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THE USE OF CUMULATIVE DUCTILITY STRENGTH SPECTRA FOR SEISMIC DESIGN AGAINST LOW CYCLE FATIGUE

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SUMMARY

Damage models that quantify the severity of repeated plastic cycling through plastic energy are simple tools that can be used for practical seismic design. The concept of constant cumulative ductility strength spectra, developed from one such model, is a useful tool for performance-based seismic design. Particularly, constant cumulative ductility strength spectra can be used to identify cases in which low cycle fatigue may become a design issue, and provides quantitative means to estimate the design lateral strength that should be provided to a structure to adequately control its cumulative plastic deformation demands during seismic response.

INTRODUCTION

Experimental and field evidence indicate that the strength, stiffness and ultimate deformation capacity of reinforced concrete elements and structures deteriorate during excursions into the plastic range of behavior. Excessive hysteretic degradation may lead to an excessive accumulation of plastic deformation that may lead to failure at deformation levels that are significantly smaller than the ultimate deformation capacity of the structure under uni-directional loading.

This phenomenon, denoted herein as low cycle fatigue, has been repeatedly observed in laboratory tests. Panagiotakis and Fardis [1] recently observed that the deformation at failure of reinforced concrete elements subjected to typical load-histories applied in laboratory tests can be estimated as 60% of their ultimate deformation capacity. Independently, Bertero [2] recommended that the maximum ductility demand a structure undergoes during ground motion should be limited to 50% of its ultimate ductility.

The importance of plastic cycling on the deformation capacity of reinforced concrete structures has been known for some time. This effect caught the attention of several researchers during the 1970's when experimental studies were carried out on the cyclic response of reinforced concrete members and beam-column subassemblages. At that time, the need to account for the effect of cycling on the performance of earthquake-resistant structures was emphasized. Some of the options that were visualized involved

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proportioning the beams to control the level of shear stress. Detailing schemes were formulated to enable structural elements to undergo several cycles of plastic deformation with stable hysteretic behavior [3,4].

In the 1980's and 1990's, the engineering profession confronted the need to design structures with predictable performance. Performance-based seismic design became a fundamental concept for the formulation of seismic design methodologies. As a consequence, proposals for design against low cycle fatigue began focusing on deformation control rather than relying exclusively on detailing recommendations to ensure stable hysteretic behavior. A key issue during the development of design methodologies to control low cycle fatigue was the recognition that the lateral strength of a structure plays an instrumental role in controlling the seismic demands that eventually induce this type of failure. Within the context of design against low cycle fatigue, it is important to emphasize that lateral strength is not supplied to enhance the deformation capacity of a structure, but as a mean of controlling maximum and cumulative plastic deformation demands, and avoiding uncontrolled and excessive degradation of its structural properties.

Low cycle fatigue should be avoided, particularly for conditions that may result in repeated plastic cycling. The complexity of low cycle fatigue has resulted in significantly different opinions regarding how to account for it during seismic design. This paper discusses a set of simple tools recently developed for practical seismic design against low cycle fatigue. Although emphasis is placed on the design of reinforced concrete structures, the tools can be calibrated for other structural materials.

ENERGY AS DESIGN REPRESENTATION OF CUMULATIVE LOADING

Significantly different methods have been proposed to estimate the severity of plastic cycling, and various design methodologies that account for the effect of low cycle fatigue have been offered. An option that has been considered attractive due to its simplicity has been the characterization of cumulative loading through energy concepts. Housner [5] offered one of the earliest discussions regarding the need to consider explicitly the effect of plastic cycling through energy concepts. Later, several attempts have been made to estimate the energy demands in simple systems, and to offer insights on how to use these demands for design purposes [6,7,8].

Design for low cycle fatigue was advanced with the formulation and calibration of damage indices [9,10], and the formalization of an energy balance equation for design purposes [11]. Based on these concepts, several design methodologies that account for low cycle fatigue have been formulated [12,13,14,15].

Today there are still significantly different approaches towards the formulation of a design representation for the energy demands. Some researchers suggest that energy spectra could be formulated and used for design purposes [8]. Other options include accounting for cumulative loading in the structure through indirect measures of the plastic energy [12], and deriving the plastic energy demands from other relevant seismic demands [16,17].

The total plastic energy dissipated by a structure during an earthquake ground motion is denoted herein as $E_{H\mu}$. The plastic energy demand can be interpreted physically by considering that it is equal to the total area under all the hysteresis loops the structure undergoes during a ground motion. In this sense, $E_{H\mu}$ provides a rough idea of the cumulative plastic deformations in the structure. Nevertheless, $E_{H\mu}$ by itself does not provide enough information to assess structural performance. Thus, it is convenient to take into account simultaneously $E_{H\mu}$, and the strength and stiffness of a system, as follows:

$$NE_{H\mu} = \frac{E_{H\mu}}{F_y \delta_y} \qquad (1)$$

where $NE_{H\mu}$ is the normalized plastic energy, and F_y and δ_y are the yield strength and yield displacement, respectively. For an elasto-perfectly-plastic system subjected to a single plastic excursion, $NE_{H\mu}$ is equal to the plastic displacement associated to that excursion normalized by δ_y. That is, for a single plastic excursion, $NE_{H\mu}$ is a direct measure of the plastic displacement.

For an elasto-perfectly-plastic system subjected to multiple plastic excursions, $NE_{H\mu}$ is the sum of all plastic displacements reached in the different cycles normalized by δ_y. In this sense, $NE_{H\mu}$ is a direct measure of the cumulative plastic displacement demands. For a system with degrading hysteretic behavior, $NE_{H\mu}$ could be defined to include all plastic excursions for which the capacity does not degrade to a value less than a specified fraction of F_y (say 0.75). Such a definition allows for rational evaluation of structural damage in reinforced concrete structures through the use of $NE_{H\mu}$.

GROUND MOTIONS

Four sets of ground motions are considered herein, three of them corresponding to the Los Angeles (*LA*) urban area and one corresponding to the lake zone of Mexico City. The ground motions for *LA*, established as part of the FEMA/SAC Steel Project [18], were grouped in sets of twenty motions as follows: A) Design earthquake for firm soil with 10% exceedance in 50 years (*LA 10in50*); B) Design earthquake for firm soil with 50% exceedance in 50 years (*LA 50in50*); C) Design earthquake for soft soil with 10% exceedance in 50 years (*LA Soft*). The set of Mexican motions (*Mexico Soft*) was formed of seven narrow banded long duration ground motions recorded in the lake zone of Mexico City. The Mexico Soft motions were scaled up in such way that their peak ground velocity was equal to that corresponding to the EW component of the motion recorded at SCT during 1985. Figures 1 and 2 show spectra for the four sets of motions, and Table 1 summarizes some of the characteristics of the motions included in *Mexico Soft*. All spectra shown were obtained for elasto-perfectly-plastic behavior and 5% of critical damping.

Table 1. Characteristics of motions included in *Mexico Soft*

Record	Date	*PGA*[1] (cm/sec^2)	*PGV*[2] (cm/sec)	T_s[3] (sec)
Alameda EW	04/25/89	46	15	2.1
Alameda NS	04/25/89	37	10	2.1
Garibaldi EW	04/25/89	52	17	2.2
Tlahuac EW	09/19/85	118	35	2.1
Tlahuac NS	09/21/85	49	13	2.0
Tlahuac EW	09/21/85	51	15	1.9
SCT EW	09/19/85	167	61	2.0

(1) Original peak ground acceleration
(2) Original peak ground velocity
(3) Corner period

Figure 1 shows strength spectra for the four sets of motions (S_a stands for pseudoacceleration). The circles identify the location of the corner period (T_s), defined as the period at which the strength spectra decreases after peaking either at a single point or at a plateau. Note that *LA 10in50* has a corner or dominant period around 0.3 sec, while those of *LA 50in50*, *LA Soft* and *Mexico Soft* are around 0.3, 1.0 and 2.0 sec, respectively.

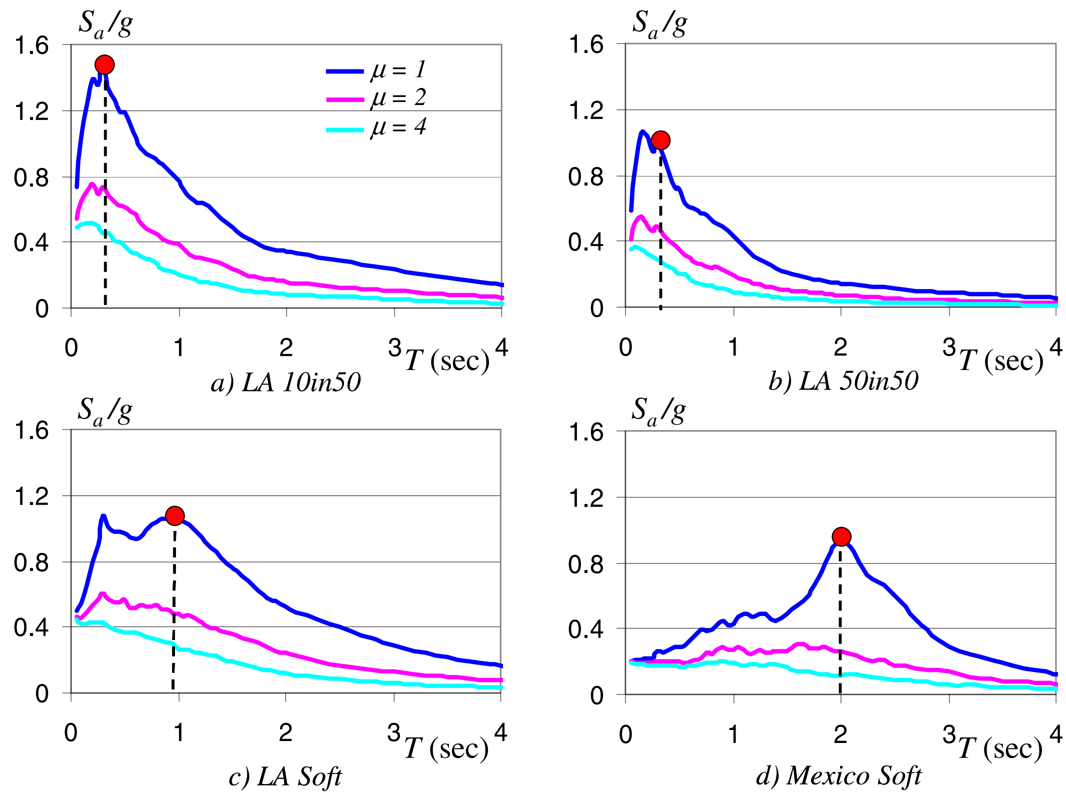


Figure 1. Strength spectra, 5% critical damping

Figure 2 shows $NE_{H\mu}$ spectra. There is a distinctive feature in the $NE_{H\mu}$ spectra corresponding to the sets of *LA* motions: starting from very small *T*, the $NE_{H\mu}$ demand tends to increase until *T* reaches the value of the corner period, after which it remains fairly constant. For the *Mexico Soft* set, $NE_{H\mu}$ tends to increase until *T* reaches the value of the corner period. After that, it tends to gradually decrease with a further increase in *T*. Note that the corner period defined according to Figure 1 delimits two distinctive zones in the $NE_{H\mu}$ spectra, and that the maximum $NE_{H\mu}$ demands for *Mexico Soft* are about two to three times larger than those corresponding to the *LA* motions. For constant ductility, *LA 10in10* and *LA Soft* are considered to have low energy content; *LA 50in50*, moderate energy content; and *Mexico Soft*, very large energy content.

LOW CYCLE FATIGUE MODELS

Although using energy-derived parameters as a representation of repeated cumulative loading allows the formulation of relatively simple seismic design methodologies, this approach should be carefully assessed. The plastic energy dissipating capacity of a reinforced concrete structure does not depend exclusively on its mechanical characteristics, but also on the specifics of its loading history. It has been repeatedly observed that the plastic energy dissipated up to failure by an element or structure can change significantly as a function of the amplitude of the plastic cycles. In particular, the plastic energy dissipated by a large number of small amplitude cycles can significantly exceed that dissipated up to failure through the application of a few large amplitude cycles.

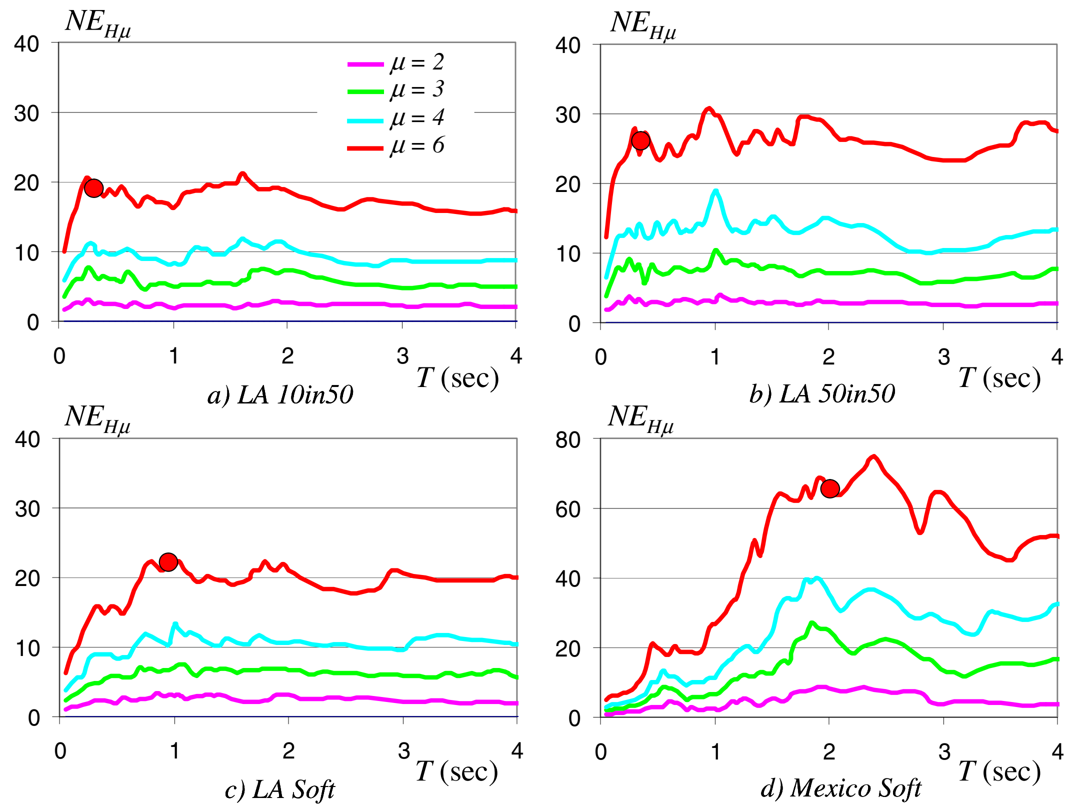


Figure 2. Normalized plastic energy spectra, 5% critical damping

Two low cycle fatigue models are used in this paper. One of these models is well-known and has been used extensively to formulate seismic design methodologies that account for low cycle fatigue. The second model is a simple energy-based model introduced in a companion paper by Teran and Jirsa [19].

Park and Ang damage index

Park and Ang [20] have formulated a damage index to estimate the level of damage in reinforced concrete elements and structures subjected to cyclic loading:

$$DMI_{PA} = \frac{\mu_{max}}{\mu_u} + \beta \frac{NE_{H\mu}}{\mu_u} \qquad (2)$$

where μ_{max} is the maximum ductility demand, μ_u is the ultimate ductility and β is the structural parameter that characterizes the cycling or cumulative deformation capacity of the element or structure (i.e., the stability of its hysteretic behavior). The work done by several researchers suggest that β of 0.15 corresponds to systems that exhibit fairly stable hysteretic behavior; while values of β ranging from 0.2 to 0.4 should be used to assess damage in systems exhibiting substantial strength and stiffness deterioration [10,21]. Under the presence of repeated cyclic loading into the plastic range, 1.0 represents the threshold value at which low cycle fatigue is expected to occur.

A simple model to predict low cycle fatigue

Teran and Jirsa [19] have recently proposed a simple model to assess the occurrence of low cycle fatigue. Basically, this model represents a simplification of the linear cumulative damage theory through the assumption of a fixed shape for the distribution of the plastic excursions that occur during the ground motion:

$$DMI_{MH}^{S} = (2-b)\frac{NE_{H\mu}}{\mu_{ucp}} \tag{3}$$

where $NE_{H\mu}$ is the ground motion parameter that quantifies the severity of the plastic demands, μ_{ucp} is the ultimate cyclic plastic ductility, and b is the structural parameter that characterizes the stability of the hysteretic cycle. b equal to 1.5 can be considered to be a reasonable conservative value to be used for seismic design of ductile structures [19]. DMI_{MH}^{S} equal to one implies incipient failure due to low cycle fatigue. For incipient collapse (DMI_{MH}^{S} = 1), Equation 3 can be reformulated in terms of μ_u as [19]:

$$NE_{H\mu} = \frac{2r}{a(2-b)}(\mu_u - 1) \tag{4}$$

where r is a reduction factor (less than one), and the value of a equals to one in case a response-control seismic design approach is used (such as the one discussed in this paper). The value of $NE_{H\mu}$ estimated from Equation 4 establishes the maximum plastic energy demand that a structure can accommodate before failure due to low cycle fatigue.

Considering the effect of the amplitude of the plastic excursions in the estimates of DMI_{MH}^{S}, Equation 3 yields, relative to the linear cumulative damage theory (Miner's Hypothesis)[19]: A) Lower damage estimates when applied to structures subjected to motions with low energy content; B) Similar estimates of damage when applied to motions with moderate and large energy content; C) Higher estimates of damage when applied to motions with very large energy content. As a consequence, the use of DMI_{MH}^{S} to assess incipient failure due to low cycle fatigue yields adequate results for the design of structures subjected to ground motions with moderate to very high energy content. In case of structures subjected to low energy demands, DMI_{MH}^{S} yields unsafe assessment of failure; and thus, needs to be complemented with other design criteria.

After extensive studies on the seismic performance of single-degree-of-freedom (SDOF) systems, it was observed that for motions having moderate to very high energy content, Equation 4 with r equal to 0.75 yields similar assessment of the occurrence of low cycle fatigue as Equation 2 [19]. Based on this observation, Equation 4 can be rewritten for design purposes as:

$$NE_{H\mu} = \frac{1.5}{(2-b)}(\mu_u - 1) \tag{5}$$

In the case of ductile structures b = 1.5, in such way that:

$$NE_{H\mu} = 3(\mu_u - 1) \tag{6}$$

CUMULATIVE DUCTILITY STRENGTH SPECTRA

The estimation of the lateral strength of a structure within the format of current seismic design methodologies is usually based on the use of constant *maximum* ductility pseudo-acceleration (strength) spectra. A constant *maximum* ductility strength spectrum corresponding to a ductility μ is defined in such way that the pseudo-acceleration (S_a) evaluated at any value of T will result in a lateral strength that is capable of controlling the *maximum* ductility demand on a SDOF system within a threshold value of μ. Within a practical design procedure that considers the static method of analysis; constant *maximum* ductility strength spectra can be used as follows:

1. Determine the design values of T and μ associated to the structure to be designed. In general, the value of μ is established according to the ultimate ductility capacity (μ_u); and thus, according to the detailing to be used in the structure.
2. Evaluate at T the design constant *maximum* ductility S_a spectrum corresponding to μ.
3. Provide the structure with a minimum base shear corresponding to $S_a(T,\mu)\ W$, where W is the reactive weight of the structure.

Concept of *cumulative* ductility strength spectra

A constant *cumulative* ductility strength spectrum corresponding to a *cumulative* ductility $NE_{H\mu}$ is defined so that its ordinates evaluated at any value of T will result in a lateral strength that is capable of controlling the *cumulative* ductility demand on a SDOF within a threshold value of $NE_{H\mu}$. As in the case of *maximum* ductility strength spectra, the ordinates of constant *cumulative* ductility strength spectra correspond to pseudo-acceleration. Note that although $NE_{H\mu}$ is a normalized plastic energy demand, spectra corresponding to constant $NE_{H\mu}$ have been denoted herein as constant *cumulative* ductility strength spectra. In the case of elasto-perfectly-plastic systems, $NE_{H\mu}$ is actually equal to the *cumulative* plastic ductility demand. For systems exhibiting deterioration of their hysteretic behavior this notation is not strictly correct, but the concept is directly applicable for their seismic design. The term constant *cumulative* ductility strength spectrum has been used herein to establish a parallelism between the concepts of constant *maximum* ductility strength spectra and constant *cumulative* ductility strength spectra.

The use of *cumulative* ductility strength spectra within the context of the static method of analysis is similar to the current use of strength spectra:

1. Determine the design values of T and $NE_{H\mu}$ associated to the structure to be designed. The value of $NE_{H\mu}$ can be established from Equation 5 according to the ultimate and cumulative ductility capacities of the structure (μ_u and b, respectively); and thus, according to the detailing to be used in the structure.
2. Evaluate at T the design constant *cumulative* ductility S_a spectrum corresponding to $NE_{H\mu}$.
3. Provide the structure with a minimum base shear corresponding to $S_a\,(T,NE_{H\mu})\ W$.

As may be concluded, the design of the lateral strength of a structure using *cumulative* ductility strength spectra follows the exact same steps currently used in seismic design. The only difference would be that the use of constant *cumulative* ductility S_a spectrum requires the definition of strength reduction factors that take into consideration the effect of the expected cumulative plastic demands.

Use of *cumulative* ductility strength spectra in seismic design

Within the context of performance-based seismic design for low cycle fatigue, adequate structural performance implies the prevention of failure or collapse of the structure due to excessive plastic deformation demands. To achieve reliable seismic design, it is necessary to provide adequate lateral strength and detailing in such way that the structure can adequately control and accommodate its maximum and cumulative plastic deformation demands. In this paper, it will be assumed that the detailing of the structure is a given (i.e., the decision process involved in selecting a particular detailing scheme will not be discussed herein), and that design for low cycle fatigue implies estimating the required lateral strength for the structure once its T is established.

Consistent with the format of current seismic design methodologies, it is suggested that strength design for low cycle fatigue be carried out through the use of pseudo-acceleration spectra. From an extensive study on the seismic performance of SDOF systems subjected to ground motions with different frequency content and duration, it has been concluded that the lateral strength to be provided to a ductile structure to avoid failure due to excessive plastic demands should satisfy the following two conditions:

1. $\mu_{max} \leq 0.7\ \mu_u$. First, it is necessary to revise that the *maximum* ductility demand does not exceed the ultimate deformation capacity of the structure. Note that this condition can be satisfied through the use of constant *maximum* ductility strength spectra, and that even for motions with low energy content, μ_{max} should not be too close to μ_u.

2. $NE_{H\mu} \leq \frac{1.5}{(2-b)}(\mu_u - 1)$. Second, and particularly for sites that generate long duration motions with narrow-frequency content, it is necessary to revise that the cumulative ductility demand on the structure does not exceed the threshold value given by Equation 5. Note that this condition can be satisfied through the use of constant *cumulative* ductility strength spectra.

The first condition depends on the ultimate deformation capacity of the structure, which is numerically characterized through μ_u once the detailing of the structure is defined. The second condition depends not only on the ultimate deformation capacity of the structure, but on the stability of its hysteretic cycle, which are numerically characterized through μ_u and b, respectively, once the detailing has been defined. The design strength to be provided to the structure should be the larger of the values derived from the two conditions.

Figures 3, 5 and 6 compare *maximum* and *cumulative* ductility strength spectra for the different sets of ground motions. Both types of spectra correspond to ductile structures in such way that b = 1.5 and μ_u = 4 and 6. While the *maximum* ductility strength spectra (black circles) were defined in such way that the value of μ associated to them is equal to 0.7 μ_u, the *cumulative* ductility strength spectra (white circles) were defined so that the value of $NE_{H\mu}$ associated to them is equal to 3 (μ_u – 1). The figures also include elastic strength spectra (black line), and incipient collapse strength spectra (gray line) derived from the Park and Ang damage index with β of 0.15 (ductile structures). Note that the Park and Ang incipient collapse spectra provide the minimum lateral strength required by SDOF systems so that their level of damage after a ground motion is incipient failure or collapse (DMI_{PA} = 1). Because of the extensive calibration of DMI_{PA} with experimental and field data, the incipient collapse strength spectra derived from it will be used as a benchmark strength design level against which the pertinence of using the two design conditions introduced in this paper will be assessed.

Figure 3 shows strength spectra for the two sets of firm soil motions corresponding to *LA*. In the case of *LA 10in50*, set formed by motions with low energy content, the ordinates of the constant *maximum* ductility strength spectra are larger than those of constant *cumulative* ductility strength spectra, and are very similar to those derived from the Park and Ang damage index. The results shown for *LA 10in50* suggest that for ductile structures subjected to low energy motions generated in firm soil, seismic design should focus on controlling the maximum ductility demand. Although not shown, similar conclusions were derived from the results obtained for *LA Soft*, set that corresponds to soft soil motions with low energy content. The similitude of conclusions derived from *LA 10in50* and *LA Soft* indicate that in the case of motions with low energy content, seismic design should focus, independently of the type of soil, on controlling the maximum ductility demand.

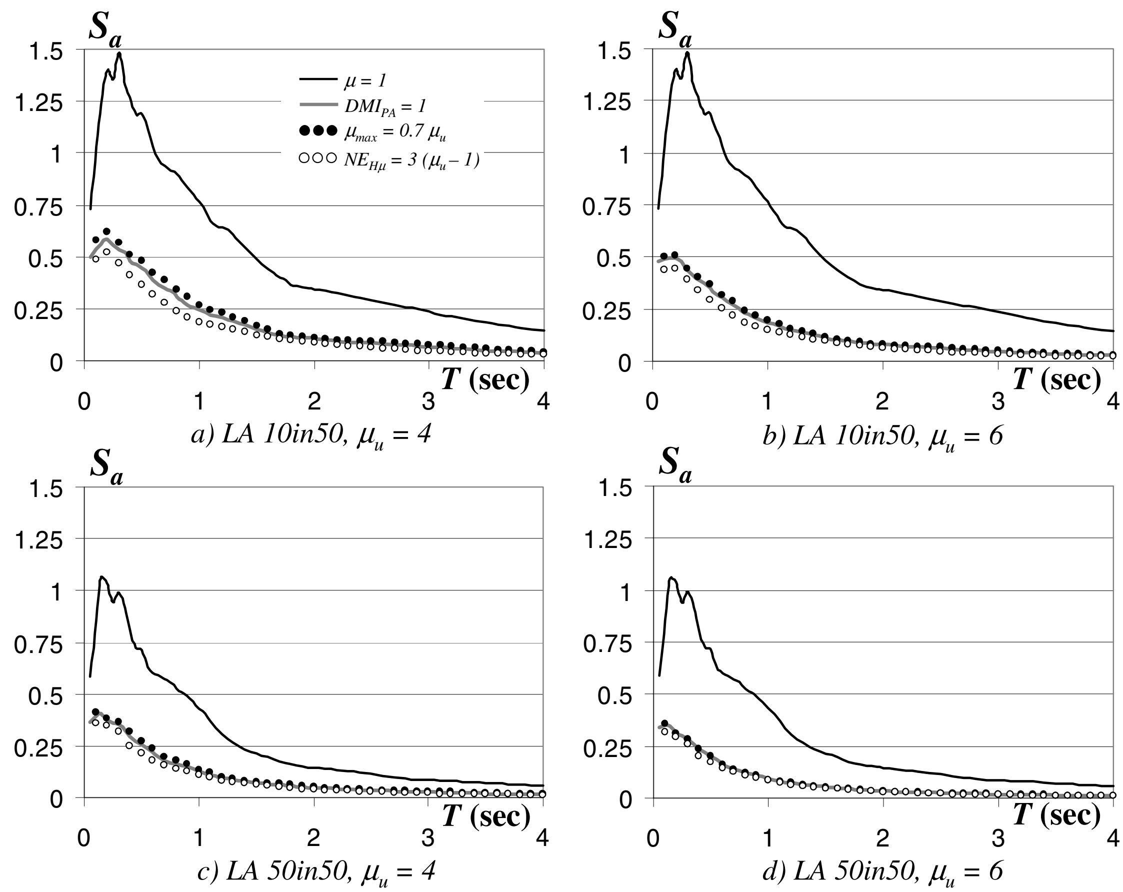
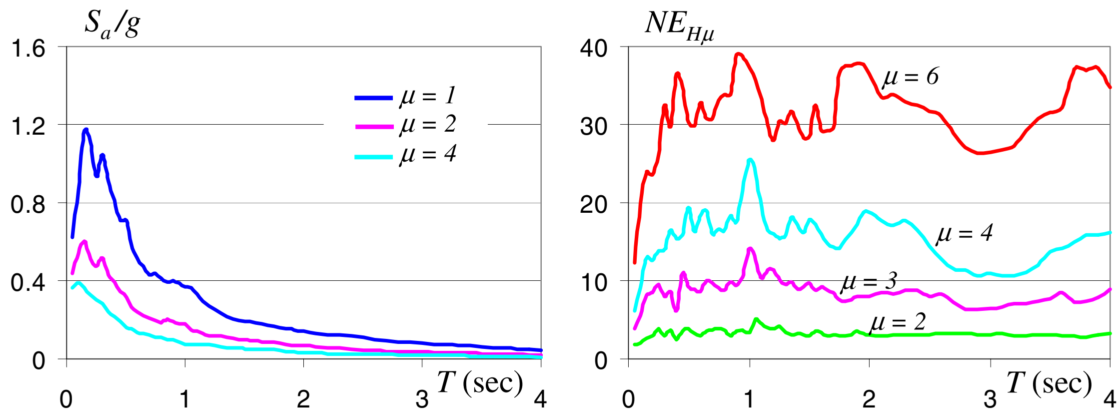


Figure 3. Strength spectra corresponding to firm soil motions, 5% critical damping

Figures 3c and 3d were obtained for *LA 50in50*, set that includes motions with moderate and high energy content. The comparison of results derived from *LA 10in50* and *LA 50in50* suggest that as the energy content of the motion is increased, the ordinates of *cumulative* ductility strength spectra increase relative to those of *maximum* ductility strength spectra. Nevertheless, the ordinates of *maximum* ductility strength spectra are still larger than those of *cumulative* ductility strength spectra, and still very similar to those derived from the Park and Ang damage index. The results obtained for *LA 50in50* tend to confirm that seismic design of ductile structures in firm soil should focus on controlling their maximum ductility demand.

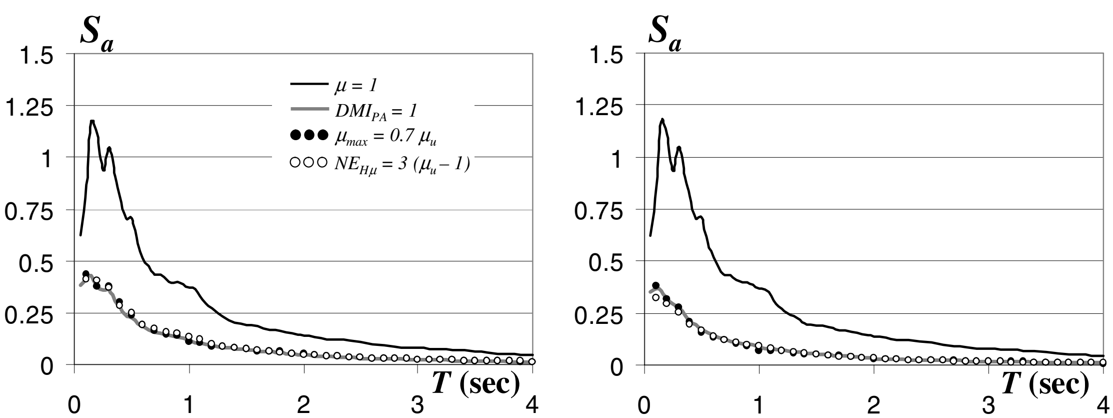
To explore the possibility of low cycle fatigue being an issue for seismic design of structures located in firm soil, a new and fifth set of ground motions was established. The fifth set, denoted *LA MaxEnerg*, is formed by the twelve motions with the highest energy content in *LA 50in50*. Figure 4 shows constant *maximum* ductility S_a and $NE_{H\mu}$ spectra for *LA MaxEnerg.* After an extensive study of normalized energy spectra corresponding to simulated and recorded firm soil ground motions, it was observed that the energy demands shown in Figure 4b are about the largest expected in firm soil.

Figure 5 shows strength spectra for *LA MaxEnerg*. A further increase in the energy content of the motions from moderate to high, results in a new increase in the ordinates of *cumulative* ductility strength spectra relative to those of *maximum* ductility strength spectra. The increase is so that for *LA MaxEnerg*, both the *maximum* and *cumulative* ductility criteria yield similar lateral strength, which in turn is very similar to that derived from the Park and Ang damage index. Although for *LA MaxEnerg* the *cumulative* ductility strength spectra have in some cases slightly larger ordinates than the *maximum* ductility strength spectra, the small difference in the design lateral strength yielded by both criteria does not seem to justify the consideration of *cumulative* ductility during seismic design of ductile structures located in firm soils.



a) Strength spectra *b) Normalized energy spectra*

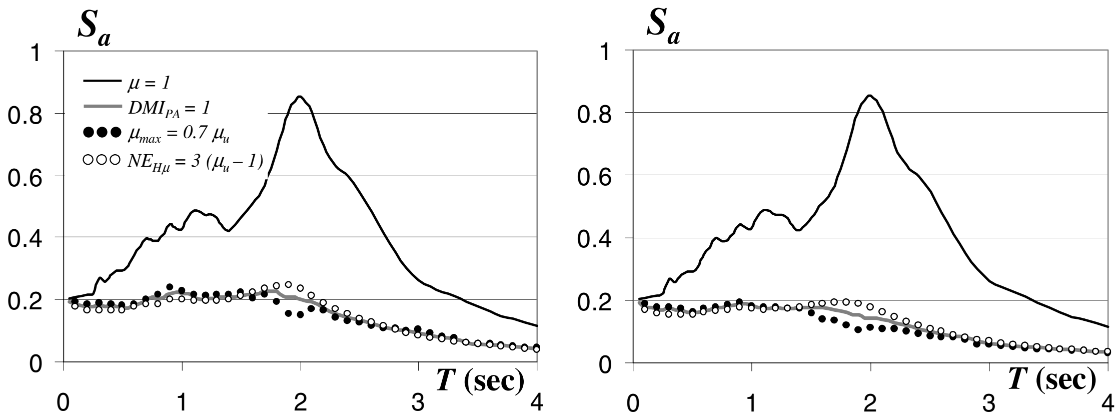
Figure 4. Spectra corresponding to *LA MaxEnerg*, 5% critical damping



a) LA MaxEnerg, μ_u = 4 *b) LA MaxEnerg, μ_u = 6*

Figure 5. Strength spectra corresponding to *LA MaxEnerg*, 5% critical damping

Figure 2d shows that for *Mexico Soft*, the energy content is very large for systems having T close to 2 sec, value that corresponds to the corner period of this set of motions. In the spectral zone with large energy demands for *Mexico Soft* (T close to T_s), Figure 6 shows that the *cumulative* ductility criteria yields considerably larger lateral strength than the *maximum* ductility criteria. As the value of T departs from T_s, the maximum ductility criteria may yield slightly larger lateral strength requirements, particularly for moderate and small values of T. Note that an envelope of the largest ordinates of both *maximum* and *cumulative* ductility strength spectra yields very similar lateral strength than the Park and Ang damage index, except for T close to T_s. Particularly, in this range of T, the *cumulative* ductility strength spectra yields higher estimates of lateral strength than the Park and Ang damage index. In this respect, recent studies suggest that DMI_{PA} underestimates, relative to the linear cumulative damage theory, the lateral strength required to prevent low cycle fatigue in SDOF systems having T close to T_s and subjected to motions with narrow frequency content [22].



a) Mexico Soft, $\mu_u = 4$ *b) Mexico Soft, $\mu_u = 6$*

Figure 6. Strength spectra corresponding to *Mexico Soft*, 5% critical damping

PERSPECTIVES FOR PERFORMANCE-BASED SEISMIC DESIGN

Within the context of current seismic design codes, the design lateral strength is obtained by reducing the design elastic strength spectra evaluated at *T* by an appropriate strength reduction factor. Because of the need to rationalize the use of strength reductions factors within performance-based design formats, significant research effort has been devoted in recent years to the formulation of transparent and reliable strength reduction factors. The strength reduction factor, *R*, is defined as:

$$R(\mu,T) = \frac{S_a(1,T)}{S_a(\mu,T)} \qquad (7)$$

where $S_a\ (\mu,\ T)$ denotes spectral pseudo-acceleration evaluated at μ and *T*. μ equal to 1 implies elastic behavior, and $S_a\ (1,T)$ is the seismic coefficient corresponding to the minimum strength that would keep a structure with 5% critical damping in the elastic range.

Equation 7 should be differentiated from strength reduction factors used in current seismic design codes. Normally, strength reduction factors used in practice implicitly consider that the actual lateral strength of a structure can be two to five times its design strength. While Equation 7 only considers reduction in strength due to inelastic behavior, a practical strength reduction factor should account for reductions due to inelastic behavior and expected over-strength in the actual structure.

The value of *R* strongly depends on μ and *T*, and is significantly influenced by the type of soil in which the design ground motion is generated. The following trends have been observed for the strength reduction factor corresponding to long duration motions with narrow frequency content [23]:

- *R* tends to one as *T* approaches zero.
- *R* increases rapidly as the value of *T* is increased, until it peaks at a value considerably larger than μ at *T* close to T_s. After it has peaked, *R* decreases at a high rate until it reaches a value of μ for large *T*.
- *R* is not particularly sensitive to the duration of ground motion or other important ground motion characteristics, such as intensity and epicentral distance.
- The values of *R* corresponding to very soft soil can be affected significantly by a variation in the frequency content of the motion.

Of particular importance to this paper is the observation that for very soft soils, such as those located in the lake zone of Mexico City, R reaches values considerably larger than μ for T close to T_s. This is illustrated in Figure 7 by the continuous black lines, which correspond to values of R for μ equal to μ_u. As shown, under the assumption that the maximum ductility demand undergone by a SDOF system is equal to μ_u, R can reach values up to $2\mu_u$.

The values of R corresponding to constant *cumulative* ductility strength spectra can be defined as:

$$R(NE_{H\mu},T) = \frac{S_a(\mu = 1,T)}{S_a(NE_{H\mu},T)} \tag{8}$$

where $S_a\ (NE_{H\mu},\ T)$ denotes spectral pseudo-acceleration corresponding to a *cumulative* ductility strength spectrum evaluated at $NE_{H\mu}$ and T. $S_a\ (\mu = 1,T)$ is the seismic coefficient corresponding to the minimum strength that would keep a structure with 5% critical damping in the elastic range. As shown in Figure 7, the values of R derived for *Mexico Soft* from the *cumulative* ductility criteria are very similar to those obtained from the Park and Ang damage index, and considerably smaller than those corresponding to the criteria in which μ is assumed equal to μ_u. In fact, the values of R derived from the *cumulative* ductility criteria do not exceed the value of μ_u, even for T close to T_s.

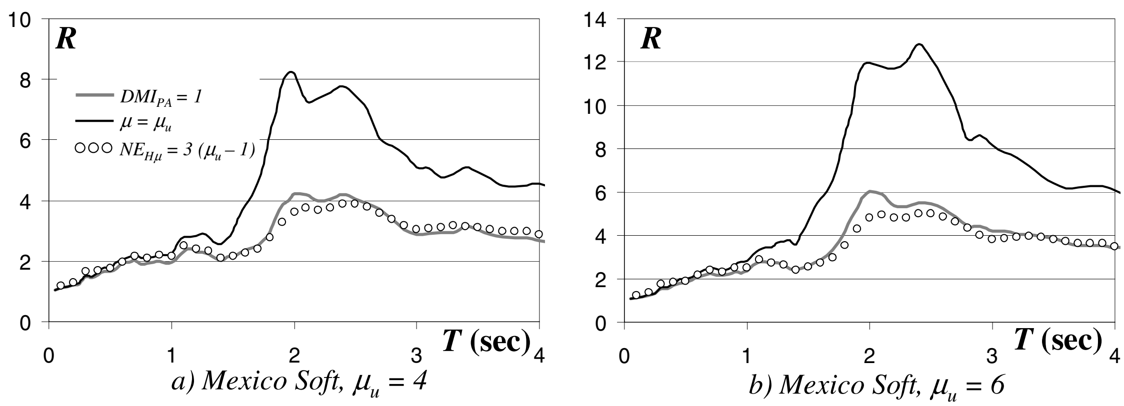


Figure 7. Strength reduction factors corresponding to *Mexico Soft*, 5% critical damping

In the short and medium terms, performance-based seismic design that accounts for the effect of low cycle fatigue should consider the following:

1. In the case of very soft soils (long duration motions with narrow frequency content), the design lateral strength should comply with the following two conditions: $\mu_{max} \leq 0.7\ \mu_u$ and $NE_{H\mu} \leq \frac{1.5}{(2-b)}(\mu_u - 1)$. Consistent with this, the value of R used for design purposes should not exceed the value of μ_u. An option to establish transparently the values of R for practical seismic design is to incorporate the use of constant *cumulative* ductility strength spectra to current codes. Within this context, *cumulative* ductility strength spectra may complement or substitute the use of constant *maximum* ductility strength spectra. In any case, it is important for current codes to allow for rational estimation of the maximum lateral displacement demand in the structure for the purpose of non-structural damage control and avoidance of structural instability.
2. In any other type of soil, seismic design should focus on controlling maximum ductility. Nevertheless, the minimum design lateral strength should be such that the maximum ductility

demand in the structure is limited to 0.7 μ_u. Perhaps and based on recommendations made by other researchers [1,2], a more stringent limit for μ_{max}, such as 0.6 μ_u, can be imposed.

As suggested before, strength reduction factors currently used in practice implicitly consider reductions due to: A) Inelastic behavior; and B) Expected over-strength. The rational use of *maximum* and *cumulative* ductility strength spectra should be the basis for the rational and transparent formulation of strength reduction factors for practical performance-based seismic design.

CONCLUSIONS

Damage models that quantify the severity of cumulative loading through plastic energy are simple tools that can be used for practical seismic design. The concept of constant cumulative ductility strength spectra, developed from one such model, is a useful tool for performance-based seismic design.

Seismic design of ductile structures located in firm soil should focus on controlling their maximum ductility demand. Nevertheless, even for motions with low energy content, the maximum ductility demand should not be too close to μ_u. The results obtained in this paper suggest that providing earthquake-resistant structures with enough lateral strength to control its maximum ductility demand within the threshold of 0.7 μ_u is enough to avoid incipient failure or collapse.

Constant cumulative ductility strength spectra can be used to identify cases in which cumulative plastic demands may become a design issue, and provide quantitative means to estimate the design lateral strength required to avoid failure due to low cycle fatigue. In the case of long duration motions with narrow frequency content, strength requirements for ductile structures should be such that they control adequately the maximum and cumulative ductility demands according to: $\mu_{max} \leq 0.7\ \mu_u$ and $NE_{H\mu} \leq \frac{1.5}{(2-b)}(\mu_u - 1)$, respectively. In this case, the value of R used for design purposes should not exceed the value of μ_u.

Studies are currently being carried out to define if constant *cumulative* ductility strength spectra should complement or substitute the use of constant *maximum* ductility strength spectra during seismic design of ductile structures located in the lake zone of Mexico City. These studies should also clarify the influence of frequency content and duration of the seismic excitation in the shape and magnitude of constant *cumulative* ductility strength spectra. As for structures that exhibit irregularities and/or exhibit rapidly deteriorating hysteretic behavior, the set of tools discussed herein become sensitive to the specifics of the local and global hysteretic behavior, and thus, its application becomes less reliable. While the tools discussed herein can be used to determine the strength and ultimate deformation requirements of ductile structures with stable hysteretic behavior, a more stringent application should be considered for structures with erratic seismic behavior. In this respect, the effects of upper modes and of stiffness and strength degradation in constant *cumulative* ductility strength spectra should be assessed. Finally, it should be considered that some type of soils other than those located in the lake zone of Mexico City (e.g., bay mud in the San Francisco Bay area) may exhibit high levels of energy content that may imply the need for using *cumulative* ductility strength spectra.

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