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CYCLIC RESPONSE OF LATERALLY LOADED TIMBER FASTENERS ACCOUNTING FOR SHAFT FRICTION

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SUMMARY

This paper is based on the extension of the program HYST, developed by Foschi for the computation of the cyclic nonlinear response of timber fasteners under lateral loading, to take into account shaft frictional forces. The wood medium is modeled using the Foschi embedment model and shaft friction is simulated with an elastic Coulomb-type friction model. The initial confining pressure surrounding driven fasteners is taken into account by a lateral shift of the wood embedment curve. The model is used to compute the cyclic response of a driven fastener and also that of a bolt connecting three wood layers when subjected to a reversed cyclic displacement history. Results from these two cases show that the value of the initial confining pressure and the coefficient of friction have only a slight effect on the computed hysteretic response. However, for the driven fastener, both quantities were found to significantly affect the computed amount of fastener withdrawal. The model is thus suitable to be used as a basic component in the modeling of shear walls with fasteners that will experience moderate withdrawal under lateral cyclic or earthquake loading.

INTRODUCTION

As a result of the shift in philosophy of earthquake-resistant design from a force-based approach to a performance-based approach, a number of researchers have been involved in the development of models for the prediction of both the component and global nonlinear response of structures. Wood is a brittle material and under earthquake loading, the response of timber structures is directly governed by the response of the fasteners, with smaller diameter fasteners more capable of responding in a ductile manner. Under cyclic or dynamic loading, wood structures undergo complex deformation mechanisms including hole size growth due to local crushing of wood resulting in pinched hysteresis loops, and degradation of stiffness and strength due to the cycling loading action.

To gain a better understanding of the response of these structures to lateral cyclic or dynamic loading, various analytical, experimental, and numerical studies have been performed. In the development of analysis programs for these structures, mechanics-based approaches, which involve modeling the wood-fastener system as a beam-on-a-linear or nonlinear foundation, have gained popularity, since they are

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computationally less intensive, and can be easily adapted and included in wood shear wall or frame analysis programs. The program HYST (Foschi [1]) is one such program, and has been used in the development of various timber shear wall and frame analyses under earthquake and wind conditions (Foschi et al. [2], He et al. [3]). The input parameters needed for HYST are basic material properties including the wood load-embedment response and the fastener nonlinear stress-strain response.

Chui et al. [4] have also developed a similar model to that of Foschi. The main differences in both formulations, however, are that in Chui et al.'s model, a set of loading and reloading rules characterized by parameters that must be defined from cyclic wood load-embedment are used to describe unload/reload paths; while in Foschi's model, unload/reload occur along a line parallel to the initial stiffness, with the formation of pure gaps (slack zone characterized by a zero force and stiffness condition). Secondly, Chui et al.'s model accounts for fastener friction-slip response, and is developed in the context of a total Lagrangian formulation to account for large fastener displacements. Even though, earlier fastener models developed for monotonic loading accounted for friction (Erki [5]; Smith [6]), apart from that by Chui et al., none of the cyclic models directly account for it. In the monotonic models, friction was characterized only by the coefficient of friction, and the frictional resistance was computed as the product of the foundation force and the coefficient of friction.

To account for the initial confining pressure around driven fasteners, different load-embedment curves have been used in the literature for modeling the monotonic response of inserted and driven fasteners (Foschi & Bonac [7]; Erki [5]). Smith [6] also used existing embedment curves and modeled the confining pressures as a foundation residual stress. None of the existing cyclic connector models directly account for the initial confining pressure, and noting that it is directly linked to the friction-slip response, in developing a connector model for lateral cyclic loading that accounts friction, there is a need to account for it.

Based on the above, the objective of this study is to extend the model by Foschi [1] to account for the effect of the initial confining pressure and side frictional forces, and to study the effect of these two parameters on nail and bolt fasteners under a representative imposed cyclic loading action. To do this, the governing equations are reformulated to account for these two factors, and an algorithm is proposed that adapts the approach used by Foschi to account for both of these effects. A parametric study is then conducted to investigate the effect of both parameters on the cyclic response of driven and inserted timber fasteners.

COMPONENT MODELLING

Figure 1 shows a schematic of a generic beam-foundation system under both axial and lateral loading. This loading is resisted by the extension/compression in the beam element, side frictional forces and lateral foundation pressure. A bilinear elastic-perfectly-plastic model is used to model the fastener. More advanced models that account for strain-hardening could be used; however, since we are mainly interested in assessing the effect of the initial confining pressure and friction, this is thought to be suitable.

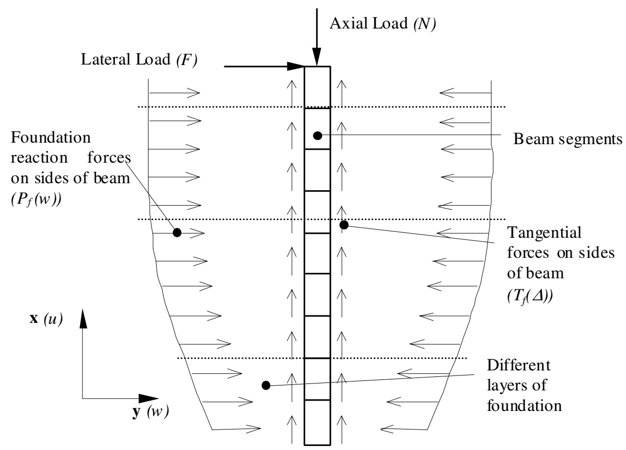

Figure 1: Schematic of wood-fastener system

Modeling of Embedment Response

The load-embedment curve used to model the medium is the 6-parameter curve by Foschi [1] shown in Figure 2 and described by the Eq. (1). Typical values of the parameters for different types of wood under various conditions can be found in Foschi et al. [8].

$$P_e = \begin{cases} (Q_0 + Q_1 w)\left(1.0 - e^{-Kw/Q_0}\right) & if\ w \le D_{\max} \\ P_{\max} e^{-Q_4 (w - D_{\max})^2} & if\ w > D_{\max} \end{cases} \tag{1}$$

in which $P_{\max} = (Q_0 + Q_1 D_{\max})\left(1.0 + e^{-KD_{\max}/Q_0}\right)$ and $Q_4 = \ln(Q_2)/[D_{\max}(Q_3 - 1.0)]^2$.

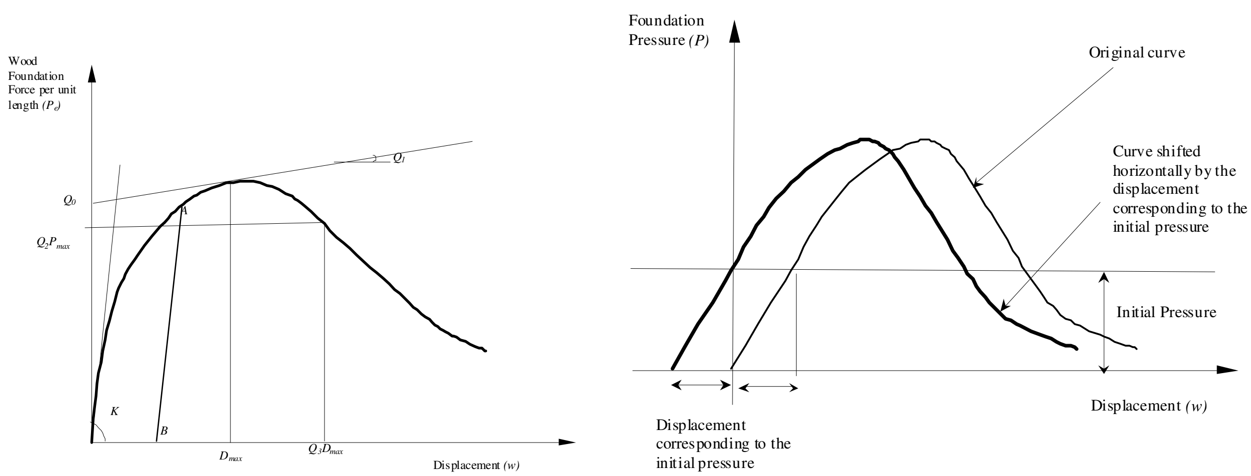

Figure 2: Schematic of Foschi's wood load-embedment curve and embedment curve horizontal shift

The embedment action is only compressive in nature, and results in the formation of gaps between the beam and the medium, when the load is removed. When an initial confining pressure exists, this must first be overcome before the formation of gaps is possible.

Since the main difference between driven and inserted fastener is the initial confining pressure, it is desirable to have a general modeling technique that can be used for both cases. It is proposed here to model the system with a single embedment curve, but with an initial pressure at zero displacement, i.e., the initial volume of wood displaced is modeled as a pre-strain. With this approach, the initial confining pressure has the effect of shifting the load-embedment curve in the horizontal direction as shown in Figure 2. The foundation pressure at a given point along the fastener can then be represented by the sum of the independent load-embedment responses on the right and left sides of the fastener. Similar to Foschi's approach, loading on each side proceeds along the embedment curve and unloading also occurs along a line parallel to the initial stiffness. Reloading of the medium to a previously experienced pressure also occurs along a line parallel to the initial stiffness, and pure gaps form on a given side when the fastener separates from the wood medium. With this approach, problems with or without initial confining pressure can be solved using the same load-embedment curve.

Modeling Tangential Response

Withdrawal of driven fasteners under lateral loading has been observed by a number of researchers during the lateral cyclic testing of shear walls, and is governed mainly by frictional effects. Friction is generally modeled using plasticity-type formulations. Classical Coulomb friction herein referred to as rigid Coulomb friction is not able to effectively model the deformations of the junctions between two deformable bodies in contact. Withdrawal experiments have shown that friction models with a finite stiffness better represent the withdrawal response. As such, in this study, an elastic Coulomb friction model, characterized by both the coefficient of friction (μ), and an effective elastic tangential stiffness (K_f) is used to model the effect of friction. Figure 3 shows a schematic of the model, which is described by Eq. (2). From Eq (2), it can be noted that when the normal force is zero, the corresponding tangential force is also zero.

$$T_f(\Delta) = \begin{cases} 0 & \text{if } P = 0 \\ T_{f_o} + K_f(\Delta - \Delta_o) & \text{if } -\mu P \le T_f \le \mu P \\ \mu P & \text{if } T_f > \mu P \\ -\mu P & \text{if } T_f < -\mu P \end{cases} \tag{2}$$

in which T_{f_o} and Δ_o are the tangential force and slip at a prior state.

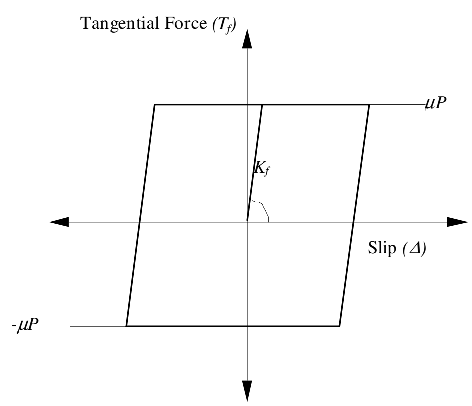

Figure 3: Schematic of tangential load – slip curve

GOVERNING EQUATIONS AND FINITE ELEMENT IMPLEMENTATION

Figure 4 shows a schematic of an elemental portion of the fastener with the various imposed forces. Similar to the model by Foschi [1], in deriving the governing equations, small displacement theory is assumed and the frame of reference is not updated.

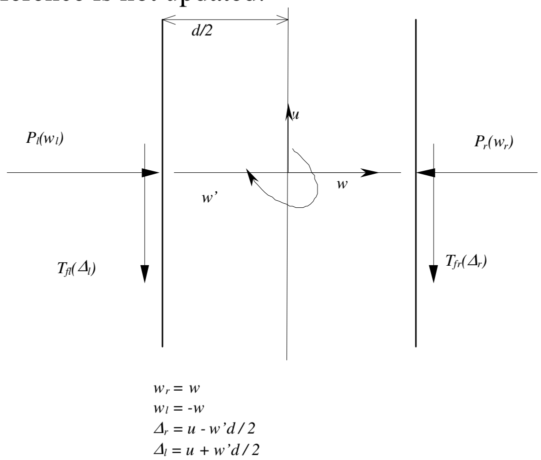

Figure 4: Free-body diagram of an elemental fastener length

Equilibrium Equations

The fastener is assumed to follow the Euler-Bernoulli hypothesis of cross-sections perpendicular to the centroidal axis, remaining plane and perpendicular to cross-section after bending. The only strain of interest is thus the longitudinal strain, and based on Von Karman's strain-displacement relationship, this is given by:

$$\varepsilon = \frac{\partial u}{\partial x} - y\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{3}$$

in which u and w are the axial displacement and lateral deflection of the fastener. From Eq. (3) moderate strains and P-delta (P-δ) effects are accounted for. From the principle of virtual work for a kinematically admissible variation in the displacement $(\delta\mathbf{u})$, the internal and external virtual work are given by:

$$\delta W_{\text{int}} = \int_v \sigma(\varepsilon)\delta\varepsilon\, dv \tag{4}$$

$$\delta W_{ext} = -\int_0^L \left(P_r(w_r)\delta w_r + P_l(w_l)\delta w_l\right)dx - \int_0^L \left(T_{fr}(\Delta_r)\delta\Delta_r + T_{fl}(\Delta_l)\delta\Delta_l\right)dx - N\delta u_{x=L} + F\delta w_{x=L} \tag{5}$$

where v and L are the volume and length of the fastener, respectively. Substituting for w_r, w_l, Δ_r, Δ_l and moving the normal and tangential terms to the internal virtual work side of the equation gives:

$$\int_v \sigma\delta\varepsilon\, dv + \int_0^L \left(P_r\delta w - P_l\delta w\right)dx + \int_0^L \left(T_{fr}\delta\left(u - {}^{d}\!/_{2}\, w'\right) + T_{fl}\delta\left(u + {}^{d}\!/_{2}\, w'\right)\right)dx = -N\delta u_{x=L} + F\delta w_{x=L} \tag{6}$$

From Eq. (6), the tangential forces on left and right sides of the fastener (T_{fr}, T_{fl}) element depend on the normal forces on either side (P_r, P_l). This accounts for coupling between the normal and tangential responses.

Finite Element Implementation and Solution Procedure

Similar to the approach by Foschi [1], the fastener displacements $u(x)$ and $w(x)$ are represented, respectively, by cubic and fifth-order polynomials. The vector of degrees of freedom for a fastener element with nodes i and j is thus given by:

$$\mathbf{a}^T = \left(w_i, w_i', w_i'', u_i, u_i', w_j, w_j', w_j'', u_j, u_j'\right) \tag{7}$$

The finite element approximations to the displacements and their corresponding derivatives are:

$$u(x) = \mathbf{N}^T \mathbf{a}\,,\ u'(x) = \mathbf{N}'^T \mathbf{a}\,, w(x) = \mathbf{M}^T \mathbf{a}\,,\ w'(x) = \mathbf{M}'^T \mathbf{a}\,,\ w''(x) = \mathbf{M}''^T \mathbf{a} \tag{8}$$

where **N** and **M**, represent the vector of shape functions.

Following standard finite element techniques, the Newton-Raphson (NR) procedure given in Eq. (9) is used to solve for the global solution vector.

$$\mathbf{a} = \mathbf{a}_o + \mathbf{K}_t^{-1}(-\mathbf{\Psi}) \tag{9}$$

$\mathbf{a}_o$ is a previous displacement vector, and $\mathbf{\Psi}$ and $\mathbf{K}_t = \partial\mathbf{\Psi}/\partial\mathbf{a}$ are the out-of-balance force vector and the consistent tangent stiffness matrix, respectively. The solution procedure is augmented with a line search technique to enhance the NR convergence procedure. Both displacement and force convergence criteria are used to check for convergence. From the NR implementation, the expression for the out-of-balance force vector is given by:

$$\mathbf{\Psi} = \mathbf{\Psi}_l + \mathbf{\Psi}_\sigma + \mathbf{\Psi}_e + \mathbf{\Psi}_f + N\mathbf{N}_{x=L} - F\mathbf{M}_{x=L} \tag{10}$$

where, $\mathbf{\Psi}_l$, $\mathbf{\Psi}_\sigma$, $\mathbf{\Psi}_e$, and $\mathbf{\Psi}_f$ represent the linear, geometric, normal and tangential parts of the force vector. Similarly, the expression for the consistent tangent stiffness matrix is:

$$\mathbf{K}_t = \mathbf{K}_l + \mathbf{K}_\sigma + \mathbf{K}_e + \mathbf{K}_f \tag{11}$$

where, again $\mathbf{K}_l$, $\mathbf{K}_\sigma$, $\mathbf{K}_e$ and $\mathbf{K}_f$, represent the linear, geometric, normal and tangential parts of the stiffness matrix.

PARAMETRIC STUDY

Based on the above formulation, another version of HYST (Foschi [1]) has been developed for the cyclic response of laterally loaded timber connections. To study the effect of friction and initial confining pressure, two example problems previously studied by Foschi [1] are used. These are a single driven fastener nail in wood, and a single bolt connecting three wood layers subjected to a varying reversed cyclic input loading history. These are shown in Figure 5. The new program was verified by comparing the results for the case of no friction and initial confining pressure with a fixed base, with those obtained by Foschi. These were found to exactly match Foschi's results.

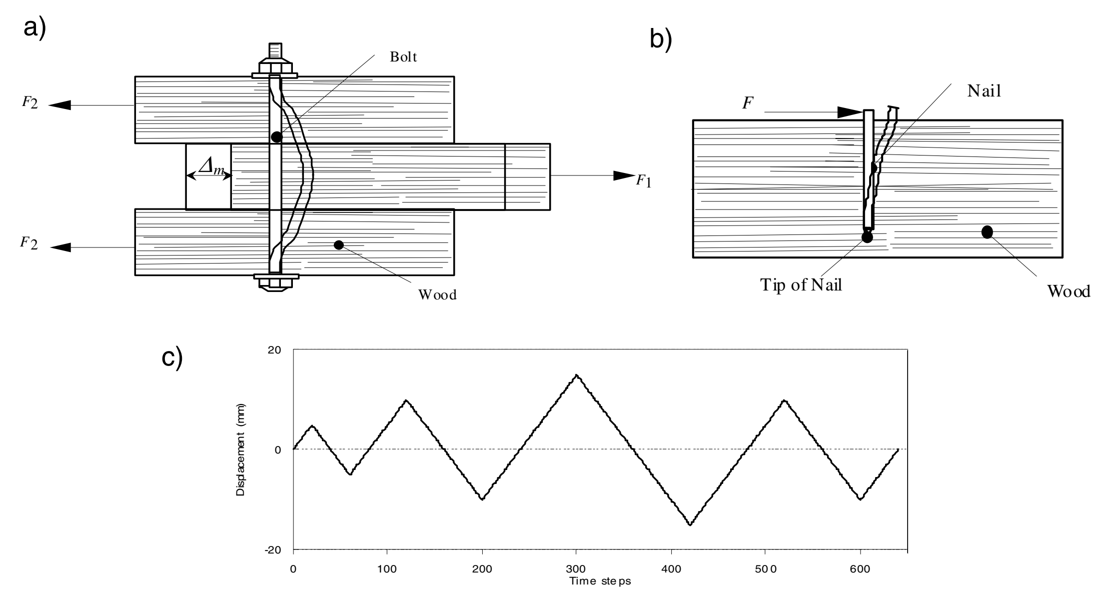

Figure 5: Schematic of a) three wood layer bolted connection; b) nail fastener in wood medium; c) imposed reversed cyclic loading history

From a review of relevant literature, the coefficient of friction was observed to vary over a wide range, and as such, the range of μ chosen for the study was between 0 – 0.8. The effective elastic stiffness was approximately estimated from nail withdrawal tests, assuming the fastener to be axially inextensible. K_f was found to range between 1000 – 30,000 kN/m^2, and a range of K_f 10^3 - 10^6 kN/m^2, (i.e., a three order of magnitude difference) was used. Very large values of K_f were selected to approximately model a rigid Coulomb friction model. The same assumptions used in estimating K_f, were also used in estimating the initial confining pressure. A maximum pressure of 130 kN/m was estimated based on an assumption of a minimum coefficient of friction of 0.1. The range of initial confining pressure used in the study was thus from 0 - 250 kN/m.

Single Bolt Connection

In this problem, a hole of the same size as the bolt is pre-drilled in the wood, as a result, the bolt does not experience any initial confining pressure. The only variables of interest are therefore μ, and K_f. The bolt is clamped at both ends and a lateral displacement (Δ_m) is applied to the middle layer, causing the bolt to deform in double shear as shown in Figure 5. The diameter of the bolt is 9.52 mm and the thickness of each piece of wood is 50 mm. The Young's modulus and yield stress of the bolt are 200 GPa and 250 MPa, respectively. The 6 parameters of the Foschi's load-embedment curve for this wood specimen with a specific gravity of 0.67 are: Q_0 = 500 kN/m; Q_1 = 1.50 MN/m^2; $Q2$ = 0.5; Q_3 = 1.5; D_{max} = 7.5 mm and K = 400 MN/m^2. Due to symmetry, only half of the bolt was discretized using a total of 6 elements; 2 in the central layer and 4 in the outside layer. Example results of the simulated hysteretic response of the three-wood-layer bolted connection for different combinations of μ, and K_f are shown in Figure 6.

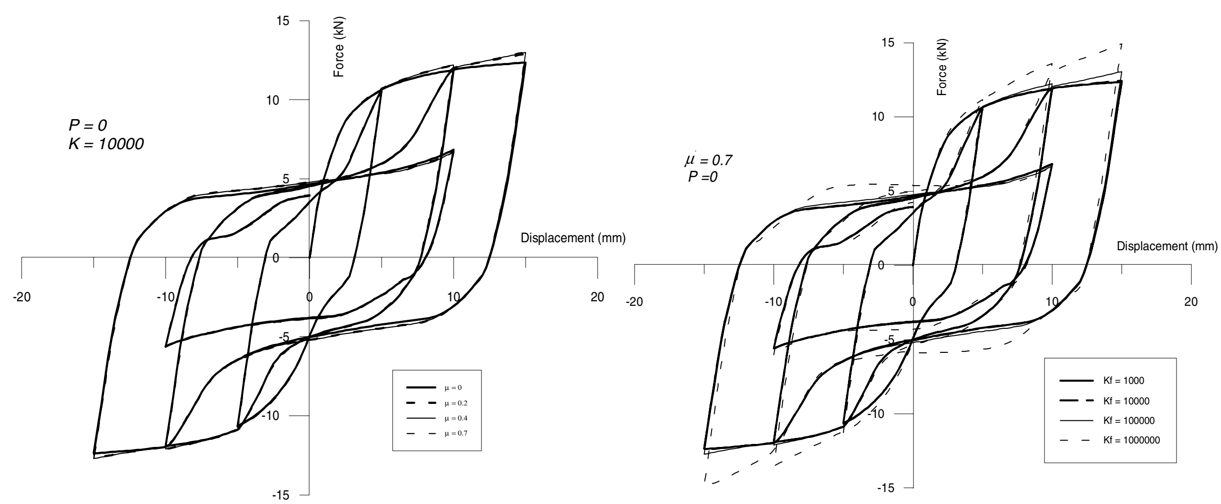

Figure 6: Effect of coefficient of friction and effective elastic tangential stiffness on cyclic hysteretic response of bolted connection

Both figures show that the basic shapes of the hysteresis loops do not change with a variation in any of the variables. It can be seen that for lower values of K_f, which represent the expected stiffness range, friction does not significantly affect the computed response. However, for higher values of K_f, an increase in the ultimate load can be observed. This is more significant for higher values of the coefficient of friction. Similar observations were made in the single nail example and will be discussed in detail in that section.

Single Nail Connection

In this problem the nail is driven into the wood and experiences an initial confining pressure, which mobilizes tangential frictional forces that keep the nail in place. Under lateral loading, the tip of nail (bottom of the nail) is able to displace vertically, i.e., withdraw from the wood. Since the model is based on small displacement theory, only relatively moderate axial displacements can be accurately predicted. The variables of interest in this case are: μ, K_f and P_i. The length of the nail is 63.5 mm with a diameter of 9.52 mm. The specifications for all other variables are the same as in the bolted connection example. The nail was discretized into 5 elements. Example cyclic hysteretic responses and corresponding tip displacements are shown is shown in Figures 7.

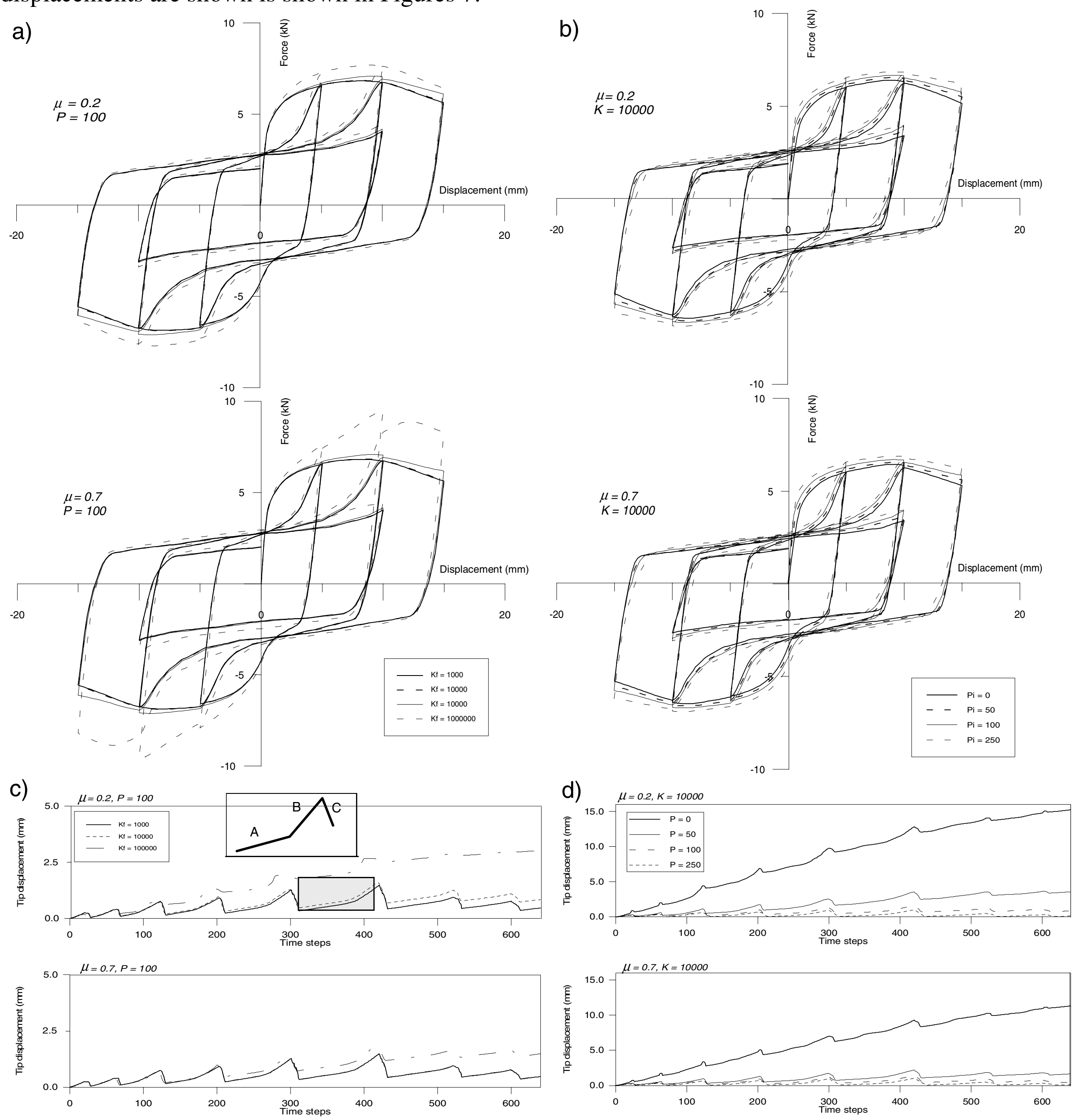

Figure 7: Effect of initial confining pressure and effective elastic tangential stiffness on the cyclic hysteretic response and corresponding tip displacement history of single nail in wood medium

From Figure 7b, for $K_f = 10^4$ kN/m^2 the initial confining pressure is seen not to significantly influence the hysteretic response of the system. The initial lateral stiffness of the system, is seen to increase with increasing initial confining pressure, on the other hand, only slight differences in the ultimate load are observed. The estimated increase in energy dissipated by the system (area enclosed by curves), when compared to the case of no initial confining pressure is less than 5%. Noting that most of the estimated confining pressures are below 100 kN/m, it can be inferred that the initial confining pressure does not significantly affect the hysteretic response. In contrast to this, from Figure 7a, increasing K_f seems to result in an increase in the corresponding ultimate load, similar to what was observed in the bolt example. For large values of K_f, as will be explained below, slippage occurs more than straining; as such, friction is activated more, and depending on the coefficient of friction, can result in a considerable increase in the ultimate load. Smaller values of K_f , which fall in the stiffness range estimated from the literature do not show this significant increase. It can thus be said that K_f therefore does not have a significant effect on the response; however, this result shows that the use of a rigid Coulomb model could lead to conflicting results.

Figure 7c and 7d also show that for all the cases studied a cumulative withdrawal of the nail under the imposed loading history occurs, which comprises of three main portions. From a comparison between the input displacement history, hysteresis curves and the tip displacement history, it can be seen that the localized decrease in tip displacement (section C) corresponds to the points where unloading of the fastener occurs. The tip displacement is a summation of elastic and inelastic fastener strain displacement, and that due to slip between the fastener and wood medium. Of the three, only the elastic strain displacements are recoverable, and from the observation made above, it can be rightfully said that the localized decreases in displacement are the elastic parts of the tip displacement history. It can also be observed that with increase in K_f this phenomenon decreases and eventually disappears. This can be explained by noting the fact that as K_f increases, slippage occurs at lower displacements, and under a displacement-controlled loading scheme, would result in increased slippage as opposed to increased straining of the fastener. In such cases therefore, the coefficient of friction has a significant effect on the response as seen from the hysteretic response. Oden and Pires [9] also noted a similar occurrence in their study of the influence of a wide range of values of K_f on the elastic contact stress distribution between a deformable half-space and a rigid foundation.

Section B is linked to the rising portions of the hysteresis curves and represents the stage where the fastener begins to bear again on wood medium and straining of the fastener increases. Most of the inelastic strain displacement of the fastener occurs at this stage. Section A also refers to the stage where the fastener is moving in the slack region, where most of the slippage of the fastener occurs. This phenomenon is clearly noticed from sample plots of the shape of the nail and corresponding hole existing at given displacement steps (see Allotey [10]).

For P_i = 0.0 kN/m maximum displacements are about 20% of the length of the nail, and are probably not accurately predicted, due to the model not being developed for large displacements. It can be seen from the figure that an increase in the confining pressure leads to a large reduction in the maximum tip displacement. This can be attributed to the confining pressure restraining slippage and causing more straining of the fastener. Under such conditions, the ratio of the contribution of inelastic strain displacement to the tip permanent displacement increases. The figure also shows that increase in the coefficient of friction results in decreased tip displacements, which is common-knowledge.

CONCLUSIONS

Foschi's connector model has been extended in this paper to account for friction and an initial confining pressure (P_i). A procedure based on a horizontal shift of the embedment curve was used to account for the initial confining pressure around the fastener. Friction was also modeled using an elastic Coulomb friction model, characterized by an effective elastic tangential stiffness (K_f) and the coefficient of friction (μ).

From a parametric study based on the response of a single bolt connecting three wooden pieces, and a nail joint, subjected to reversed cyclic lateral loading, the following results were obtained.

i. The initial confining pressure around driven timber fasteners does not have a significant effect on their hysteretic cyclic response under laterally loaded fasteners, but plays a very important role in reducing the amount of nail withdrawal.

ii. For estimated practical ranges of the coefficient of friction and effective elastic tangential stiffness, friction does not significantly influence the hysteretic response, but considerably affects the amount of nail withdrawal under lateral cyclic loading.

iii. For large value of K_f, which can be thought to approach a rigid Coulomb friction model, the cyclic response of the system becomes more sensitive to the coefficient of friction; implying that the choice of the friction model is important in such analysis.

iv. The tip displacement history can be divided into three main sections comprising a section where the fastener is in the slack zone and experiences predominantly slippage displacement, a portion where the fastener begins to strain and where any inelastic strain displacement occurs, and a portion related to fastener unloading, where the elastic strain displacement is recovered. The combination of these three actions results in a cumulative withdrawal action superimposed with a periodic displacement decrement.

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