



## **CORRELATION STUDIES ON AN RC DOUBLE T-SHAPED SHEAR WALL SPECIMEN**

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### **SUMMARY**

A three-dimensional small strain constitutive model for concrete based on an orthotropic hypoelastic formulation is used to simulate the nonlinear behaviour of several concrete and reinforced concrete (RC) structures. In particular it is utilized to perform an extensive correlation study on an RC double T-shaped shear wall specimen subjected to a constant vertical load and a horizontal increasing load up to failure. The analyses are performed both at global and local level. The nonlinear global response is studied in terms of top panel displacement. At local level the crack distribution for the different load levels in the three panels constituting the considered RC specimen is studied. The good correspondence between analytical and experimental results obtained in the performed correlation studies indicates that the adopted constitutive model is able to accurately describe the nonlinear behaviour of RC structures.

### **INTRODUCTION**

Numerous constitutive models have been proposed and used in the last four decades for analyzing concrete structures. Among them, nonlinear elasticity models, plasticity models, damage and fracture mechanics models, and models based on the endochronic theory of plasticity have been found adequate in describing the macroscopic stress-strain behaviour of concrete. More recently, attempts have been made to unify some of these theories through the introduction of nonlinear elastic-plastic models and of elasto-plastic fracturing/damage models.

Many of the above-mentioned models have a conceptual importance greater than their practical importance. They can describe only certain aspects of concrete behaviour and therefore their practical use is still limited to situations of scarce engineering importance. Beside these limitations, it should be appreciated that many of the existing constitutive models are difficult to implement for practical analyses of concrete structures.

A constitutive model for concrete which, on one hand, is simple enough to be easily implemented in a finite element code and used for practical calculations but, on the other hand, retains the more important

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features of the gross engineering behaviour of concrete has been proposed by D'Ambrisi and Balan [1,2]. This model, implemented in a finite element code, is used in the following to simulate the nonlinear behaviour of several concrete and RC structures.

The model capabilities are first tested by performing the nonlinear analysis of a concrete cylinder subjected to a split-tension test. The results are compared with those obtained with the linear elastic analysis and the limit analysis. The model is then used to perform a correlation study on a concrete pipe subjected to an external hydrostatic pressure. The model is finally utilized to perform an extensive correlation study on an RC double T-shaped shear wall specimen subjected to a constant vertical load and an horizontal increasing load up to failure.

## CONCRETE MATERIAL MODEL

The concrete material model proposed by D'Ambrisi and Balan [1,2] is a three-dimensional, small strain constitutive model based on an orthotropic hypoelastic formulation. It takes into account triaxial nonlinear stress-strain behaviour, strain-softening and cyclic load reversal conditions. The incremental constitutive law is orthotropic, with the directions of orthotropy defined by the current principal stress directions [3,4,5,6,7,8].

The model is completely defined by conventional material parameters, e.g., the initial Young modulus  $E_0$ , the initial Poisson ratio  $\nu_0$ , the uniaxial compressive strength  $f_c$ , the uniaxial tensile strength  $f_t$ , and the corresponding strains  $\epsilon_c$  and  $\epsilon_t$ , respectively.

The major features of the proposed constitutive model are briefly described in the following. An incremental stress-strain relationship is assumed for concrete of the type:

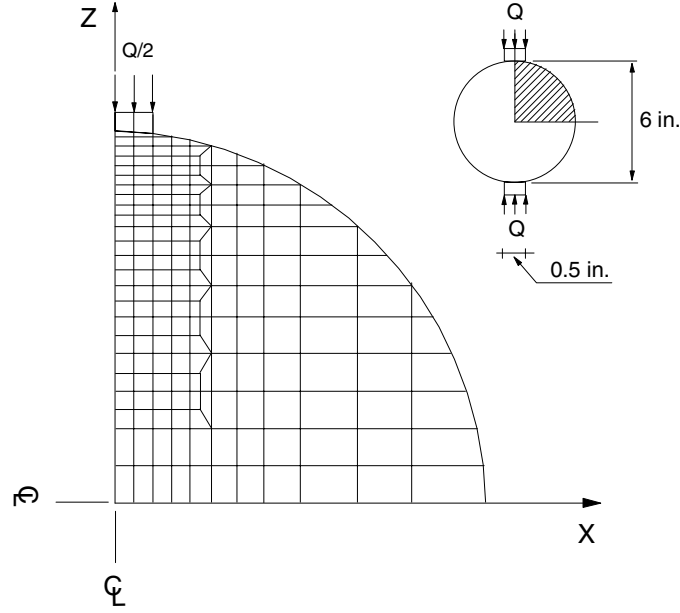
$$d\sigma = C(\sigma)d\epsilon \quad (1)$$

i.e. a piecewise linear incremental relation between stress increments  $d\sigma$  and strain increments  $d\epsilon$ . The incremental constitutive tensor  $C$  depends on the stress tensor  $\sigma$ . In particular the terms of the tensor  $C$  are defined as functions of the stress state by generalizing to the triaxial case the concept of equivalent uniaxial strain [3,4,6,7,8]. The constitutive relation (1) is written with reference to the direction of orthotropy, defined by the current principal stress directions [3,4,5,6,7,8].

## CONCRETE CYLINDER SUBJECTED TO A SPLIT-TENSION TEST

In this paragraph the nonlinear analysis of a concrete cylinder subjected to a split-tension test (ASTM [9]) is performed. The results are compared with those obtained with the linear elastic analysis and with the limit analysis.

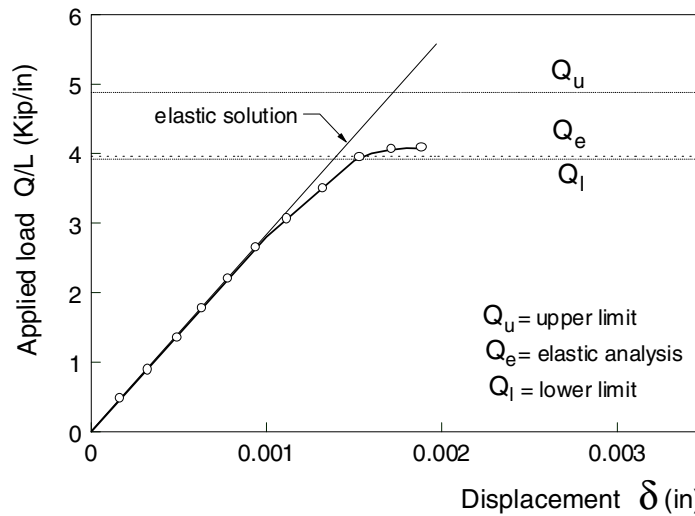
In the numerical analysis the load is applied uniformly on the loading surface. Moreover, it is assumed that there is no slip between the steel loading plate and the concrete. A plane strain state is considered. This assumption is valid throughout the concrete cylinder, except in the zones close to the bases. Utilinzing the symmetries only a quarter of the cylinder is considered, as shown in Figure 1. The mesh for the finite element analysis is constituted by four node plane strain isoparametric quadrilateral elements. In the upper part of the vertical diameter a more refined mesh is used (Figure 1), since a large stress gradient is expected in this zone.



**Figure 1. Finite element mesh utilized to simulate a split-cylinder test.**

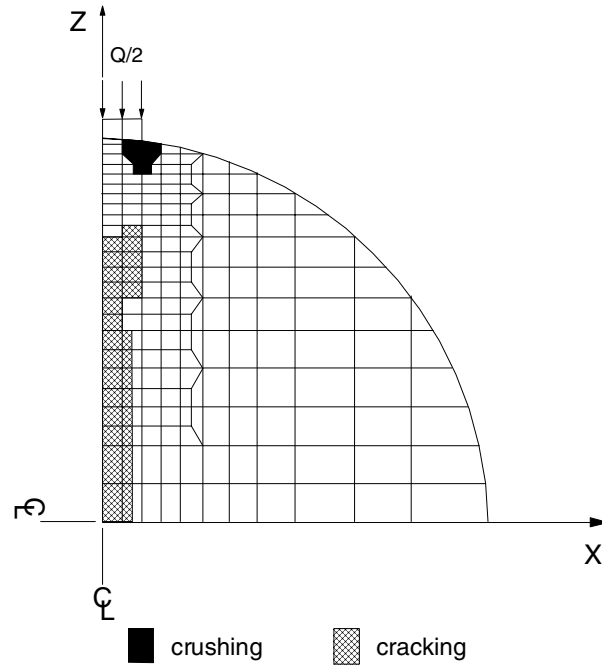
The concrete is characterized by the following mechanical parameters (Kupfer [10]):  $E_0 = 4600 \text{ ksi}$ ,  $\nu_0 = 0.19$ ,  $f_c = 4.65 \text{ ksi}$ ,  $f_t = 0.46 \text{ ksi}$ ,  $\epsilon_c = 0.002$  and  $\epsilon_t = 0.0002$ ; while the steel loading plate is characterized by:  $E_0 = 30000 \text{ ksi}$  and  $\nu_0 = 0.3$ .

The obtained load-displacement response is reported in Figure 2. In this figure the results relative to the maximum load obtained with the linear elastic analysis ( $Q_e = 3.96 \text{ kips}$ ) and those relative to the upper limit ( $Q_u = 4.88 \text{ kips}$ ) and the lower limit ( $Q_l = 3.92 \text{ kips}$ ) of the limit analysis are also reported. The collapse load obtained with the performed nonlinear analysis is  $Q = 4.12 \text{ kips}$ , that is approximately 4% larger than  $Q_e$ .



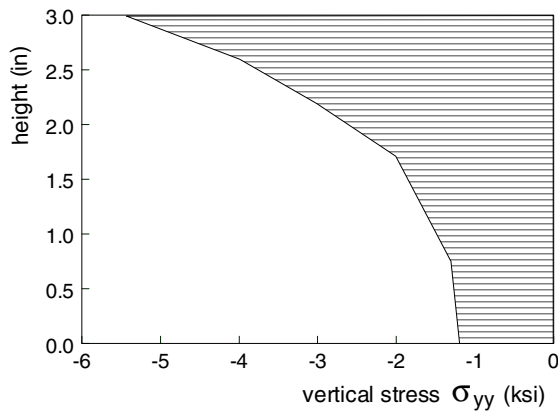
**Figure 2. Load-displacement response of the split-cylinder test and maximum loads obtained with the elastic analysis and with the limit analysis.**

Figure 3 shows the distribution of crushing and cracking zones in correspondence of the collapse load. It has to be noticed that crushing originates below the edge of the loading plate, for a load of 1.2 kips ( $0.3 Q_e$ ). With the increasing of the load, the plastic zone spreads throughout the concrete cylinder until the collapse load is reached.

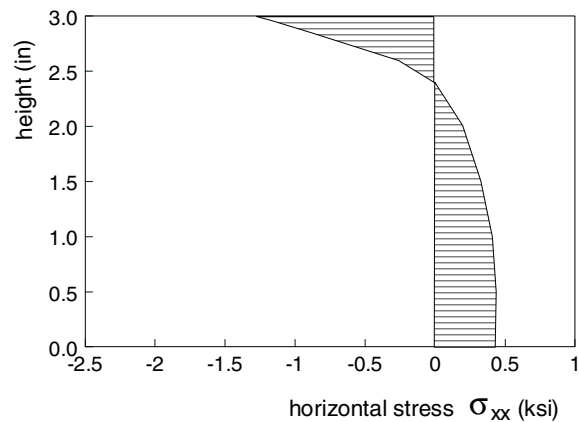


**Figure 3. Distribution of crushing and cracking zones in correspondence of the collapse load.**

In Figures 4 and 5 the distribution of the vertical stress component  $\sigma_{yy}$  and of the horizontal stress component  $\sigma_{xx}$  along the vertical diameter at the collapse load  $Q$  are reported, respectively. The vertical stress is a prevalently compression stress. The horizontal tension stress is approximately uniform along the three-quarters of the vertical diameter. In the zones right below the loading plate, both the vertical and the horizontal stress have a large gradient.



**Figure 4. Distribution of the vertical stress  $\sigma_{yy}$  along the vertical diameter in correspondence of the collapse load.**

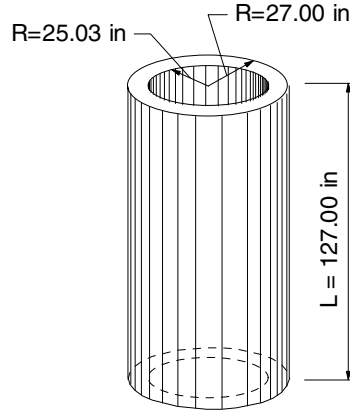


**Figure 5. Distribution of the horizontal stress  $\sigma_{xx}$  along the vertical diameter in correspondence of the collapse load.**

## CONCRETE PIPE SUBJECTED TO AN EXTERNAL HYDROSTATIC PRESSURE

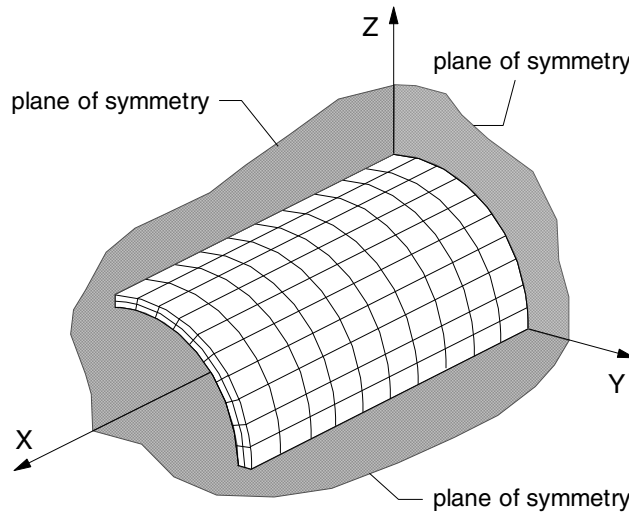
In the present paragraph the nonlinear behaviour of one of the concrete pipes tested by Runge and Haynes [11] is simulated. The obtained results are then compared with the experimental data.

The pipe is subjected to an external hydrostatic pressure. In the analysis it is assumed that the pipe has a perfectly circular cross section (Figure 6), that is the effects produced by the initial imperfections are ignored (Hsieh et al. [12]). The mechanical parameters utilized for concrete are those reported in Runge and Haynes [11]:  $E_0 = 3660$  ksi,  $\nu_0 = 0.19$ ,  $f_c = 7.22$  ksi,  $f_t = 0.722$  ksi,  $\epsilon_c = 0.002$ , and  $\epsilon_t = 0.0002$ .

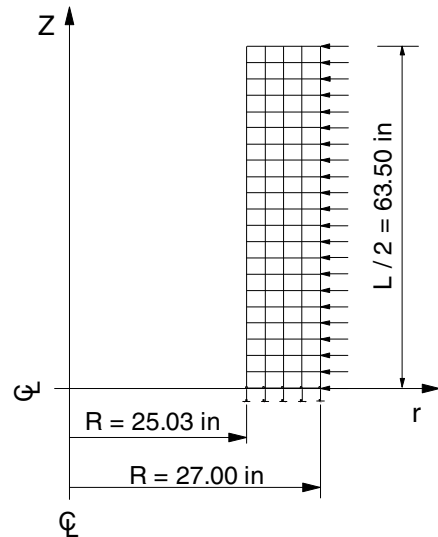


**Figure 6. Geometry of the concrete pipe subjected to an external hydrostatic pressure.**

The nonlinear analysis is performed with reference to two different finite element discretizations. In one case, utilizing the symmetries shown in Figure 7, only one eighth of the concrete pipe is studied. Since the pipe has a significant thickness, the mesh is realized with two layers of eight node isoparametric brick elements (Figure 7). In the second case, utilizing the axial symmetry and the symmetry with respect to the midspan transversal plane, only the longitudinal section of one fourth of the pipe is studied (Figure 8).

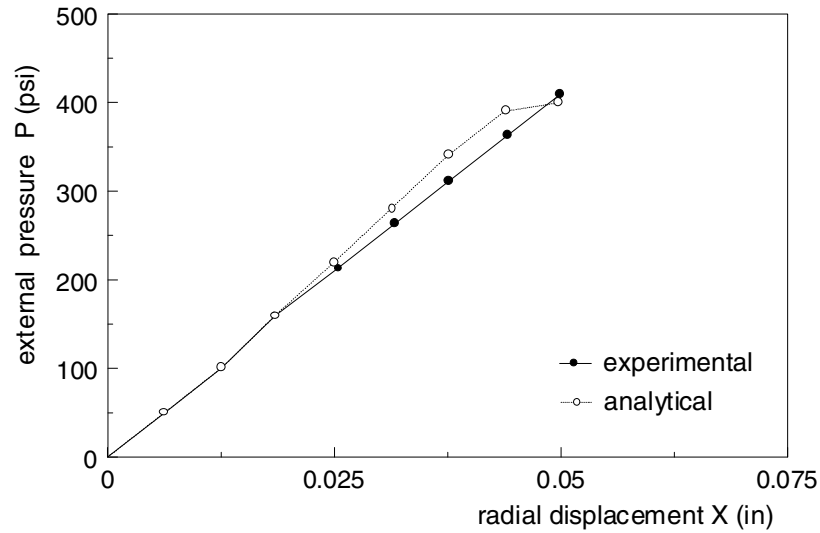


**Figure 7. Discretization of one eighth of the concrete pipe with eight node isoparametric brick elements.**



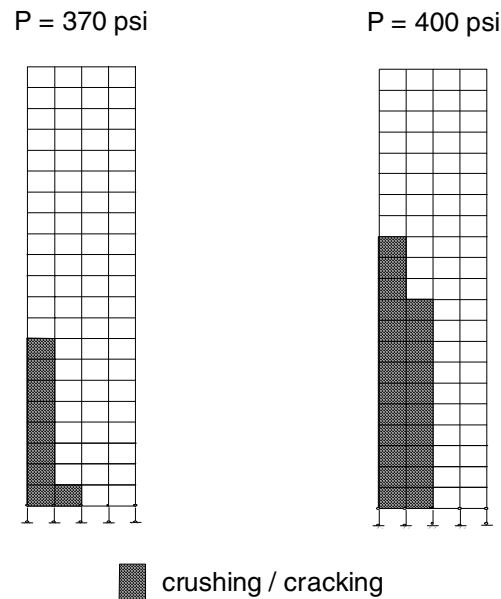
**Figure 8. Discretization of the longitudinal section of one fourth of the concrete pipe with four node axisymmetric elements.**

The mesh is realized with four node axisymmetric elements (Figure 8). The load-displacement responses obtained in the two cases are approximately coincident. In Figure 9 the load-displacement response relative to the axisymmetric case is compared with the experimental data. A good agreement between analytical and experimental results can be observed. The calculated collapse load is slightly lower than that experimentally measured.



**Figure 9. Load-displacement response of the concrete pipe tested by Runge and Haynes [11].**

Figure 10 shows the distribution of crushing and cracking zones in correspondence of an external pressure  $P=370$  psi and an external pressure  $P=400$  psi (collapse load). Cracking originates from the internal surface of the concrete pipe and spreads throughout the structure until its collapse.



**Figure 10. Distribution of crushing and cracking zones in the concrete pipe in correspondence of an external pressure  $P=370$  psi and an external pressure  $P=400$  psi (collapse load).**

## RC DOUBLE T-SHAPED SHEAR WALL SPECIMEN

In the following the nonlinear behaviour of an RC structure constituted by three shear panels arranged as a double T, that was tested at the NUPEC's Tadotsu Engineering Laboratory of Tokyo, is simulated. The analyses are performed both at global and local level. The nonlinear global response is studied in terms of top panel displacement. At local level, the crack distribution for the different load levels in the three panels constituting the considered RC specimen is studied. The obtained results are then compared with the experimental data.

### Geometrical characteristics and material parameters

The geometry of the structure, named U-1 specimen in the original report, is illustrated in the Figures 11, 12, and 13. The web panel is 75 mm thick, 3000 mm long from center to center of the lateral panels, and has a clear height of 2000 mm. The lateral panels are 100 mm thick, 2980 mm long and have a clear height of 2000 mm.

The distribution of the reinforcing bars in the panels is illustrated in Figures 14 and 15. The vertical and horizontal reinforcement of the web panel is constituted by D6 type bars (nominal diameter of 6.345 mm according to the Japanese Industrial Standard) disposed to a constant distance of 70 mm (Figure 15). The vertical reinforcement of the lateral panels is constituted by D6 type bars disposed every 175 mm, while the horizontal reinforcement is constituted by D6 type bars disposed every 70 mm (Figure 14).

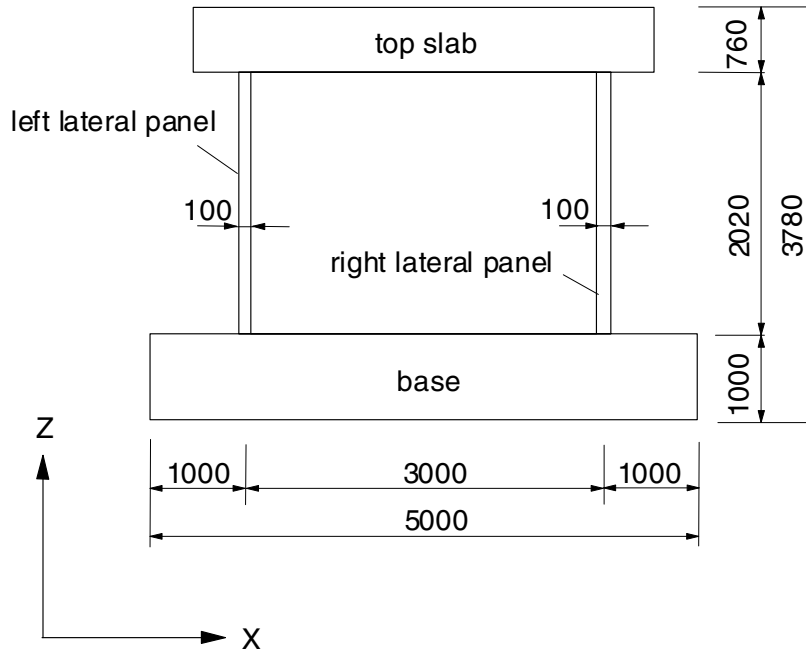
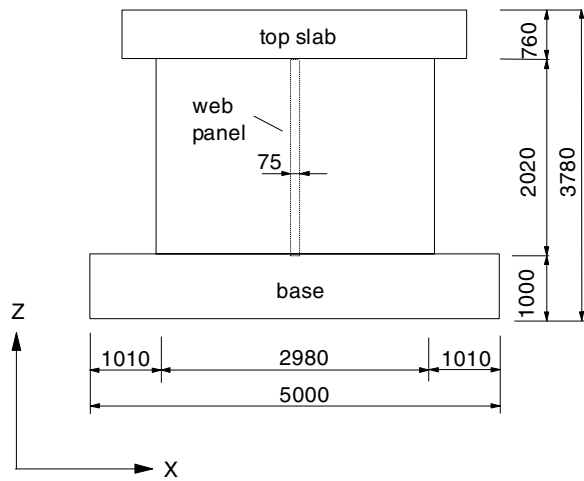
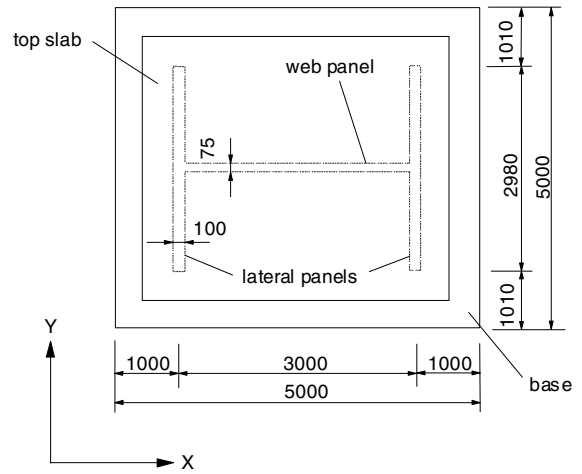


Figure 11. Frontal view.

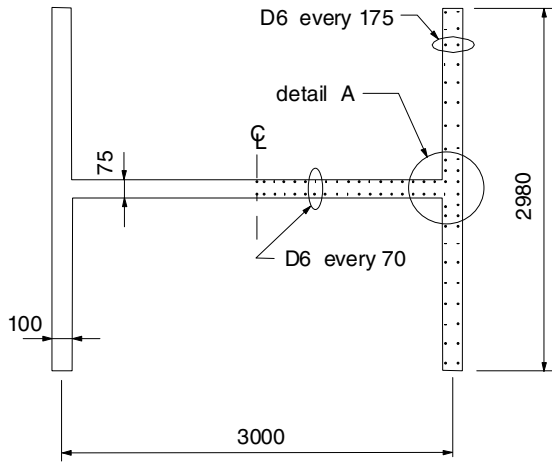
The mechanical parameters characterizing the reinforcing steel are:  $E_s = 18.9 \cdot 10^3 \text{ kgf/mm}^2$ ,  $\sigma_y = 38.7 \text{ kgf/mm}^2$ ,  $E_h = 9.45 \cdot 10^2 \text{ kgf/mm}^2$  and  $A_s = 32.0 \text{ mm}$ ; while those characterizing the concrete are:  $E_0 = 23.2 \cdot 10^2 \text{ kgf/mm}^2$ ,  $\nu_0 = 0.167$ ,  $f_c = 2.880 \text{ kgf/mm}^2$  and  $f_t = 0.244 \text{ kgf/mm}^2$ .



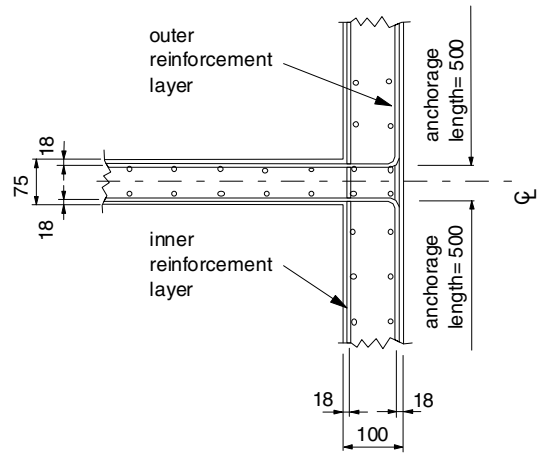
**Figure 12. Lateral view.**



**Figure 13. Plan view.**



**Figure 14. Transversal section.**



**Figure 15. Detail A.**

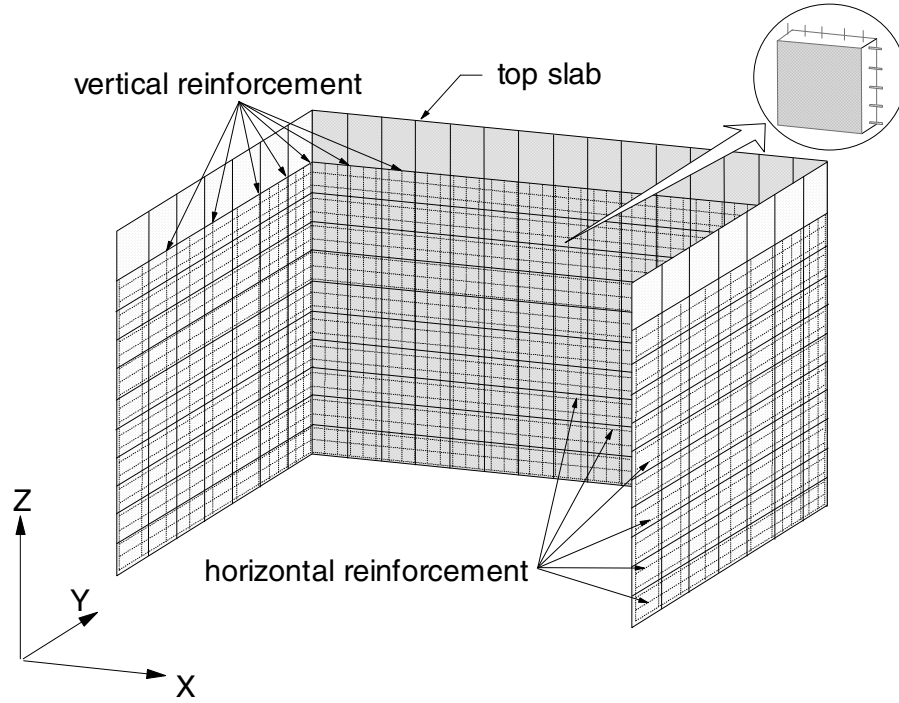
### Finite element modeling

Considering the geometrical symmetry and the loading symmetry, only a half of the structure is analyzed. The finite element discretization is reported in Figure 16. The mesh is realized with the four node rectangular plane element shown in Figure 17. It is assumed that there is no slip between the reinforcing bars and the concrete. The concrete is modeled with the D'Ambrisi and Balan [1,2] constitutive model, while the steel is modeled with the constitutive law shown in Figure 18.

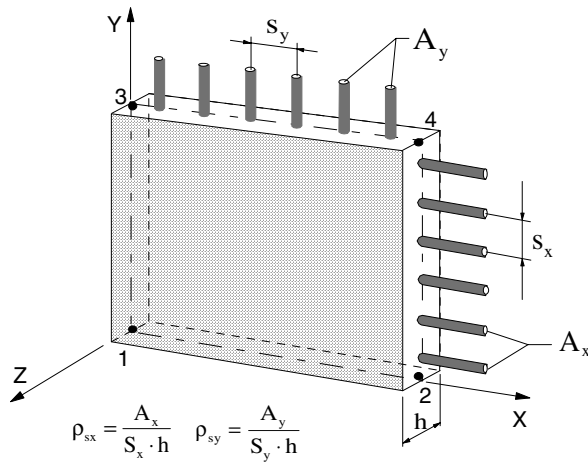
In the numerical analysis the steel and the concrete mechanical parameters are assumed coincident with the experimental parameters reported in the NUPEC's Tadotsu Engineering Laboratory report (see previous paragraph).

To take into account the influence of the top slab on the behaviour of the structure and to reproduce the actual loading condition, a layer of four node rectangular plane elements is introduced at the top of the finite element model (Figure 16). It is assumed that these elements are constituted by an isotropic linear elastic material, characterized by the following mechanical parameters:  $E_0 = 23.2 \cdot 10^2 \text{ kgf/mm}^2$ ;  $\nu_0 = 0.2$ .

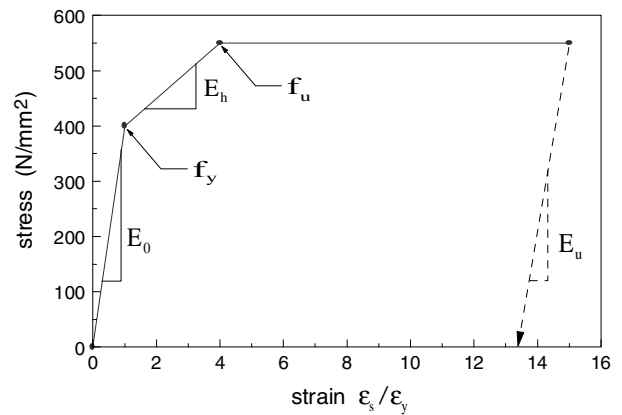




**Figure 16. Finite element discretization.**



**Figure 17. Four node rectangular plane element.**

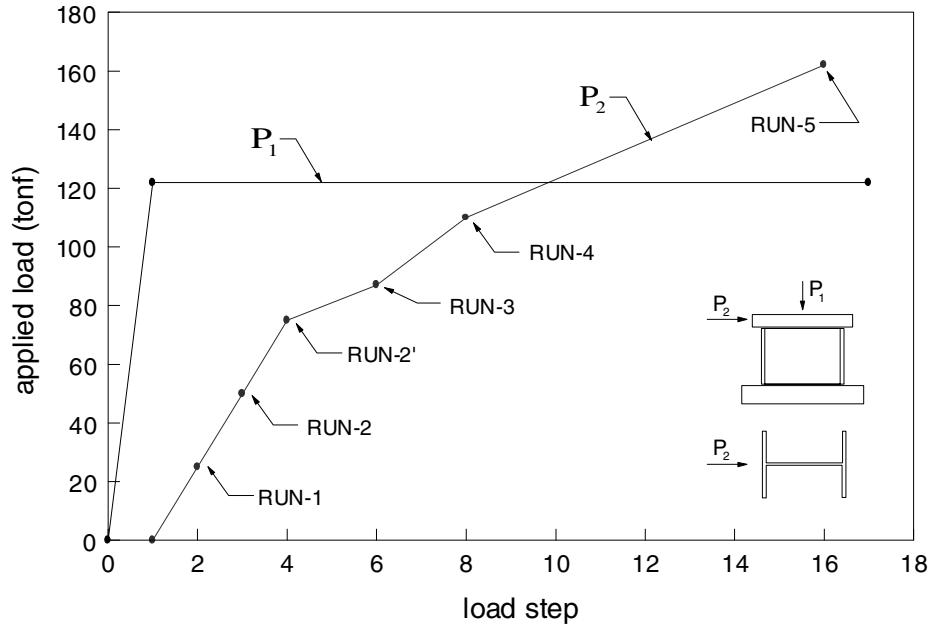


**Figure 18 Constitutive model for steel.**

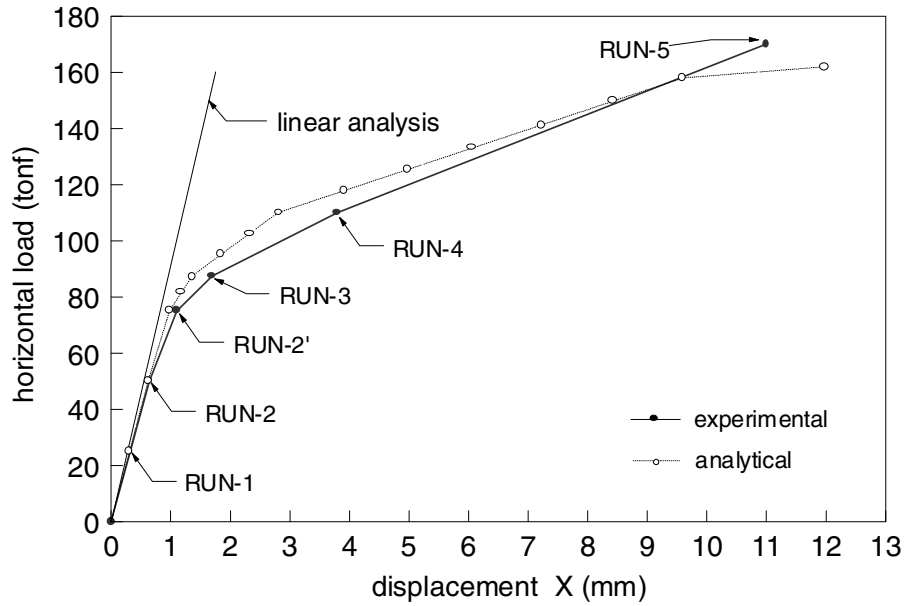
### Correlation between analytical results and experimental data

The loading history utilized to test the considered specimen is shown in Figure 19. Initially only a vertical load  $P_1 = 122 \text{ tonf}$  (total weight of the top slab and of the additional loading) is applied. In the subsequent steps, a horizontal monotonic load in the X direction up to  $P_2 = 158 \text{ tonf}$  is applied.

The comparison between the predicted load-displacement response and the experimental data is reported in Figure 20. A good agreement between analytical and experimental results can be observed.



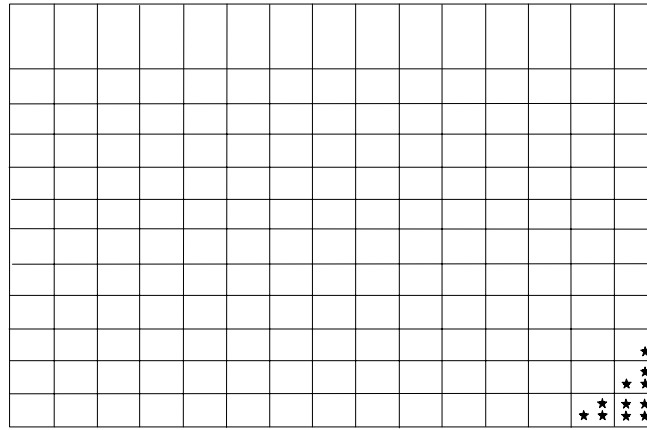
**Figure 19. Loading history.**



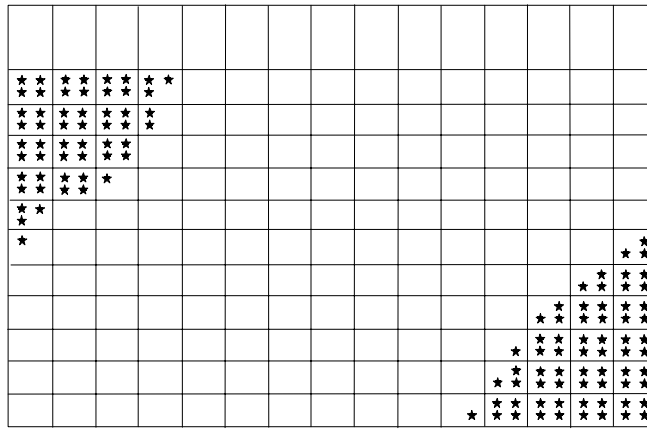
**Figure 20. Load-horizontal displacement response.**

The distribution of the crushing and the cracking zones in the web panel for  $P_2 = 50 \text{ tonf}$ ,  $P_2 = 75 \text{ tonf}$  and  $P_2 = 158 \text{ tonf}$  is reported in Figure 21. Figure 22 reports, instead, the distribution of the crushing and the cracking zones in the lateral panels for  $P_2 = 110 \text{ tonf}$  and  $P_2 = 158 \text{ tonf}$ .

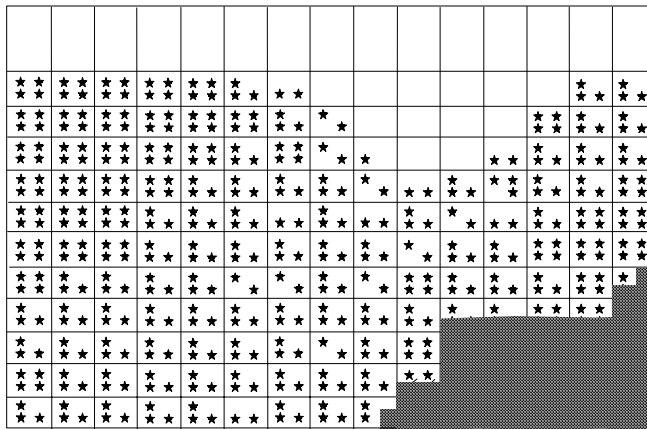
The results shown in Figures 21 and 22 are in good agreement with the crack patterns observed during the experiment.



(a)



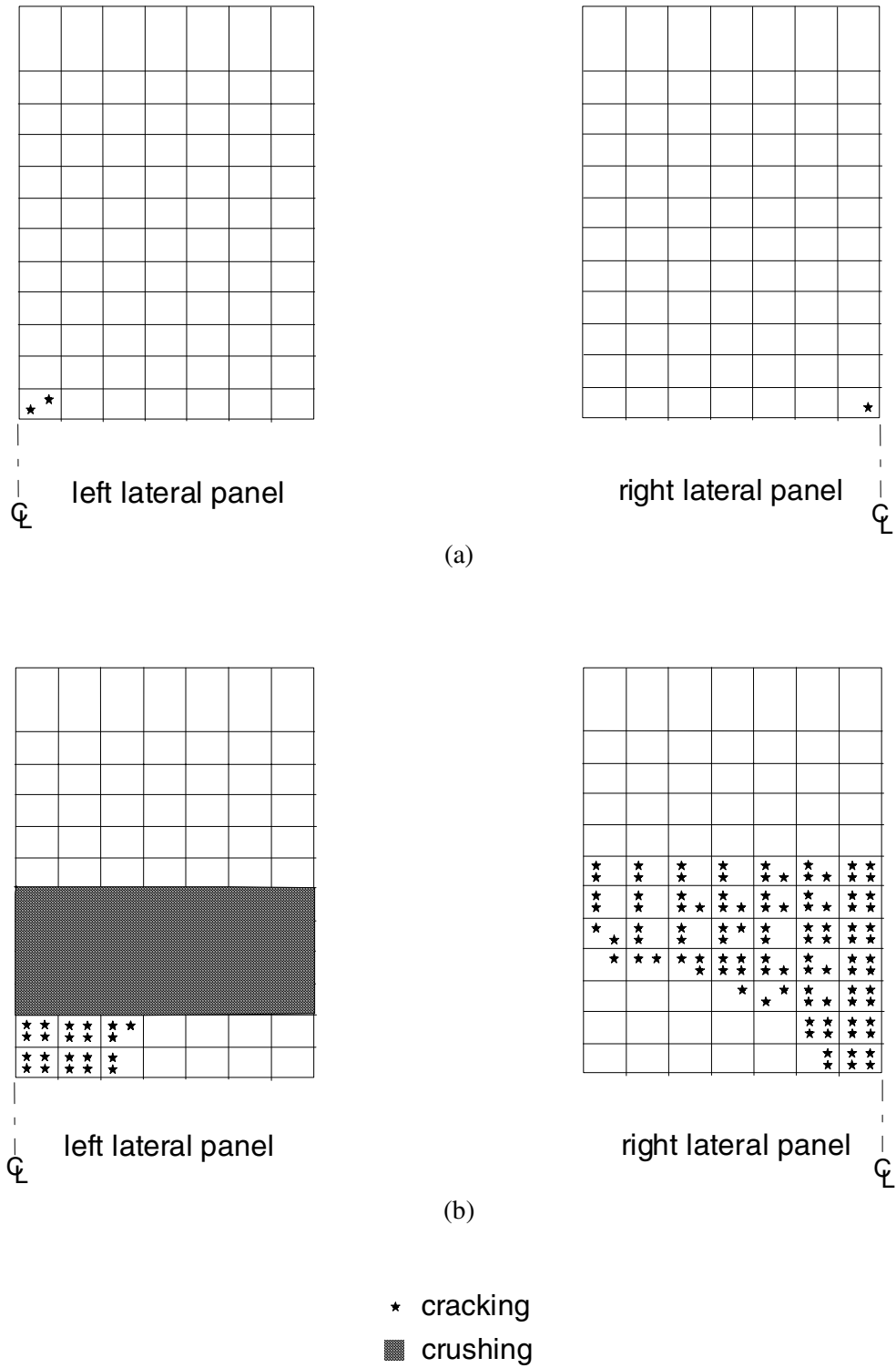
(b)



★ cracking      ■ crushing

(c)

Figure 21. Distribution of the crushing and the cracking zones in the web panel for a horizontal load of: (a)  $P_2 = 50 \text{ tonf}$  ; (b)  $P_2 = 75 \text{ tonf}$  ; (c)  $P_2 = 158 \text{ tonf}$  .



**Figure 22. Distribution of the crushing and the cracking zones in the lateral panels for a horizontal load of: (a)  $P_2 = 110 \text{ tonf}$  ; (b)  $P_2 = 158 \text{ tonf}$  .**

## CONCLUSIONS

A three-dimensional small strain constitutive model for concrete based on an orthotropic hypoelastic formulation has been used to simulate the nonlinear behaviour of several concrete and RC structures.

In particular it has been utilized to perform an extensive correlation study on an RC double T-shaped shear wall specimen subjected to a constant vertical load and an horizontal increasing load up to failure. The analyses have been performed both at global and local level.

The good correspondence between analytical and experimental results obtained in the performed correlation studies indicates that the adopted constitutive model is able to accurately describe the nonlinear behaviour of RC structures.

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