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LIMITS OF APPLICATION OF SIMPLIFIED DESIGN PROCEDURES TO NON-REGULARLY ASYMMETRIC BUILDINGS

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SUMMARY

In this paper the Authors describe the results of a study devoted to the definition of the limits of application of an approximated design method of non regularly asymmetric systems.
Simplified design procedures proposed by many codes are generally based on the elastic behavior of single-storey systems and, therefore, strictly valid with reference to multi-storey structures belonging to a special class of buildings, commonly referred to as regularly asymmetric. In spite of such strict theoretical limits, the above-mentioned approximated methods of analysis are generally used by engineers, with reference to a wide range (not precisely defined) of real non regularly asymmetric structures. It is opinion of the Authors that such a dilemma has to be resolved, defining clear limits in the application of the simplified methods of analysis.
In order to catch such a goal the seismic behavior of a wide set of non-regularly asymmetric structures, characterized by different values of the structural eccentricity e_s and level of in-plan irregularity along the height of the building, has been investigated. At first, some parameters quantifying the in-plan irregularity of structures have been defined: some derive from the definition of the optimum torsion axis, which aims at minimizing the sum of the squares of the deck rotations produced by a distribution of lateral forces; others take into account the irregularity caused by the variation of the radius of gyration of stiffness along the height of the building. Finally, with the aim of defining proper limits of application of a simplified approach, the aforementioned parameters have been related to the maximum error committed in the evaluation of the seismic response.

INTRODUCTION

The use of static analysis has been firstly conceived for plane frames, on the basis of the observation that, for such schemes, the distribution of forces corresponding to the first mode of vibration is not so much different from an inverted triangular one. Much more questionable is the application of static analysis to three-dimensional schemes, the dynamic response of which presents torsional rotations inconsistent with plane models. The analysis of the modal response of single-storey spatial schemes has suggested a simple way to safely evaluate the displacements of the deck, or the use of eccentricities in the application of static forces, calibrated so as to catch the dynamic increase of torsional rotations, Anastassiadis [1],

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Calderoni [2]. Simplified formulations of these eccentricities have been included in most seismic codes such as Eurocode8 [3], NBCC [4], UBC [5]. In the eighties Chopra demonstrated the validity of this approach for a particular class of multi-storey buildings, named "regularly asymmetric". Their dynamic behavior may be exactly described by coupling the results of a modal planar analysis to the normalized torsional response of a single-storey system, Hejal [6]. Therefore, the use of static analysis is possible, provided that static forces are applied with eccentricities defined with reference to the single-storey coupled system. Unfortunately, in reality very few buildings fulfill all the conditions required to regularly asymmetric buildings, because structural properties (e.g. the location of center of mass C_M, center of rigidity C_R and the radius of gyration of mass r_m and stiffness r_k) often vary from one floor to another. Nevertheless, static analysis is commonly applied to buildings which present small irregularities, on the basis of the intuition that a small non-correspondence to the theoretical requirements cannot dramatically modify the behavior of buildings. Intuition is, doubtlessly, a powerful tool of the human kind but, if we accept this, which is the limit? How can we define a level of irregularity that makes reliable the use of static analysis? This paper tries to answer these questions, derives parameters which numerically define the in-elevation irregularity and show how the parameters are related to the errors committed in the evaluation of the seismic behavior by means of the simplified analysis.

ANALYSIS OF REGULARLY ASYMMETRIC SYSTEMS

In regularly asymmetric systems the in-plan distribution of the maximum displacements at each floor, evaluated by means of spatial modal analysis, describes proportional curves (Figure 1a).

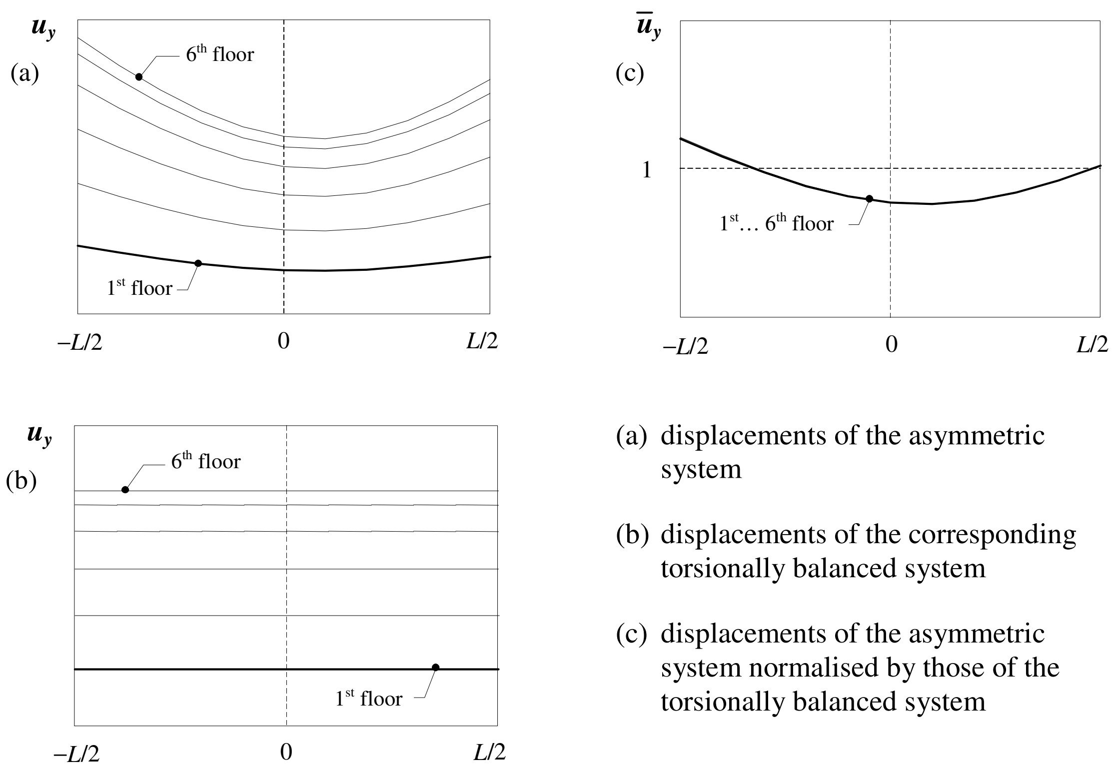


Figure 1. Application of modal analysis to a multi-storey regularly asymmetric system (Ω_θ=1.0; T_y=1 s; e_s=0.10 L)

The ratio of these displacements over those of the corresponding balanced system (Figure 1b) provides the same curve at each floor (normalised displacements, Figure 1c). Consequently, a single-storey system that describes the effects of the lateral-torsional coupling exists, Hejal [6], Marino [7]. Planar modal analysis of the balanced system, corrected by means of the normalized response of the corresponding single-storey asymmetric system, can be used, obtaining the same results as the spatial modal analysis.

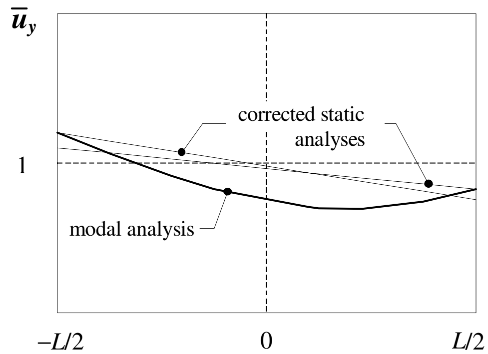
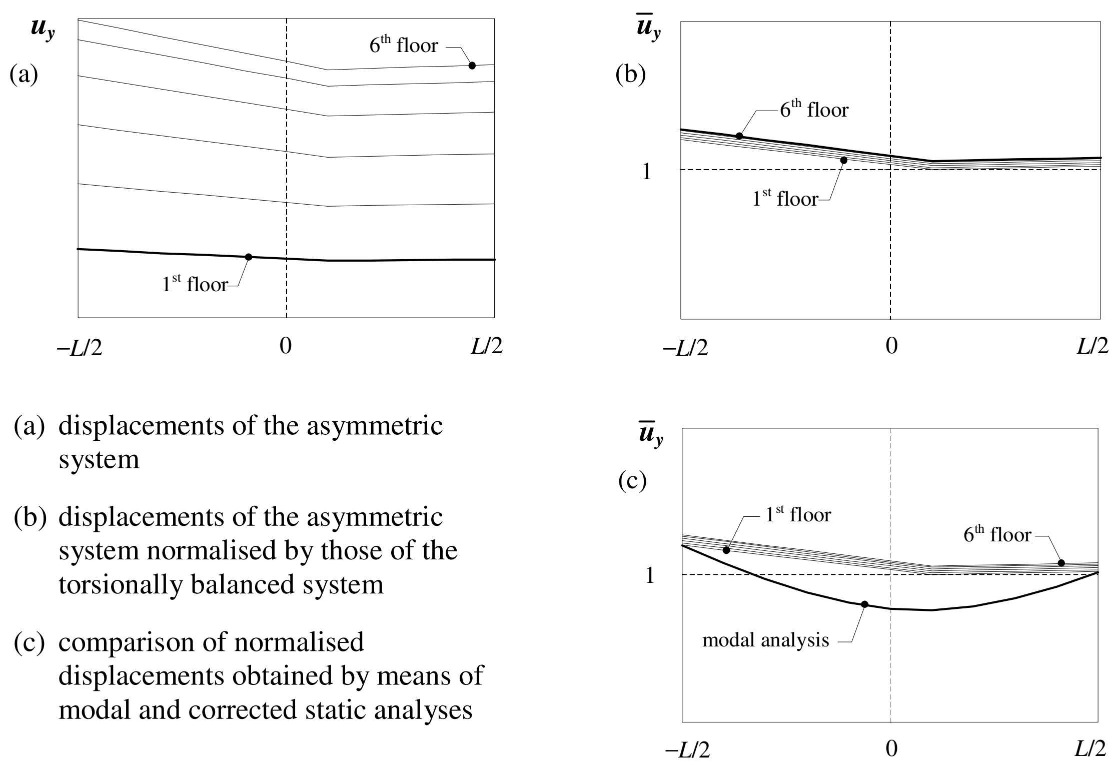


Figure 2. Application of corrected static analyses to a one-storey asymmetric system (Ω_θ=1.0; T_y=1 s; e_s=0.10 L)

The maximum modal displacements of both flexible and stiff sides of the single-storey system may be evaluated by means of two static analyses (Figure 2) by means of appropriate eccentricities e_{d1} and e_{d2}, Anastassiadis [1], Calderoni [2]. Using the same eccentricities in the static analysis of the three-dimensional scheme, we obtain displacements of the regularly asymmetric structure (Figure 3a) that, when normalized with respect to those represented in Figure 1b, provide curves that slightly differ from one floor to another (Figure 3b).



(a) displacements of the asymmetric system

(b) displacements of the asymmetric system normalised by those of the torsionally balanced system

(c) comparison of normalised displacements obtained by means of modal and corrected static analyses

Figure 3. Application of corrected static analysis to a multi-storey regularly asymmetric system (Ω_θ=1.0; T_y=1 s; e_s=0.10 L)

The corrected static analysis is conservative at the upper floors, while at the lower floors it provides values closer to those of the modal analysis (Figure 3c). These small differences are connected to the different distribution of forces applied in the case of modal or static analysis. Note that the comparison has been carried on by using the same base shear in the two approaches, as some seismic codes suggest, in order to avoid larger differences.

ANALYSIS OF NON REGULARLY ASYMMETRIC SYSTEMS

Equivalent single story system

Regularly asymmetric buildings are characterized by some properties, which are necessary to define the corresponding single-storey system: it has an elastic axis; the mass centers of all floors are lined up in vertical; the radii of gyration of stiffness and mass do not vary along the height, Hejal [6], Marino [7]. Unfortunately, in most actual buildings the in-plan distribution of stiffness varies from one storey to another. As a consequence, they do not have an elastic axis; different positions of generalized centers of rigidity, twist and shear centers may be evaluated, with reference to different distributions of the horizontal actions, Anastassiadis [1], Calderoni [2], Ghersi [8], Marino [9], Tso [10]. In order to define a corresponding single-storey system, it is possible to refer to the optimum torsion axis proposed by Makarios [11],[12], defined as the vertical line that joins the points of the floors where the equivalent seismic forces must be applied in order to minimize the sum of the squares of the deck rotations. Such an axis coincides with the elastic axis when this one exists; furthermore, its position is only in a minor way influenced by the distribution of the horizontal forces, differently from what has been observed for other reference points (particularly the center of rigidity), Ghersi [8], Marino [9].
As regard as the radius of gyration of stiffness, in a regularly asymmetric system it is unique and may be evaluated by the following expression,

$$r_{k,eq} = e_1 \sqrt{\frac{u_{F,i}}{\theta_{M,i}\, e_1} - \left(\frac{\theta_{F,i}}{\theta_{M,i}}\right)^2} \tag{1}$$

where u_{Fi}, θ_{Fi} are the displacement and rotation of the deck of the i^{th} floor produced by a set of horizontal forces $\boldsymbol{F}$ and θ_{Mi} is the rotation of the same deck produced by a set of torsional couples $\boldsymbol{M}$ obtained multiplying $\boldsymbol{F}$ by an eccentricity e_1. In irregular buildings, this expression gives a different value at each floor. We suggest assuming, as radius of gyration of stiffness of the corresponding single-storey system, the mean of the values provided by Eq. (1) at all the floors. Finally, if the mass centers do not lie on a vertical axis or if their radii of gyration vary along the height, we may use the mean value of these quantities in order to characterize the corresponding single-storey system. Therefore, it is possible to evaluate the eccentricities e_{d1} and e_{d2} necessary to perform a *corrected static analysis*.

Parameters to measure the in-plan irregularity along the height of buildings

With the above-mentioned assumptions, it is possible to evaluate also in irregular schemes structural displacements and internal actions by means of simplified analyses. Anyway, it is necessary to know the entity of the errors produced by the use of approximate methods and to relate it to simple parameters that take into account the level of irregularity.

First couple of parameters

In order to find a measure of the in-plan irregularity of structures, two parameters have been defined with the aim of taking into account two different aspects, Bosco[13]. The first parameter comes out from the definition of the optimum torsion axis, which aims at minimizing the sum of the squares of the deck rotations produced by a distribution of lateral forces. Noting that this sum is null for regularly asymmetric schemes, in which optimum torsion axis and elastic axis coincide, and that is greater than zero for irregular schemes, we may assume as measure of non-regularity the parameter:

$$\Theta_1 = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\theta_i^{\,2}} \tag{2}$$

where θ_i is the deck rotation caused by a distribution of forces **F** applied to the optimum torsion axis and N is the number of storeys of the building.

The effectiveness of the parameter Θ_1 is limited by the fact that it is not able to properly catch the effect of the variation of radius of gyration of stiffness along the height; e.g., it is null for mass-eccentric buildings having stiffness centers lined along the symmetry axis but presenting at the same time relevant variation of torsional stiffness at different floors, Marino [7], [9]. In order to overcome this problem, we may use a second parameter Θ_2, which takes into account the in-plan irregularity caused by the variation of the radius of gyration of stiffness along the heiht of buildings. In single-storey systems the translation u_F of the corresponding balanced systems produced by a force F and the rotation θ_M induced by a couple $M = F{\cdot}e_1$ may be easily calculated by means of the following expressions:

$$u_F = \frac{F}{K} \tag{3}$$

$$\theta_M = \frac{F\ e_1}{K\ r_k^{\,2}} \tag{4}$$

being K the translational stiffness of the system. The deck rotation is thus given by the expression:

$$\theta_M = \frac{u_F\ e_1}{r_k^{\,2}} \tag{5}$$

The previous equation, which expresses the rotation of the deck θ_M as a function of the displacement u_F, is valid for regularly asymmetric systems but not for non regularly asymmetric systems. Such an observation suggests a second parameter able to quantify the vertical irregularity of buildings.

In fact, with reference to an actual multi-storey system, in where deck rotations have been restrained, the displacements $u_{F,i}$ of the floors due to a distribution of forces F_i may be evaluated. Therefore, the deck rotations of actual buildings, caused by the application of a distribution of couples $F_i\ e_1$, may be evaluated by means of Eq. (5). The radius of gyration of stiffness is calculated according to the above mentioned formula valid for regular asymmetric systems (Eq. 1). Only if the analyzed buildings was regularly asymmetric systems, the obtained rotations would be equal to those evaluated applying the couples $F_i\ e_1$ at the spatial model. With the aim of defining a structural parameter able to quantify the non regularity along the height the difference between the rotation $\theta_{M,i}$ produced in the examined multi-storey systems by a considered distribution of couples and that evaluated by means of Eq. (5) is calculated at each floor. The sum of the squares of such differences has been assumed as a measure of the in-plan irregularity along the height of the building and the parameter Θ_2 ha been defined as:

$$\Theta_2 = \frac{\sqrt{\sum_{i=1}^{N}\left(\theta_{M,i} - \frac{u_{F,i}e_s}{r^2_{k\,med,i}}\right)^2}}{N} \tag{6}$$

where the radius of gyration of stiffness $r_{k\,med,i}$ is the mean value between those obtained at the $i+1,\ldots,N$ floors.

Both indices Θ_1 and Θ_2 are equal to zero in regularly asymmetric systems and greater than zero in non regularly asymmetric systems.Unfortunately these two parameters are not linearly independent and for this reason two other parameters have been proposed.

Second couple of parameters

In this section another couple of parameters that may be used as a measure of structural irregularity along the height is proposed.

The first parameter concerns with a structural eccentricity varying from one storey to another. It is known that, for a single-storey system the normalized structural eccentricity may be evaluated by means of the following expression:

$$e = -\frac{\vartheta_F}{\vartheta_M} e_1 \tag{7}$$

where θ_F is the rotation of the deck caused by a force F, θ_M is the rotation of the same deck produced by a torsional couple M obtained multiplying F by an arbitrary eccentricity e_1. In irregular buildings, this expression gives a different value at each floor.

$$e_i = -\frac{\vartheta_{F,i}}{\vartheta_{M,i}} e_1 \tag{8}$$

We suggest assuming as parameter of irregularity the standard deviation of values provided by Eq.(8):

$$\sigma(e_i) = \sqrt{\frac{\sum_{i=1}^{N} (e_i - e_m)^2}{N}} \tag{9}$$

where e_m is the mean structural eccentricity.

The second parameter has been introduced to take into account the irregularity caused by the variability of the radius of inertia of stiffness along the height. It is represented by the standard deviation of the values of the radius of inertia obtained by applying equation (1):

$$\sigma(r_{k,i}) = \sqrt{\frac{\sum_{i=1}^{N} (r_{k,i} - r_{k,m})^2}{N}} \tag{10}$$

A positive aspect connected to the use of these two parameters is their negligible dependence. For this reason it is possible to sum them by means of a linear combination.

Last Parameter

In order to have a complete measure of the irregularity of the structure a parameter has been introduced taking into account both the irregularity caused by the vertical variation of radius of gyration of stiffness and structural eccentricity.

At first, the seismic response of the equivalent single storey system is evaluated by means of analytical formulae and the maximum displacement u_{eq} at the outermost frames of the deck has been calculated, Calderoni [2].

Then, eight single-story systems have been generated by varying structural eccentricity and radius of gyration of stiffness:

1. $e_s = e_{m,eq}\,;\, r_s = r_{k,eq} + 1.64\,\sigma(r_{k,i})$
2. $e_s = e_{m,eq}\,;\, r_s = r_{k,eq} - 1.64\,\sigma(r_{k,i})$
3. $e_s = e_{m,eq} + 1.64\,\sigma(e_i)\,;\, r_s = r_{k,eq}$
4. $e_s = e_{m,eq} + 1.64\,\sigma(e_i)\,;\, r_s = r_{k,eq} + 1.64\,\sigma(r_{k,i})$
5. $e_s = e_{m,eq} + 1.64\,\sigma(e_i)\,;\, r_s = r_{k,eq} - 1.64\,\sigma(r_{k,i})$
6. $e_s = e_{m,eq} - 1.64\,\sigma(e_i)\,;\, r_s = r_{k,eq}$
7. $e_s = e_{m,eq} - 1.64\,\sigma(e_i)\,;\, r_s = r_{k,eq} + 1.64\,\sigma(r_{k,i})$
8. $e_s = e_{m,eq} - 1.64\,\sigma(e_i)\,;\, r_s = r_{k,eq} - 1.64\,\sigma(r_{k,i})$

where $e_{m,eq}$; $\sigma(e_i)$; $r_{k,eq}$; $\sigma(r_{k,i})$ have been defined in previous paragraphs.

Also in these cases it is possible to evaluate the maximum displacement u_{max} at the outermost frames of the deck of single storey systems.

The parameter of irregularity is given by the relationship:

$$\Gamma = \frac{u_{\max} - u_{eq}}{u_{eq}} \tag{11}$$

and expresses the increment of the displacements caused by the variability of the in-plane irregularity along the height.

NUMERICAL MODELS

In order to validate the use of static analysis for irregular structures, four typologies of asymmetric buildings have been considered in the present paper. Irregularity has been introduced by means of random but limited modifications of the stiffness of the resisting elements of four regularly asymmetric schemes.

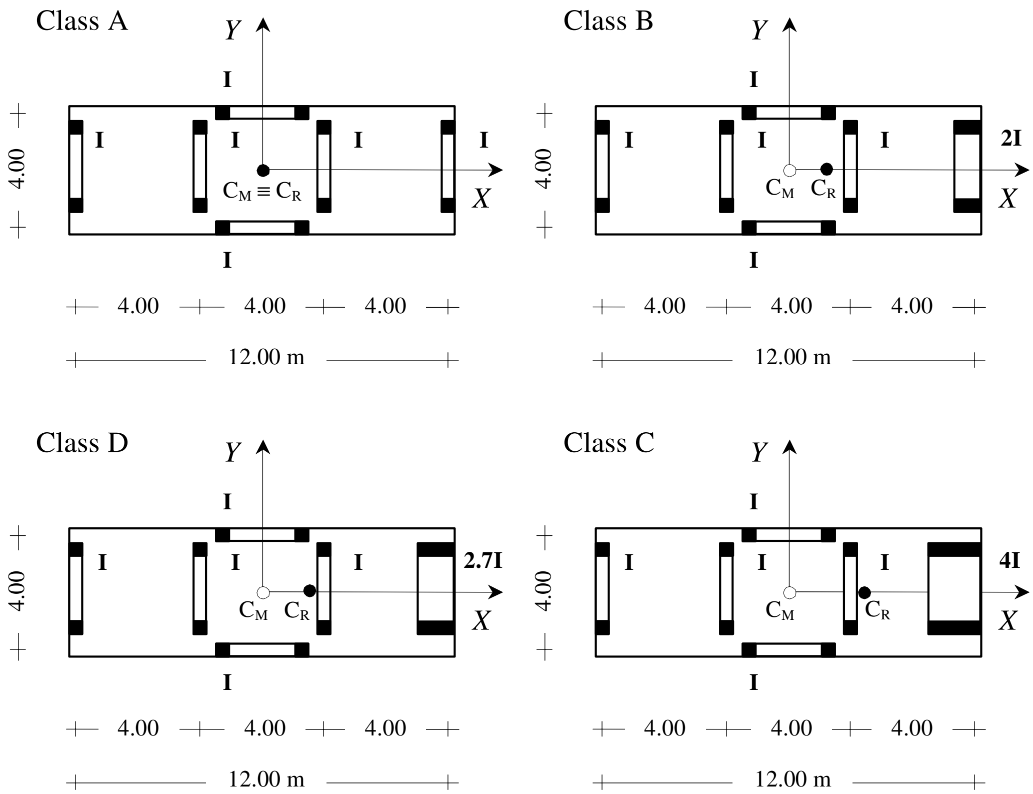


Figure 4. Reference systems for class A, B, C, D irregular buildings

The structure of the reference systems (Figure 4) presents shear-type frames symmetrically disposed with respect to the *x* and *y*-axes (four in *y*-direction and two in *x*-direction). All the *y*-direction frames are equal to each other in the first scheme, from which a set of irregular buildings named “class A” has been generate. In the second scheme, giving irregular buildings of “class B”, one of the outermost frames has rigidity double than the others. In third and fourth schemes one of the outermost frames has respectively rigidity 2.7 (irregular buildings of “class D”) and 4 times (irregular buildings of “class C”) greater than the others. All the schemes have six storeys and present *x*-direction frames equal to each other, mass and radius of gyration of mass with the same value at all the floors, centers of mass disposed along a vertical axis (*z* axis).

Starting from these four basic systems, four sets of 1500 irregular buildings have been generated, by randomly imposing:

- the number of resisting elements to be modified, in the range one to six;
- the storey and position of the elements to be varied (more than one modification may occur at the same element);
- the entity Δ of the variation of the lateral rigidity of each element, in the range $-50\ \% \leq \Delta \leq 400\ \%$.

CORRELATION BETWEEN ERROR AND IRREGULARITY

When the corrected static analysis is used, the results do not coincide to those of the spatial modal analysis. The maximum and minimum differences (non-conservative and conservative errors) may be plotted versus an irregularity parameter, obtaining a couple of points for each analyzed building (Figure 5a) and a large number of points for the whole set of buildings (Figure 5b). These points are enveloped with two curves corresponding to 95% fractile (Figure 5c), which may be used from now on to discuss the correlation between error and irregularity, e.g. to determine the value of the irregularity parameter Θ corresponding to a given error (Figure 5d).

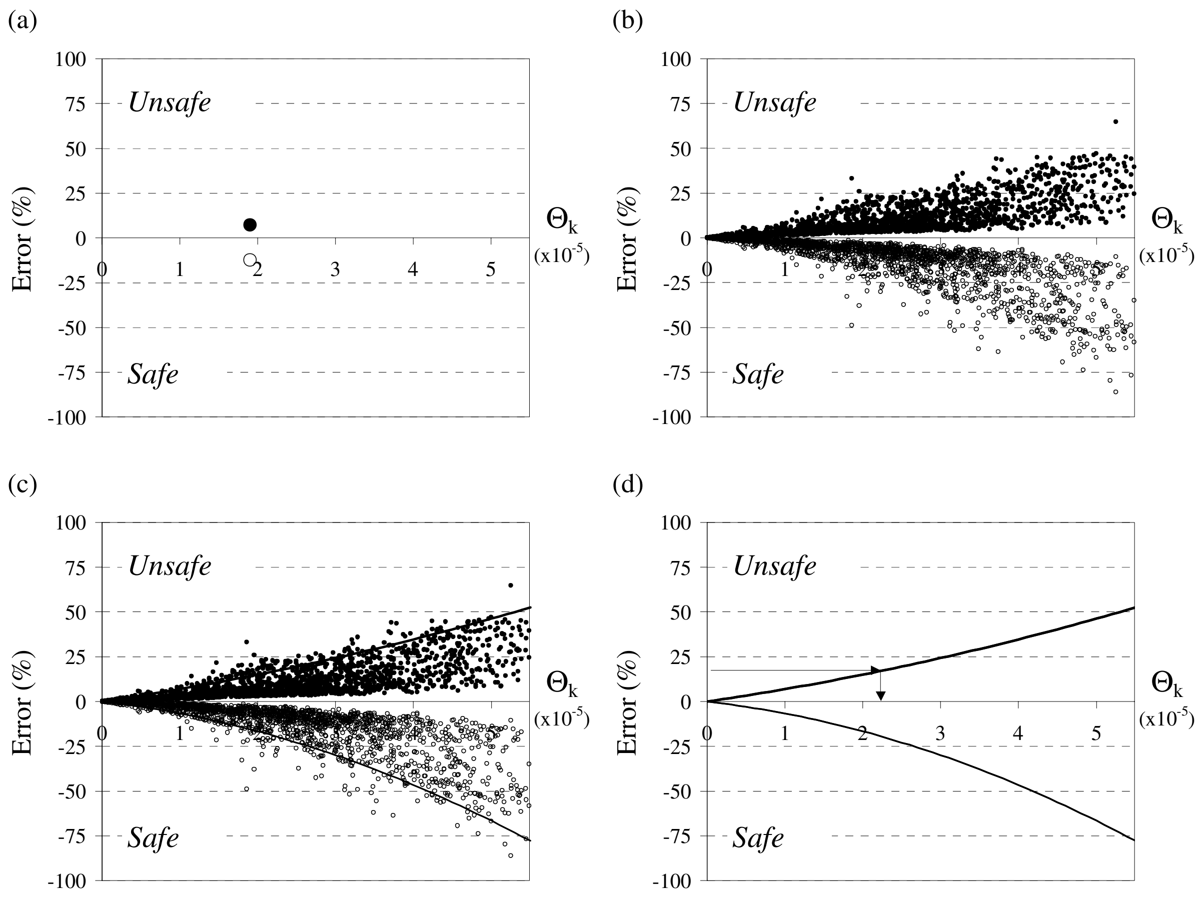


Figure 5. Correlation between error and irregularity parameter

ANALYSIS OF THE RESULTS

For each group of irregular buildings errors have been plotted versus a single parameter of irregularity given by linear combination of the two parameters firstly defined.

Coefficients of linear combination have been defined solving a linear programming problem aiming at minimizing the distance between the tendency lines corresponding to the irregular classes of buildings

(with the exception of "Class A"). This choice derives from the observation that irregular buildings of class A are characterized by small errors thanks to the regularity of the basic scheme.
At first, errors have been plotted versus the parameter:

$$\Theta_k = \Theta_1 + 0.2\Theta_2$$

where coefficient 0.2 comes from the solution to the above mentioned linear programming problem. It is evident (Figure 6) that there is a good correspondence between the curves of classes B, C and D buildings.

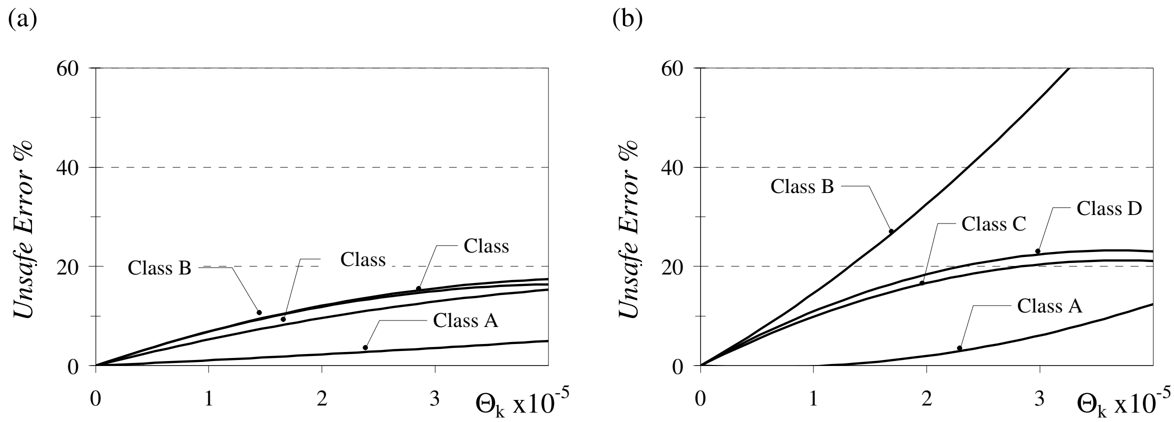


Figure 6. Unsafe errors versus parameter Θ_k in the evaluation of: a) absolute displacements; b) relative displacements.

Then errors have been also plotted versus the second couple of parameters:

$$\Sigma_k = \sigma(e_i) + 0.24\sigma(r_{ki})$$

where coefficient 0.24 comes from the solution to the linear programming problem. Also in this case there is a good correspondence between the curves of classes B, C and D buildings (Figure 7).

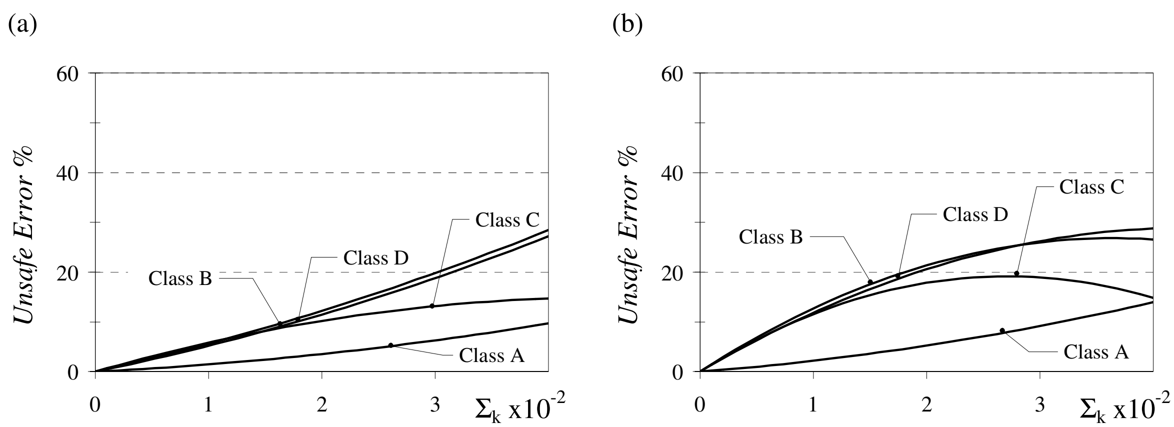


Figure 7. Unsafe errors versus parameter Σ_k in the evaluation of: a) absolute displacements; b) relative displacements.

At least errors have been plotted versus the parameter Γ

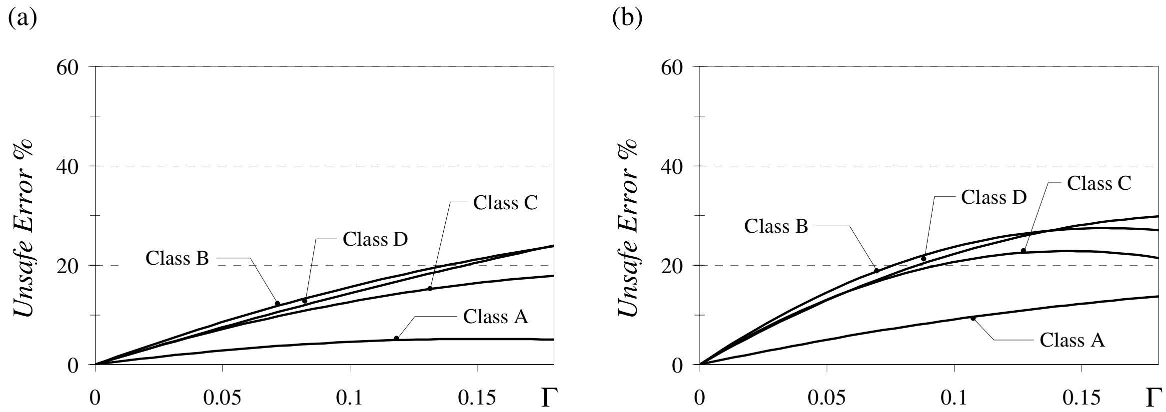


Figure 8. Unsafe errors versus parameter Γ in the evaluation of: a) absolute displacements; b) relative displacements.

Therefore if we accept an error up to 10% it is possible to define a range of application of simplified analysis common to all the analyzed irregular buildings belonging to classes B, C, and D.

CONCLUSIONS

The paper demonstrates that it is possible to find a correlation between some parameters of irregularity and the maximum error connected to the application of approximate methods of analysis in the evaluation of dynamic response of non regularly asymmetric structures. This result has a quantitative validity based on the analysis of a large set of buildings with random variation of stiffness. Furthermore, the proposed parameters are quite simple to be evaluated also in practice. On the basis of these results, it will be possible to define in seismic codes clear limits to the applicability of simplified approaches, related to the maximum acceptable error and to the level of irregularity of the structure. The field of application of corrected static analysis is slightly different as a function of the typology of the buildings; for this reason a deeper analysis has to be done to generalize the results of this study.
The extension of seismic analysis to the inelastic range is necessary.

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