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THE EFFECT OF THE LENGTH AND LOCATION OF YIELD ZONES ON THE ACCURACY OF THE SPREAD PLASTICITY MODELS

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SUMMARY

The Newton Raphson iteration procedure that uses these moment curvature hysteretic relationship to solve the displacement equation is unable to recognise a change in yield zone length after yielding because it does not result into a change of slope. Also it is unable to recognise yielding that starts within the span. Thus it is important to determine the location and length of the yield zone for realistic application of the spread plasticity model.

A method to determine, more accurately, the length, and location of yield zones is presented together with an Example showing the effect of determining more accurately the lengths and locations of yield zones on the structural deformations. The example among other points demonstrates that by recognising the location of yield zones in the span before they extend to the support, a 77% improvement in the accuracy of the moment rotation curve for the joint under sagging moment is achieved. Also a 25% improvement in the load deflection curve is achieved.

INTRODUCTION

Hysteretic relationship, show the variation of a load and a given deformation during cyclic loading for a given member, joint or structure. Typical load deformation relationships include; the force–displacement, moment-rotation, and moment–curvature and shear force versus shear deformation relationships.

Hysteretic loops drawn from experimental data provide information on mechanical properties, load deformation paths for analysis and correct detailing of members and connections for resisting cyclic loading. Also the area enclosed by loops measures the energy dissipation capacity of a structure, which plays a dominant role in reducing its vibration. Popov [1].

Based on the observed hysteretic response of members to cyclic loading during experiments, rules have been formulated to model the dynamic response. However most of these experiments are for the simplified beam column frames without slabs and those with slabs lack gravity loads of the magnitude expected in normal building structures. Whereas it is recognised that one of the limitation of the discrete time history analysis models is the use of a set of predefined phenomenological rules or hysteretic relationships to confine and define the complex behaviour of a member during a complex loading such as that imparted by earthquake. Saadeghraziri, [2]. It is equally important to realise that the hysteretic rules based on experiments where the gravity load is not considered might not accurately represent real practical situations.

The concentrated plasticity models which concentrate non-linear behaviour in springs at member ends use moment rotation hysteretic relationships. The existing spread plasticity models, which assume that yielding starts at beam-ends and when unloading the last yielded part is at beam-ends use moment curvature hysteretic relationships for the sections at the beam column interfaces. Thus a joint or a section controls or represents the hysteretic behaviour of a member.

To explain the limitation of the moment curvature hysteretic relationship, consider the Newton Raphson iteration procedure illustrated in Figure 1. Since this relationship is defined by straight lines. The iteration is very simple. In the figure;

M_{Rn}, ϕ_{Rn} = Moment and curvature at the beginning of the current time step.

$M_{R(n+1)}, \phi_{R(n+1)}$ = Moment and curvature at the end of the current time step.

M_{cr}, ϕ_{cr} = Cracking moment and curvature.

M_y, ϕ_y = Yielding Moment and curvature

M and ϕ refer to the moment and curvature respectively.

K_{oa} is the slope of curve between the origin and the cracking point given by line OA.

K_{ab} is the slope of curve after cracking but before yielding given by line AB.

K_{bc} is the slope curve after yielding given by line BC.

ΔM_o is the initial moments.

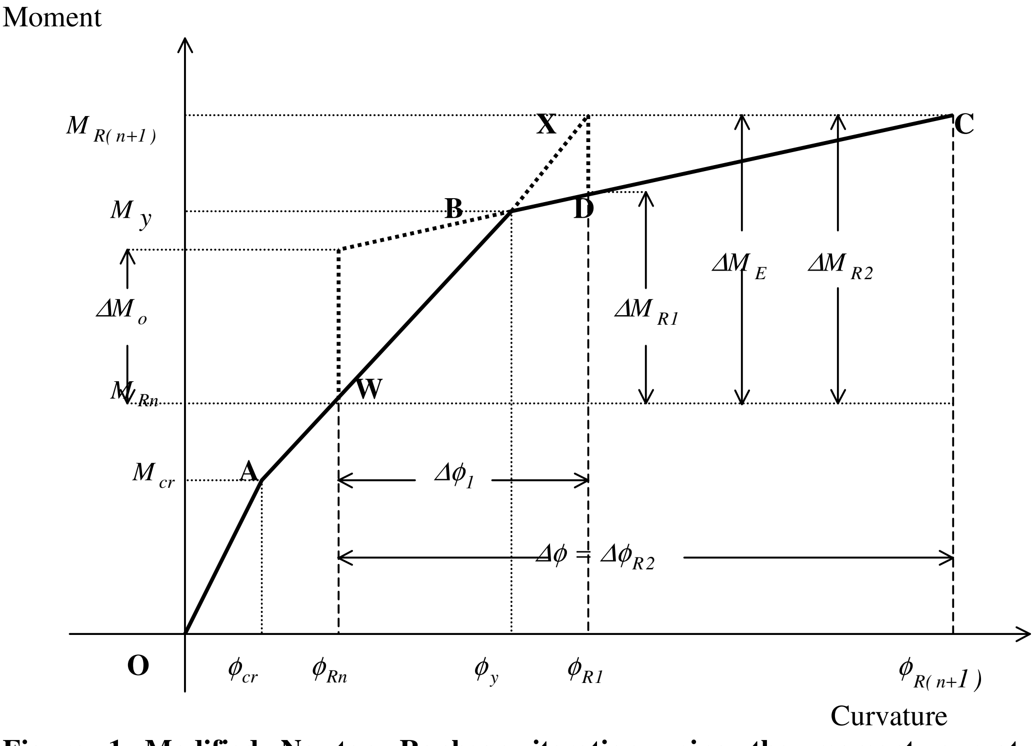

Figure 1 Modified Newton Raphson iteration using the moment curvature hysteretic relationship.

Assuming that at the beginning of the current time step, (point W), the moment and curvature, are less than the yield moment and curvature, but greater than the cracking moment and curvature. If during the time step the external moment increases by an incremental moment ΔM_E, the initial calculation of curvature increment $\Delta\phi_1$ is given by;

$$\Delta\phi_1 = \frac{\Delta M_E}{K_{ab}} \qquad (1)$$

Curvature increments are added to curvatures at the end of the previous step to obtain the current curvatures ϕ_{R1}.

$$\phi_{R1} = \phi_n + \Delta\phi_1 \qquad (2)$$

If the current curvatures are less than the yield curvature; ($\phi_{R1} \le \phi_y$), the current internal moment is given by:

$$M_{R1} = M_{Rn} + K_{ab}\Delta\phi_1 \qquad (3)$$

If the current curvatures are greater than the yield curvatures, $\phi_{R1} > \phi_y$, the current internal moment is given by:

$$M_{R1} = M_y + K_{bc}\left(\phi_{R1} - \phi_y\right) \qquad (4)$$

The moment corresponding to a curvature of ϕ_{R1} is the one at D, $M_{R1} = M_D$. The incremental vector of internal resisting moments ΔM_{R1} is then determined by subtracting from the current internal resisting moments those at the end of the previous time step.

$$\Delta M_{R1} = M_{R1} - M_{Rn} \qquad (5)$$

Comparing the internal moments increments ΔM_{R1} to the external moments increments, ΔM_E, it is determined whether a change of slope took place in the moment curvature hysteretic relationship. If the difference between the external and internal moment increments is less than a given tolerance, then it is assumed that there was no change in stiffness.

$$\Delta M_E - \Delta M_{R1} < Tolerance. \qquad (6)$$

If $\left(\Delta M_E - \Delta M_{R1}\right) > Tolerance.$ then for the second iteration, the incremental curvature is given by;

$$\Delta\phi_2 = \frac{\left(\Delta M_E - \Delta M_o\right)}{K_{bc}} \qquad (7)$$

As long as the value of initial moment ΔM_o is correct, the current incremental curvature $\Delta\phi_2$ that is calculated from equation (7) is equal to the true value of incremental curvature $\Delta\phi$. The current curvatures ϕ_{R2} are given by adding the current incremental curvature to the converged value of curvature ϕ_{Rn} from the previous time step.

$$\phi_{R2} = \phi_{Rn} + \Delta\phi_2 \qquad (8)$$

The current internal moment M_{R2} is determined from the hysteretic moment curvature relationship. It is given by;

$$M_{R2} = M_y + K_{bc}\left(\phi_{R2} - \phi_y\right) \qquad (9)$$

The incremental internal moment is given by:

$$\Delta M_{R2} = M_{R2} - M_{Rn} \qquad (10)$$

$\Delta M_{R2} = \Delta M_E$, as illustrated in Figure 1 and the iteration is complete, M_{R2} becomes $M_{R(n+1)}$.

It is therefore clear that an increase in yield zone length after yielding can not be recognised by the moment curvature relationship because there is no change of slope of the hysteretic curve. On the other hand, any change in the slope of moment curvature hysteretic relationship from elastic to yielding or yielding to elastic will be accompanied by a change in the yield zone length.

Also yielding in the span will not be recognised by the moment curvature hysteretic relationship for the sections at the beam column interface before it extends to the beam-ends.
Thus provided the yield zone length is calculated accurately, it is a better indicator of changes in the spread plasticity model than the hysteretic moment curvature relationships.

From the fore going, it is necessary to carry out an investigation into whether there has been a change in the plastic zone length or not irrespective of the results obtained from the hysteretic relationship.

That is whether the difference between the incremental external moment ΔM_E, and the corresponding internal moment increment ΔM_R at the beam column interface sections are less or greater than the specified tolerance, the yield zones locations and length still need to be computed and their extension investigated.

Determination of the location and length of yield zones;
The accuracy of the spread plasticity model depends on the accurate determination of the location and length of yield zones. The existing spread plasticity models assumed that yield zones formed only at beam-ends and spread in wards from there. The yield zone length Z_c is given by;

$$Z_c = \frac{M_j - M_{yj}}{V} \tag{11}$$

Where;

M_j, M_{yj} = The applied moment and yield moment at end j.

V = Actual shear force calculated for the end whose yield zone is being calculated, Soleimani [3].
To improve on calculation of the yield zone length, Filippou [4] proposed that if both ends of the beam have yielded, the shear force V is given by:

$$V = \left(\frac{M_i + M_j}{L} \right) \tag{12}$$

Where:

M_i and M_j are total moments at beam column interfaces i and j.

L is the clear span length.

The yield zone lengths are limited to a maximum value Z_{max} given by:

$Z_{max} = 0.25L$ Filippou [4], Soleimani [3].

The assumption that yield zones start spreading from beam ends is only true for hogging moments yield zones and for sagging moments yield zones of beams in lower storeys of the frame where the seismic load is greater than the gravity load. In upper and middle stories, sagging moments yield zones start forming within the span and spread towards either support. Even if yield zones form at beam-ends, for a beam carrying a gravity load w, equation (11) overestimates the actual yield zone length Z_c by a length ΔZ_c. Kyakula [5]

$$\Delta Z_c = \frac{w}{2V_i} (Z_C)^2 \tag{13}$$

The expression in equation (12) assumes that the shear forces at both the beam column interfaces are equal. This is true if only the earthquake load is acting on a structure with elastic rectangular beams having equal top and bottom reinforcement. The shear force for the section under action of hogging moments is the sum of the shear force due to gravity and earthquake load, while that for a section under action of sagging moments is the difference between that due to earthquake and gravity load. Therefore equation (12) overestimates the yield zone length due to hogging moments and under estimates the yield zone length due to sagging moments. Also limiting the maximum yield zone length to $0.25L$ makes it difficult to predict or explore the maximum deformation of the structure before collapse, unless it is assumed that once one of the beams reaches a yield zone length of $0.25L$ the structure is presumed to have collapsed.

A more accurate method for determination of yield zone length and location is proposed. In this method, the record of the distribution of the bending moment due to gravity load is kept by storing only the values of the gravity load acting on the beam, and the shear force and bending moment of one of the beam column interfaces.
Only the total end moments of the member due to the earthquake load at the end of the previous time step needs to be stored. At the end of the current time step, this is updated by adding the incremental

end moments to it. The total current moments due to the earthquake load and the gravity load is each computed separately at any chosen intervals along the length of the beam. For each of these intervals, the moment due to the gravity and total current moment due to the earthquake load are added. Also for each interval, the yield moment is subtracted from the total moment and a change in the sign of the difference indicates that the yield point has been exceeded. This gives an approximate location of the yield point, which is accurate to a fraction of the chosen interval. For each value of approximate yield point obtained, interpolation within the interval is carried out. This resulted in a very stable program demonstrated by the fact that changing the increment from 0.0001m to 0.76m for a 7.6m clear span gave a difference between the two results of 0.1%.

For a yield zone within the span, the first point where the difference between the total moment and the sagging yield moment shows a change in sign, identifies one end of the yield zone, the second point identifies the other end. The difference in length of these points gives the length of the yield zone. For hogging yield zone, the point where the difference between the total moment and the hogging yield moment changes signs also identifies the end of the yield zone. The difference in length between this point and the end of the beam under hogging moments gives the length of the yield zone.
When sagging moments at the beam end are equal to or greater than the yield moment, the length from the beam column interface to the point where the total sagging moment is equal to the yield sagging moment then gives the length of the yield zone.

The expressions for the bending moments M_a and M_g due to the earthquake and gravity load at length y along the beam are given by equation (14) and (15) respectively.

$$M_a = M_{ai} - \left(M_{ai} - M_{aj}\right)\frac{y}{L} \quad (14)$$

$$M_g = V_{gi}\, y - \frac{w}{2} y^2 - M_{gi} \quad (15)$$

Where;
M_{ai} is the moment due to earthquake load at end i.
M_{aj} is the moment due to earthquake load at end j.
V_{gi} is the shear force due to gravity load at end i.
M_{gi} is the moment due to gravity load at end i.
is the clear span
The total moment M at length y is given by equation (16):

$$M = M_a + M_g \quad (16)$$

Thus starting at one end of the beam column interface and incrementing the points along the beam in any desired steps Δy, the total moment along the beam at any desired interval is computed. Typical length increments considered varied from 0.0001m to 0.76m.

Determination of yield points;
Figure (2) represents the general case of the total bending moment due to gravity and seismic load. The yield point is defined as the point at which the total applied moment is exactly equal to the yield moment. From Figure (2):
y_1 is the length from the left hand side beam column interface to first sagging yield point.
y_2 is the length from the left hand side beam column interface to the second sagging yield point.
y_3 is the length from the left hand side beam column interface to hogging moment yield point.
At each of the chosen interval along the beam, sagging and hogging yield moments M_{yb} and M_{yt}, are subtracted from the total applied moment M to obtain the differential moments ΔM_b and ΔM_t.

$$\Delta M_b = M - M_{yb} \;\&\; \Delta M_t = M - M_{yt} \tag{17}$$

At the sagging yield points, $\Delta M_b = 0.0$, and at the hogging yield point, $\Delta M_t = 0.0$. These points are identified as follows;

When $y = 0.0$; At the left hands side beam column interface,

$$\Delta M_{bo} = \Delta M_b \;\&\; \Delta M_{to} = \Delta M_t \tag{18}$$

At any length y of the beam other than $y = 0.0$, ΔM_{bo} is equal to the value of ΔM_b at length $(y - \Delta y)$ and ΔM_{to} is equal to the value of ΔM_t at length $(y - \Delta y)$

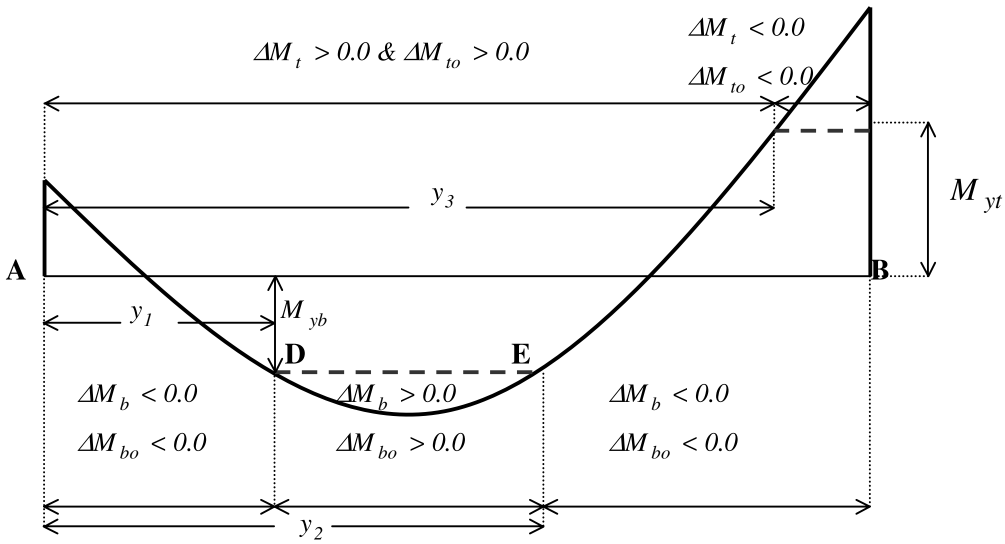

Figure 2 Identification of yield points

Also when $y = 0.0$, the maximum positive moment (sagging moment) M_{max} and minimum moment, (maximum hogging moment) M_{min} are each set equal to the total moment M.

At any point along the beam if the applied moment M is greater (more positive), than the most positive moment so far, the applied moment becomes the most positive moment or largest sagging moment and its location is denoted y_c. Similarly at any point along the beam, if the applied moment M is less (more negative), than the most negative moment so far, the applied moment becomes the most negative moment or largest hogging moment and its location is denoted y_n.

$$M > M_{max}, \Rightarrow M_{max} = M, y_c = y \;\&\; M < M_{min} \Rightarrow M_{min} = M, y_n = y \tag{19}$$

In this way the maximum sagging and hogging moments and their respective length from the left hand side beam column interface y_c and y_n are determined, as the length is incremented along the beam.

Determination of sagging yield points

Determination of the sagging moment yield points is illustrated in Figure 2. If the total applied moment is less than the sagging yield moment; $(M < M_{yb})$, then: $\Delta M_b < 0.0$, and $\Delta M_{bo} < 0.0$. On the other hand if the total applied moment is greater than the sagging yield moment; $(M > M_{yb})$, then; $\Delta M_b > 0.0$, and $\Delta M_{bo} > 0.0$. It is difficult for the length y to exactly coincide with the length to the yield point, but if it does then $\Delta M_b = 0.0$. What normally happens is that at the first yield point when the applied moment M at a length y along the beam changes from being less to greater than the yield moment M_{yb}, the length y is just a fraction of the chosen interval Δy, greater than the length to the first yield point. And at the second yield point, when the applied moment M at a length y along the beam changes from being greater to less than the yield moment M_{yb}, the length y is also just a fraction of the chosen interval Δy, greater than the length to the second yield point.

Therefore before yielding, the product $(\Delta M_b \Delta M_{bo})$ is greater than zero because both ΔM_b and ΔM_{bo} are negative. At the yield point it is equal to zero. At a point where y is just a fraction of the chosen interval greater than the length to the yield point, it is less than zero because ΔM_b is positive and ΔM_{bo} is negative. At other points between the first yield point and the second yield point, $(\Delta M_b \Delta M_{bo})$ is greater than zero because both ΔM_b and ΔM_{bo} are positive. At the second yield point, it is again zero. And at a point where y is just a fraction of the chosen interval greater than the length to the second yield point, $(\Delta M_b \Delta M_{bo})$ is less than zero because ΔM_b is negative and ΔM_{bo} is positive. Between the second yield point and the right hand support, it is positive. Therefore finding the sagging yield point involves tracing the point where the sign of the product $(\Delta M_b \Delta M_{bo})$ changes.

If y_{12} is the length to the sagging moment yield points calculated based on the chosen interval Δy.

If $(\Delta M_b \Delta M_{bo}) \le 0.0 \Rightarrow y_{12} = y$ (20)

It is the only points where $(\Delta M_b \Delta M_{bo}) \le 0.0$. Where either, $\Delta M_b \ge 0.0$, and $\Delta M_{bo} < 0.0$ for the first yield point or $\Delta M_b \le 0.0$, and $\Delta M_{bo} > 0.0$ for the second yield point. Therefore if $(\Delta M_b \Delta M_{bo}) \le 0.0$, the locations of yield points have been identified.

The error caused by the length increments Δy not coinciding with the actual yield point are corrected for by interpolation as given in equation (21) and illustrated in Figure (3):

If $\Delta M_b \ne 0.0 \Rightarrow y_o = y_{12} - \dfrac{\Delta y}{\left(1 - \Delta M_{bo} / \Delta M_b\right)}$ (21)

If the length to the yield point y_o corrected in equation (21) is less than the length y_c to the point of the maximum sagging moment, then it is the first sagging yield point y_1. On the other hand if it is greater than y_c then it is the second sagging yield point y_2.

If $y_o < y_c \Rightarrow y_1 = y_o$ & If $y_o > y_c \Rightarrow y_2 = y_o$ (22)

From Figure 3, it is seen that the interpolation considerably reduces the error between the yield point calculated according to the chosen interval and the actual yield point. And if the chosen interval is small enough, the actual curve approximates a straight line and coincides with the assumed curve.

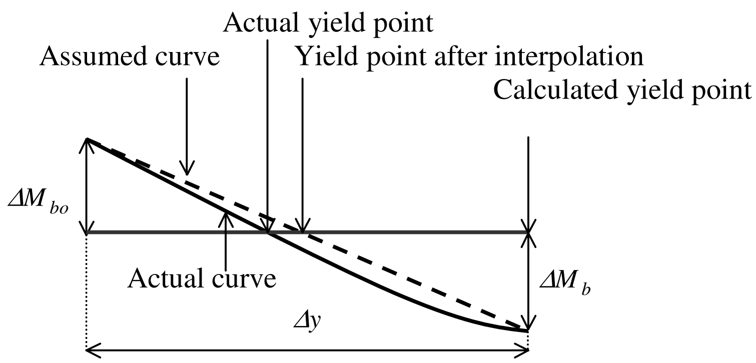

Figure 3: Interpolation within the interval at sagging yield point.

Determination of the hogging moment yield point.

If the total applied moment M is sagging or less than the hogging yield moment, M_{yt} then; $\Delta M_t > 0.0$ and $\Delta M_{to} > 0.0$. If M is exactly equal to the hogging yield moment M_{yt}, then $\Delta M_t = 0.0$. If M is greater than the hogging yield moment, then $\Delta M_t < 0.0$ and $\Delta M_{to} < 0.0$. Therefore for any value of y where the applied moment is not equal to the yield hogging moment, the product $(\Delta M_t . \Delta M_{to})$ is positive. The product is zero at the yield point, and negative if the length y is just a fraction of the chosen interval Δy, greater than the actual location of negative yield point such that ΔM_{to} is positive and ΔM_t is negative. Denoting y_3 as the length to the hogging moment yield point calculated based on the chosen interval Δy;

If $(\Delta M_t . \Delta M_{to}) \leq 0.0 \Rightarrow y_3 = y$ (23)

The location of the yield point y_o is obtained by interpolation as given in equation (24).

$$\Delta M_t \neq 0.0 \Rightarrow y_o = y_3 - \frac{\Delta y}{\left(1 - \Delta M_{to} / \Delta M_t\right)} \qquad (24)$$

Determination of the length of yield zones

There are four possible cases of bending moment distribution for a beam under the combined action of seismic and gravity loading that can be defined in terms of the bending moment at the beam column interfaces. These are:

(a) The moments at the left and right hand side beam column interfaces are hogging.

(b) The moment at the left-hand side is sagging and that at the right hand side beam column interface is hogging.

(c) The moment at the left-hand side is hogging and that at the right hand side beam column interface is sagging.

(d) The moments at the left and right hand side beam column interface are both sagging.

Cases (a), (b) and (c) have four possible scenarios, these are;

(i) The yield zone is in the span only.

(ii) The yield zone is in the span and one of the supports.

(iii) The yield zone is at only one support.

(iv) The yield zone is at both supports.

Case(d) has two possible scenarios; these are

(i) The moment at one of the beam column interfaces is greater than the yield moment and at the other it is not.

(ii) Neither of moments at the beam column interfaces is greater than the yield moment but the sagging moment in the span has reached the yield value.

The formulae for calculating the coefficients of the spread plasticity flexibility matrix are determined in terms of the length X_1, X_2, X_3, X_n and X_c from the beam column interface with the more positive moments to the point of 1st, 2nd sagging yield points, hogging yield point, maximum hogging and sagging moments respectively. Therefore the lengths y_1, y_2, y_3, y_n and y_c are converted into lengths X_1, X_2, X_3, X_n and X_c respectively. If the total sum of seismic forces on a beam is towards the right, the more positive moment is at the left-hand side beam column interface, and $X_1 = y_1$, $X_2 = y_2$, $X_3 = y_3$, $X_n = y_n$ and $X_c = y_c$. On the other hand If the total sum of seismic forces on a beam is towards the left, the more positive moment is at the right-hand side beam column interface, and $X_1 = L - y_2$, $X_2 = L - y_1$, $X_3 = L - y_3$, $X_n = L - y_n$ and $X_c = L - y_c$. The yield zone length due to the sagging moment is found by subtracting X_1 from X_2, while that due to the hogging moment is found by subtracting X_3 from X_n.

EXAMPLE

The single bay, single storey structural shown in Figure 4 was analysed. Time history analysis computer programs incorporating the existing and proposed spread plasticity models were separately subjected to a half cycle sine wave ground acceleration loading shown in Figure 5. Incremental and total floor displacements, velocities, joint rotations, moments and forces were recorded at each time step.

The Structural Frame

The structural frame shown in Figure 4 consists of columns of 4m lengths that are 0.4m square and fixed at the base and beams of 8m length that are 0.6m deep with effective flange width b = 2.1m but supporting a floor slab that is 4m wide. The depth of the slab / beam flange is 0.2m and width of the web is 0.3m. The beam carries a gravity load of $42.76 kN/m$ composed of the dead and imposed loads. The fundamental period of the structure is 0.801 seconds.

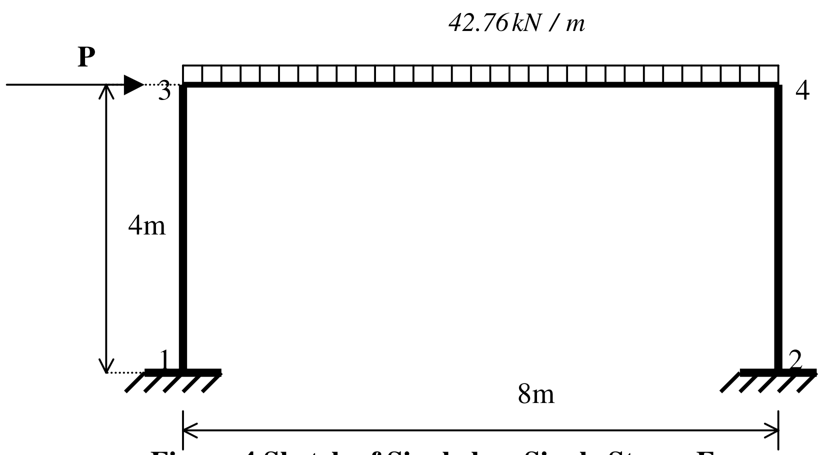

Figure 4 Sketch of Single bay Single Storey Frame

The Ground Acceleration;

A Simple half cycle sine wave ground acceleration A, used in this example is shown in Figure 5. It has a period T of 2 seconds. It is given by the expression:

$$A = K\left(0.5 - 0.5\cos\left(\frac{4\pi}{T}t\right)\right) \qquad (25)$$

K is a variable that can take any reasonable value.

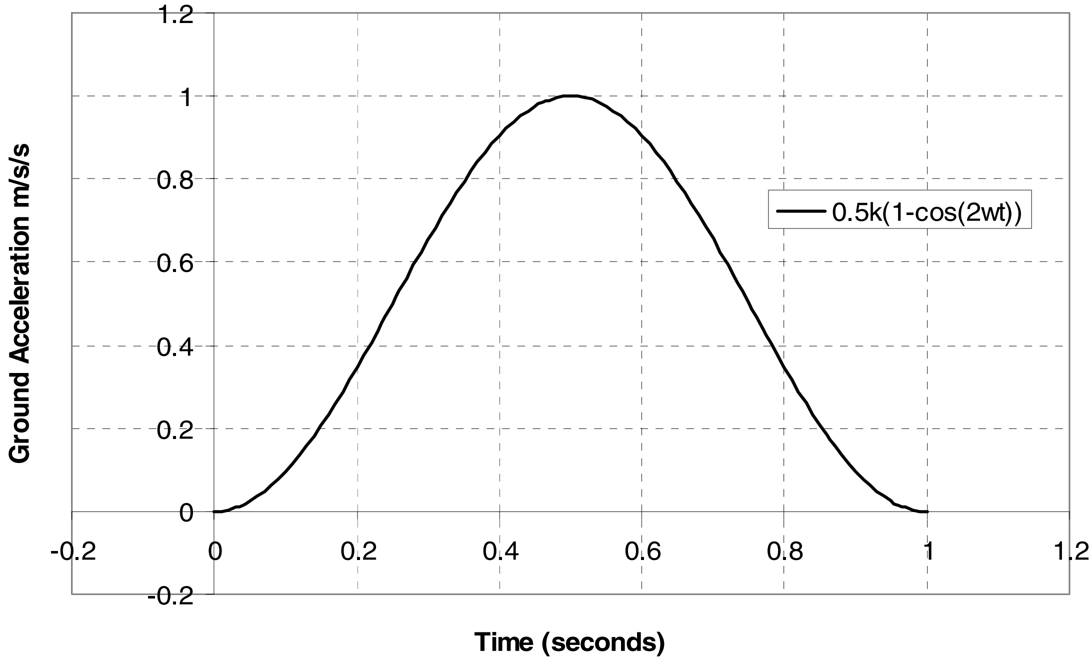

Figure 5 Half cycle sinewave Ground Acceleration

The half cycle sine wave was considered with both large and smaller magnitudes of K. The aim was to highlight the merits of the proposed spread plasticity model that incorporates the method of calculating yield zone forming in the span before they reach the beam end over existing models that assumed that yield zones are only at beam-ends. Under the action of the small ground acceleration, yielding within the span may not advance to the supports and therefore can not be recognised by the existing models. The larger ground acceleration on the other hand serves to investigate the case where yielding has reached the support. The value of K was 20.0 for the larger ground acceleration and 10.0 for the small ground acceleration.
Thus:

$$A_{Large} = 20\left(0.5 - 0.5\cos\left(\frac{4\pi}{T}t\right)\right) \tag{26}$$

$$A_{Small} = 10\left(0.5 - 0.5\cos\left(\frac{4\pi}{T}t\right)\right) \tag{27}$$

Also a sine wave with a smaller period of 0.8 seconds was applied to the structure. The major difference was in the shape of the output curves, but the difference between the results of the existing and proposed model were found to be of the same order. Kyakula [5]

In this example, deformations resulting from analysis based on the proposed and existing spread plasticity models are compared. The existing model considers two different methods of determining yield zone lengths. Thus the models and the method of determining the yield zone length were combined as follows:

(a) The proposed model incorporating the proposed method of determining the yield zone length
(b) The existing model incorporating the existing method of determining the yield zone length derived by Filippou [4], to be referred to as existing method 1
(c) The existing model incorporating the existing method of determining the yield zone length due to Soleimani [3], to be referred to as existing method 2
(d) The elastic Analysis.

Load Deflection Curves

The load deflection curves for the existing and proposed model resulting from applying the larger ground acceleration are shown in Figure 6. Before yielding, all the four curves coincide. After the hogging moment at the support has reached yield, the curves for the existing and proposed spread plasticity model slightly diverge from the elastic curve at point V. Along length VW, the curves for the existing and proposed model almost coincide. This is because this divergence is due to yielding of the hogging moment at the right hand side support, which is recognised by both models. At point W, the sagging moment in the span has reached its yield value and the curve for the proposed model sharply diverges. On the other hand the curves of the existing spread plasticity model continue with the same slope until point (X). At point X, the sagging moment at the left-hand support has reached the yield value. The curves due to the existing spread plasticity model also diverge towards and parallel to that of the proposed model. It is seen that after the moments at both supports have reached their yield values, the existing model behaves like the proposed model.

The percentage improvement $\%IM$ in accuracy of the load deflection curve of the proposed model over that of the existing model is given by;

$$\%IM = \frac{\left(\delta_p - \delta_e\right)}{\delta_p} x100$$

Where:

δ_p	=	Deflection for the proposed spread plasticity model corresponding to a given load F
δ_e	=	Deflection for the existing spread plasticity model corresponding to F

At load of about 600kN, the proposed model improves the accuracy of the load deflection curve over that given by the existing models by about 25%.

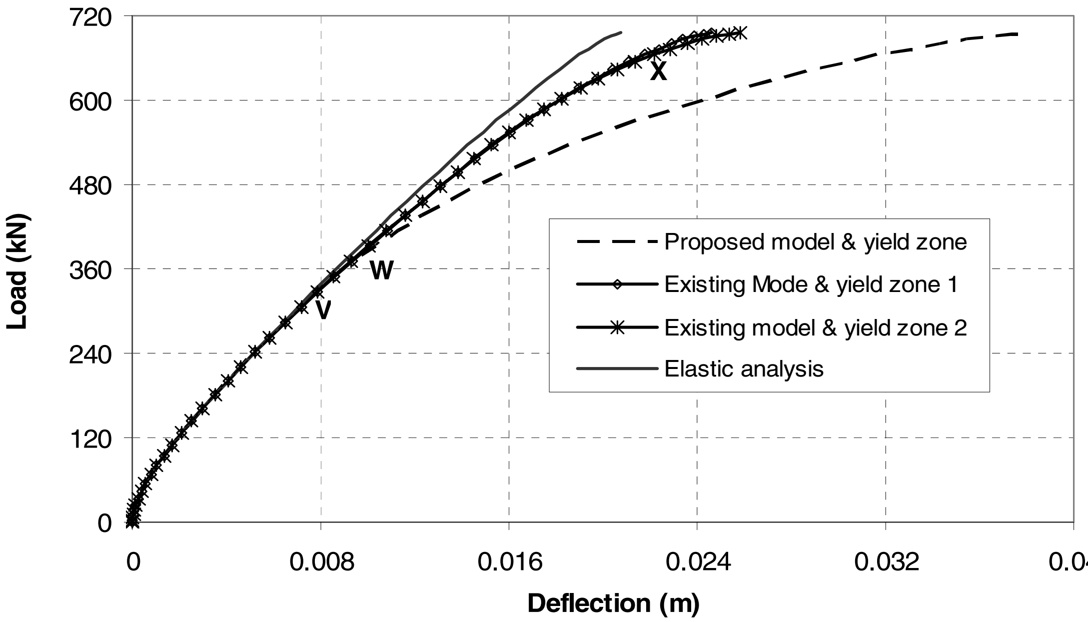

Figure 6: Load deflection curve

Moment rotation curves

The moment rotation curves for joint 3 are shown in figure 7. Between point A and B, there is no yielding and all the curves coincide. At point B, the moment at joint 4 reaches yield. Because rotations are coupled, this is recognised at joint 3 and the curves of the proposed and existing spread plasticity model diverge from that of elastic analysis. At point C, the sagging moment within the span reaches the yield value. This is recognised by the proposed spread plasticity model. Its curve diverges away from those of the existing model. The curves of the existing model continue on without divergence up to point D. At point D the sagging moment at joint 3 also reaches yield, which is now recognised by the existing spread plasticity models. This causes the curves of the existing model to diverge.

The percentage improvement in the moment rotation curve calculated by the proposed model over that calculated by existing models is given by;

$$\%IM = \frac{\left[\left(\theta_p - \theta_{t=0}\right) - \left(\theta_e - \theta_{t=0}\right)\right]}{\left(\theta_p - \theta_{t=0}\right)} x100$$

Where

$\%IM$ = Percentage improvement corresponding to any chosen moment M

θ_p = Rotation on the moment rotation curve for proposed model corresponding to the moment M

θ_e = Rotation on the moment rotation curve for existing model corresponding to the moment M

$\theta_{t=0}$ = rotation at time due to the gravity load, before application of any dynamic load

In calculating the percentage improvement, it is necessary to consider the initial rotation at the joint due to gravity load because it does not start from zero and the sign of rotation changes. For a moment value of about 300kNm, the percentage improvement increased is about 77%.

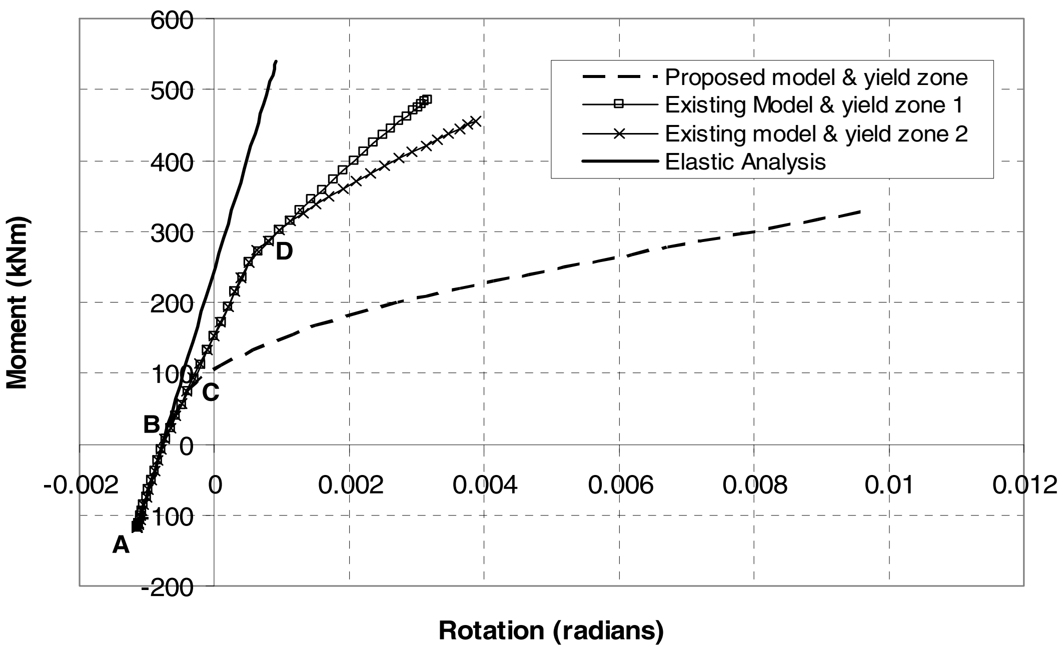

Figure 7: Moment rotation curves for joint 3

CONCLUSION

It has been shown that the moment curvature hysteretic relationship for the section at the beam column interface can not represent the non-linear behaviour in the spread plasticity model. This is because the Newton Raphson iteration procedure for solving the force displacement equation depends on recognising the difference between the internal and external moment that occurs when the hysteretic relationship changes slope at yielding or “unyielding”. Therefore it can not recognise the increase in yield zone length that occurs after yielding as these will not result in change of slope and the internal moment will be equal to the external moment. For a given section, the yielding and “unyielding “ occur a maximum of 4 times steps in a cycle of loading yet there may be several time steps in a cycle when the member has yielded and the yield zone length is changing. In the rest of the time steps, it is the change in yield zones length that is the only basis for changing the stiffness of the member. Thus it is important that the yield zone length is determined accurately. Moreover the cases when yielding due to sagging moments starts in the span and during unloading, the span is the last yielded point of the beam. The moment curvature hysteretic relationship based at the beam column interface can not recognise such a case. It is therefore important that the location of yield zones be determined accurately.

A simple yet accurate method for determining yield zone length formed anywhere in the beam has been presented. It depends on investigating the difference between the total applied moment (due to gravity and seismic load) and the yield moment at suitable intervals along the beam. Change of signs in this difference identifies the yield points. Then the error caused by the chosen interval is reduced to a minimum by simple interpolation. This method of determining yield zone length was incorporated in a spread plasticity model given in Kyakula [5, 6].

A simple example showing the effect of determining more accurately the lengths and locations of yield zones on the structural deformation has been presented. There was a maximum improvement in the load deflection and moment rotation curves of up to 25% and 77% respectively. For the moment rotation relationship, It was shown that the plastic rotation at a joint under the action of sagging moments started when the hogging moments at the other joint reached yield. This caused a slight divergence of the moment rotation curve. The curve then diverged sharply when the sagging moment in the span reached yield value although the moment at the joint was less than half the yield value. It can thus be concluded that the determination of length and location of yield zones is important to the accurate application of the spread plasticity model.

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