate and large energy content; C) Higher estimates of damage when applied to motions with very large energy content.

Because of the above, it was considered convenient to adjust Equation 12 for design purposes by introducing a parameter *a* that accounts for the energy content of the motion (and thus indirectly, for the manner in which energy is expected to be dissipated):

$$DMI_{MH}^{S} = (2-b)\frac{aNE_{H\mu}}{\mu_{ucp}} \quad (15)$$

For incipient collapse, Equation 15 can be reformulated in terms of μ_u as (DMI_{MH} = 1):

$$\mu_u = \frac{(2-b)aNE_{H\mu}}{2r} + 1 \quad (16)$$

Recommendations for the use of Equation 16 in practical seismic design are summarized in Table 2. These recommendations were obtained from extensive studies on the seismic performance of single-degree-of-freedom (SDOF) systems designed according to Equation 16 and subjected to the sets of ground motions considered in this paper (the design process will be discussed and illustrated in detail in the next section). Note that only two different values of a are suggested for practical application: 0.75 for the use of Approach B with motions with very large energy content, such as those generated in the Lake Zone of Mexico City, and 1.0 for any other case. Note that the unsafe nature of Equation 16 with a=1 for motions with low energy content can be taken into consideration by establishing minimum or maximum values for μ_{max} and μ_u (such as those indicated in Table 2).

Table 2 Considerations for the practical use of Equations 15 and 16

Energy Content	Approach A	Approach B
Low	$a = 1$, $\mu_{max} \leq \mu_u$	$a = 1$, $\mu_u \geq \mu_{max}$
Moderate or High	$a = 1$	$a = 1$
Very High (Mexico City)	$a = 1$	$a = 0.75$

PRACTICAL CONSIDERATION OF LOW CYCLE FATIGUE IN SEISMIC DESIGN

The impact of using different low cycle fatigue models during seismic design will be assessed through the use of Approach A with the three damage models presented in the previous section.

Approach A

While Table 3 shows implementation details for Approach A, Table 1 provides a general overview for its practical application. Once T is established and design values for b and μ_u are established as a function of the detailing provided to the structural elements (and their supports and connections), design against low cycle fatigue implies estimating the design base shear. The determination of V_b is carried in two steps: A) The target ductility (μ_{max}) is determined; and B) The design V_b corresponds to the minimum strength required to control the global plastic response of the structure within the threshold established by μ_{max}. For practical purposes, V_b can be established from a constant ductility pseudoacceleration (S_a) spectra, corresponding to μ_{max}, evaluated at T. Approach A requires the target ductility to satisfy the conditions formulated in the last column of Table 3. As shown, Equations 7, 4 and 16 yield estimates of target ductility for DMI_{MH}, DMI_{PA} and DMI_{MH}^{S}, respectively. Due to inconsistencies in their formulation, the following condition was imposed on the value of μ_{max} derived from DMI_{MH} and DMI_{MH}^{S} : $\mu_{max} \leq \mu_u$.

Figure 4a shows, for *LA 50in50* and μ_u of 5, a general comparison of the values of μ_{max} obtained with DMI_{MH} and DMI_{PA}. The three values of β used with DMI_{PA} are considered to characterize a wide range of structural behavior. For DMI_{MH}, r was set equal to 0.75 and b was set equal to 1.2 and 1.8. The value of μ_{max} tends to decrease when starting from zero the period increases. After the corner period, μ_{max} exhibits a fairly stable behavior with respect to T. Note that the dependence of $NE_{H\mu}$ with respect to T (see Figure 1c) helps explain this tendency. There is similarity between the values of μ_{max} obtained from DMI_{PA} with β of 0.05 and DMI_{MH} with b of 1.8, and those obtained from β of 0.15 and b of 1.2.

Table 3   Use of Approach A with three different damage models

Model	Known	Unknown Response	Target ductility should satisfy:
DMI_{MH}	$b,\ \mu_{ucp} = 2r(\mu_u - 1)$	$\sum_{i=1}^{Nexc} \mu_{pi}^{b}(\mu_{max})$	Eq. 7: $\mu_{ucp}^{b} = \sum_{i=1}^{Nexc} \mu_{pi}^{b}(\mu_{max})$
DMI_{PA}	$\beta,\ \mu_u$	$\mu_{max},\ NE_{H\mu}(\mu_{max})$	Eq. 4: $\mu_u = \mu_{max} + \beta NE_{H\mu}(\mu_{max})$
DMI_{MH}^{S}	$b,\ \mu_{ucp} = 2r(\mu_u - 1)$	$NE_{H\mu}(\mu_{max})$	Eq.16: $\mu_{ucp} = \frac{(2-b)aNE_{H\mu}(\mu_{max})}{2r} + 1$

Figure 4b shows values of μ_{max} corresponding to ductile structures with stable hysteretic behavior. Although DMI_{PA} yields slightly conservative estimates with respect to DMI_{MH} and DMI_{MH}^{S}, the three models yield similar results. For firm soil motions with moderate $NE_{H\mu}$ demands, the value of μ_{max} is not particularly sensitive to the values of β and b. DMI_{PA} yields conservative results with respect to DMI_{MH} and DMI_{MH}^{S}, so that a β of 0.30 does not seem meaningful. The results shown in Figure 4 suggest that μ_{max} should be limited to about 0.65 μ_u in structures that exhibit significant deterioration of their hysteresis loop, and to about 0.75 μ_u for structures with stable hysteretic behavior.

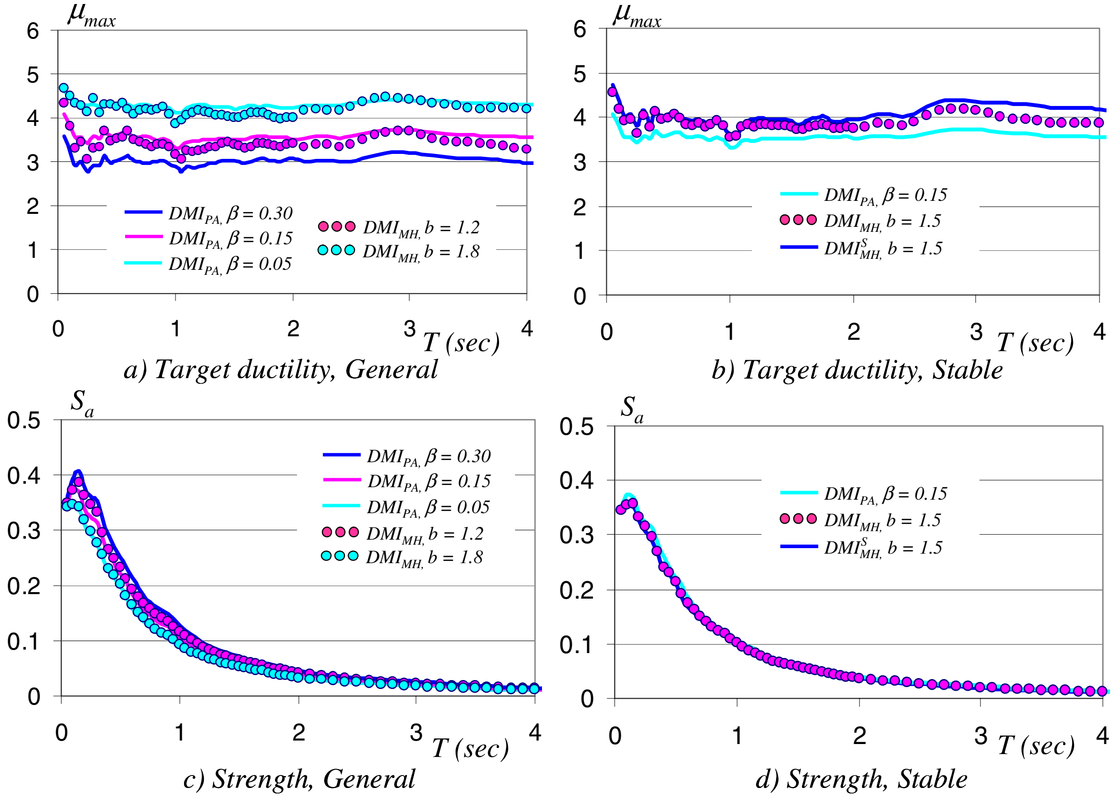
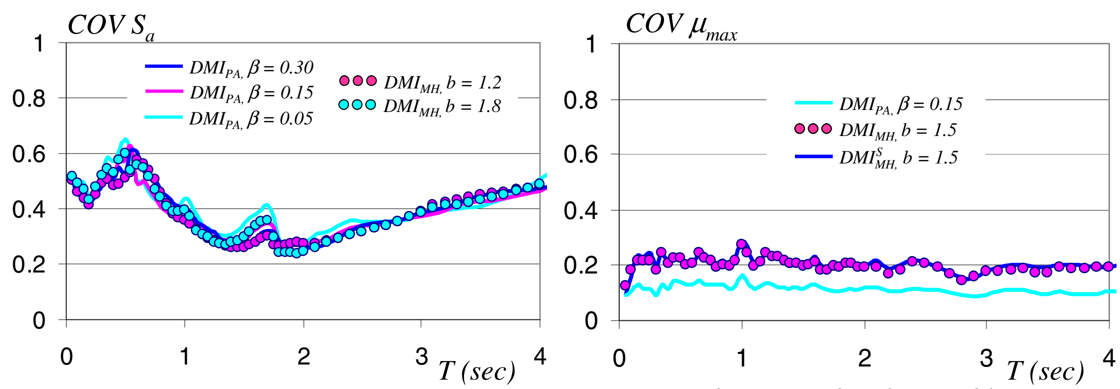


Figure 4. Design values obtained from three damage models, *LA 50in50*, μ_u = 5, ξ = 0.05

Figures 4c and 4d show the minimum strength required to control, for *LA 50in50*, the maximum ductility demand within the values shown in Figures 4a and 4b, respectively. Considering that the lateral strength is the actual structural property to be designed within Approach A, the results shown suggest that the impact of using one or another low cycle fatigue model during seismic design would be minimal.

Figure 5 shows that the *COV* of the strength demands is fairly insensitive to the values of b and β, and that this *COV* is similar to that shown in Figure 2a. This implies that if the structural parameters involved in the three models are considered deterministic, the uncertainty involved in determining the minimum strength required to avoid failure due to low cycle fatigue is similar to that involved in the determination of constant ductility strength spectra. Although not shown, results obtained for other values of μ_u (3, 4 and 6) and for *LA 10in50* and *LA Soft* are similar to those summarized in Figures 4 and 5. Nevertheless, it was observed that as the energy content of the ground motions decreases, the Park and Ang model becomes more conservative with respect to DMI_{MH}. Figure 5b shows there is a small *COV* involved in the determination of the target ductility.



a) Strength, General *b) Target ductility, Stable*

Figure 5. *COV* of S_a and μ_{max} obtained from three damage models, *LA 50in50*, $\mu_u = 5$, $\xi = 0.05$

Figure 6 shows values of μ_{max} obtained for *Mexico Soft*. Note that the correspondence between the values of β and b not only changes with respect to that observed in firm soil, but becomes sensitive to the value of T. This can be explained by the larger $NE_{H\mu}$ demands of *Mexico Soft*, and by the fact that these demands strongly depend on T (Figure 1d). The value of μ_{max} tends to decrease when starting from zero, the value of T increases up to the corner period. After that, μ_{max} tends to increase with respect to a further increase in T, until it stabilizes for large T. While Figure 6a shows that β of 0.30 yields results similar to b of 1.2 in a wide period range, Figures 6b and 6d suggest that DMI_{PA} remains conservative with respect to DMI_{MH} in period intervals where the $NE_{H\mu}$ demand is small, and becomes slightly unsafe around the corner period (2 sec). In contrast, DMI^S_{MH} yields slightly higher strength requirements than DMI_{MH} at the corner period and slightly lower strength requirements as T departs from it. As the value of μ_u increases, the strength requirements derived from DMI_{PA} around the corner period become progressively smaller than those obtained from DMI_{MH}, while the opposite occurs, under the same circumstances, to the strength requirements derived from DMI^S_{MH}.

Figure 6 suggests that for structures subjected to narrow-banded long duration motions, μ_{max} should be limited under certain circumstances to about 0.40 μ_u for rapidly degrading structures, and to about 0.50 μ_u for structures with stable hysteretic behavior. Under these circumstances, the conservatism usually involved in the deformation thresholds suggested for displacement-control methodologies would not seem enough to protect adequately structures having a T similar to the corner period.

Considering that within Approach A the base shear of the structure is the structural property to be designed, Figures 4d and 6d suggest that the impact of using one or another low cycle fatigue model would be minimal.

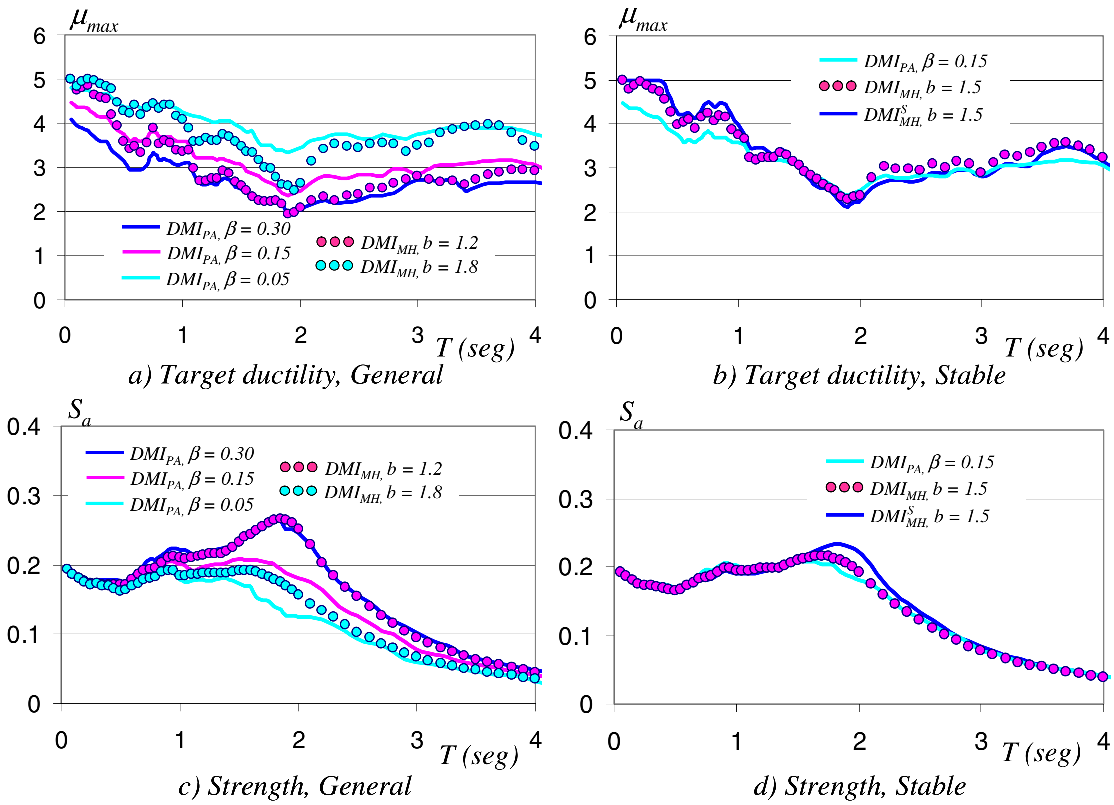


Figure 6. Design values obtained from three damage models, *Mexico Soft*, $\mu_u = 5$, $\xi = 0.05$

Implications

According to what is shown in Table 1, once T is established, and values of b and μ_u are assumed as a function of the detailing provided to the structural elements, design against low cycle fatigue implies the estimation of the lateral strength of the structure. Several methodologies recently proposed for seismic design that accounts for low cycle fatigue [3,8] formulate a two-step procedure to establish the design base shear (steps 3 and 4 in Table 1): A) Establish μ_{max} through the explicit consideration of the severity of plastic cycling and the stability of the hysteretic cycle; and B) Establish the design base shear as the minimum strength required to control the global plastic response of the structure within the threshold set by μ_{max}. Step A requires a design representation for the energy demands or plastic cycling. During step B, the design base shear can be established by evaluating at T, a constant maximum ductility S_a spectrum corresponding to μ_{max}.

The above two-step procedure for strength design differs from current practice in that the latter considers a simple and empirical basis to determine the value of μ_{max}. In this sense, current code formats can be considered one-step procedures, in which the value of μ_{max} is implicitly considered during strength design. Due to some reluctance to include a representation of the energy demands in current codes, the fairly simple two-step performance-based procedures recently proposed [3,8] have not found their way into seismic codes.

The damage model introduced in this paper allows the formulation of a rational and simple two-step procedure that closely resembles the current one-step strength design format. Table 4 draws a parallelism between current one-step and the proposed two-step formats. As shown, the two-step procedure does not only focus its attention to the ultimate deformation capacity of the structure, but to the stability of its hysteretic cycle as well. During the estimation of the design base shear, current one-step procedures concentrate on controlling the maximum ductility demand within the threshold set by μ_{max}, while the proposed two-step procedure focuses on controlling the cumulative ductility demand within the threshold set by Equation 16. While the current one-step procedure uses constant *maximum* ductility strength spectra during the estimation of the base shear, the proposed two-step procedure requires the use of constant *cumulative* ductility strength spectra. The concept of constant *cumulative* ductility strength spectra is discussed and illustrated in a companion paper [7]. The advantage of the two-step procedure introduced herein over recently proposed two-step procedures is that the former does not require a design representation for the energy demands or plastic cycling and thus, yields a strength design format that is very similar to the current one-step procedure.

Table 4 Comparison between current one-step and simple two-step approaches

Step	One-step	Two-step
1	Determine T	Determine T
2	Determine μ_u	Determine μ_u and b
3	Establish μ_{max} (implicit)	Establish maximum allowable $NE_{H\mu}$ according to Equation 16 (explicit)
4	Estimate $V_b = f\,(T,\mu_{max})$	Estimate $V_b = f\,(T,NE_{H\mu})$

Final Considerations

In general terms, assuming proportionality between the n_i and N_i curves implies: A) Unsafe estimates of damage for motions with low energy content; B) Reasonable estimates for motions with moderate and large energy content; and C) Conservative estimates for motions with very large energy content. Because of this, a parameter a was introduced to the damage model developed here (see Equation 15). The value of a requires calibration according to the circumstances of application of Equations 15 and 16 (Table 2). The value of r in Equation 16 depends on the detailing used; in such way that it may vary from rapidly degrading structures to systems with stable hysteretic behavior. An r of 0.75 seems to provide adequate results for both the design of ductile and non-ductile structures. The value of b affects the level of conservatism involved in the use DMI^{S}_{MH}. Particularly, as b increases, the level of conservatism of its damage estimates tends to decrease, in such way that the use of DMI^{S}_{MH} with values of b larger than 1.6 seems unsafe for design purposes.

Although DMI_{PA} neglects the way in which the plastic energy has been dissipated, it can not be considered equivalent to DMI^{S}_{MH}. In fact, the conservatism of the damage estimates derived from DMI_{PA} (with respect to those obtained from DMI_{MH}) exhibit opposite tendencies than those obtained from DMI^{S}_{MH}: they are conservative for low $NE_{H\mu}$ demands, and become progressively unsafe as the $NE_{H\mu}$ demand increases. To explain this, it should be considered first that the assumption of independence between μ_{max} and $NE_{H\mu}$ introduces a high level of conservatism for small $NE_{H\mu}$ demands; and second, that the calibration of DMI_{PA} did not contemplate the high levels of plastic energy demands expected during long duration narrow-banded ground motions. These inconsistencies imply some adjustments to the use of DMI_{PA}, particularly on the value of β used to characterize the structure (e.g., β of 0.30 does not seem meaningful for firm soils).

In firm soils, the strength demands obtained from Approach A do not exhibit a high sensitivity with respect to the values of b and β. As suggested before by Cosenza et al. [10] and the results obtained herein, the uncertainty in the determination of these values for design purposes would not appear to significantly impact the design results obtained from Approach A. Within this context, b of 1.5 and β of 0.15 seem to yield reasonable results for the seismic design of systems exhibiting fairly stable hysteretic behavior.

In this paper, design against low cycle fatigue has been approached in global terms. Design considerations at the local level, such as those discussed by Dutta and Mander [16], require a clear understanding of the relationship existing between the plastic deformation demands at the local and global levels. An issue that has not been considered in this paper is the effect of the hysteretic behavior on the seismic demands and capacities of an earthquake-resistant structure. In some cases, the response of a structure becomes sensitive to the specifics of its hysteretic behavior, particularly for systems that exhibit pinching. Another issue not considered explicitly is the multi-degree-of-freedom effect. Although results obtained by several researchers suggest that the response of a SDOF system can be used to obtain reasonable estimates of displacement, energy dissipation, and structural damage in regular structures [2], there are still issues to be addressed within this context, such as the effect of higher modes and of layout and structural irregularities.

CONCLUSIONS

Low cycle fatigue is, in many cases of practical interest, an issue during seismic design. Particularly, displacement control seismic design methodologies seem to provide adequate level of safety for the design of structures with stable hysteretic behavior and subjected to “typical” firm soil motions. Nevertheless, the use of low cycle fatigue models should be considered for the design of structures exhibiting rapid and excessive deterioration of their hysteresis loop, and for any type of structure subjected to long duration narrow-banded ground motion.

The concepts discussed in this paper seem to provide a robust set of tools for seismic design against low cycle fatigue. This is particularly true for well conceived regular structures that exhibit stable hysteretic behavior and controlled response during severe ground motion. The application of the principles of capacity design and performance-based design are instrumental to achieve this type of behavior. As for structures that exhibit irregularities and/or exhibit rapidly deteriorating hysteretic behavior, this set of tools become sensitive to the specifics of the local and global hysteretic behavior, and thus, its application becomes less reliable. As has been done in other contexts, the use of the tools discussed herein can be applied to determine the strength and ultimate deformation requirements of ductile structures with stable hysteretic behavior; while a more stringent application should be considered for structures with erratic seismic behavior.

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