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COLLAPSE BEHAVIOUR OF HIGH-RISE BUILDINGS – A RESPONSE-HISTORY APPROACH

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SUMMARY

The results of a series of non-linear response history analyses are presented. The non-linear model includes elasto-plastic behaviour of beam connections up to a critical moment where upon the connection 'breaks ' and suffers irrecoverable loss of strength and stiffness. This corresponds to an extreme, idealized form of material degradation and when coupled with P-Delta effects, allows the complete collapse of the structures to be investigated. Three generic frames are subjected to seven earthquake excitations. Results were obtained for both the plastic limit (i.e. where all beams remain within their plastic range) and the collapse limit (where all beams exceed their ultimate capacity) and are presented in terms of number of storeys and ductility. The results show that significant reserve capacity is achievable even in structures with minimal ductility. The results are very dependant on the correspondence between the frequency content of the earthquake and the natural periods of the building and also the building configuration. Simple pushover analysis is not capable of predicting the collapse load of structure.

INTRODUCTION

Presently, all earthquake codes use response spectra methods as the primary means of designing earthquake resistant structures. The magnitude of the forces generated by the ultimate design earthquake is so large that structures are expected to behave inelastically when resisting these forces. The various codes simplify the inelastic considerations by dividing the equivalent earthquake force by a response modification factor (also known as a behaviour factor).

Put simply, the response modification factor is a measure of the ratio of the building's ultimate capacity to its elastic capacity and is an indication of how well a building can be expected to provide energy absorption in the inelastic range. In design, this factor is obtained from various tables in the relevant earthquake code and is only a function of the structural type. Virtually all earthquake codes around the world will have a table of values of response modification factors (e.g. IBC [1], EC8 [2], SEAOC [3], SANZ [4] to name a few) but each code may have different descriptions of the structural types and significantly different values assigned to the response modification factors. It is also relevant to note that the range of response modification factors is extremely limited. The various codes only recognise a few structural types and the factors are defined entirely on this basis.

To produce a better method of determining response modification factors, it is necessary to understand their history. The response modification factors in the codes are derived using semi-empirical means. The commentary to the National Earthquake Hazards Reduction Program (NEHRP) describes the factor as *"an empirical response modification (reduction) factor intended to account for both damping and ductility inherent in the structural system at displacements great enough to approach the maximum displacement of the system"*; FEMA[5]. Whittaker [6] suggests that "*There is no technical basis for the values assigned to R [response modification factor] in either the Uniform Building Code or the NEHRP Recommended Provisions."* The values of R can be traced back to the empirical horizontal force factors adopted in the 1959 SEAOC Blue book; ATC [7].

Attempts to reduce the uncertainty in the values assigned to behaviour factors has been the subject of much research, notably Uang [8], Whittaker [6], Krawinkler [9] and Miranda [10] to name a few. Whereas no universal methodology has yet been adopted the pushover analysis has emerged as a useful tool to assess the potential performance of a building in the inelastic range.

Pushover analysis subjects the building to an inelastic static analysis. In this analysis the load is gradually increased until the collapse capacity of the building is reached. Once the collapse mechanism and loading have been determined, the rotations of all the plastic hinges are calculated and the joints detailed to ensure that these rotations can be achieved. Typically designers perform the inelastic analysis indirectly, using an equivalent sequence of elastic analyses (placing pinned connections at the positions of the plastic hinges).

Although effective for a multi-storey building, this procedure is tedious and some doubt has been raised about its accuracy. It is generally thought that if the first mode shape does not dominate the response of the building then the load distribution and subsequent results may not be valid. *"It must be emphasised that the pushover analysis is approximate in nature and is based on static loading. As such it cannot represent dynamic phenomena with a large degree of accuracy. It may not detect some important deformation modes that may occur in a structure subjected to severe earthquakes, and it may exaggerate others. Inelastic dynamic response may differ significantly from predictions based on invariant or adaptive static load patterns, particularly if higher mode effects become important."* Krawinkler [9].

A recent improvement to this technique, known as modal pushover analysis (which accounts in an approximate way for the effects of yielding) has been developed and evaluated by Chintanapakdee [11]. The combination of modal responses remains problematic, and results suggest that significant errors may arise in the analysis of tall and/or reduced-strength frames. Other recent developments of the pushover technique include that of Kim [12] where the procedure is enhanced by considering more than just the fundamental mode and recalculating mode shapes whenever yielding occurs.

This paper presents the results of a response history analysis on generic frames subjected to European earthquakes in an attempt to identify the effect of higher modes on the collapse behaviour of high-rise buildings. Full details of the model can be found in Wilkinson and Hiley [13], however a summary of the modelling methodology is presented here.

MODELLING

The idealized structure is a plane frame, with m floors and n bays, as shown (for m=n=2) in Figure 1. Axial degrees of freedom (DOFs) are neglected in both beams and columns. Thus the model has (n+2) DOFs per floor.

In the sequel: $i = 1,\ldots,m$ is the floor/storey index; $j = 0,\ldots,n$ is the column (or bay, if $j = 1,\ldots,n$) index; $k = 0,1$ is the beam-end index; and $\mathbf{A}_{(1,2)}$ refers to element (1,2), for example, of any matrix $\mathbf{A}$.

Members

The stiffness matrix for a column (superscript C), subject to a *known* axial compression force P (due in our case to gravity i.e. P-Delta effects), is given by Krenk [14].

$$\mathbf{K}_{i,j}^{C} = \frac{\psi_{i,j} EI_{i,j}^{C}}{L_{i,}} \begin{bmatrix} 12\varphi_{i,j}/L_{i,}^{2} & -6/L_{i,} & -12\varphi_{i,j}/L_{i,}^{2} & -6/L_{i,} \\ -6/L_{i,} & 3+\varphi'_{i,j} & 6/L_{i,} & 3-\varphi'_{i,j} \\ -12\varphi_{i,j}/L_{i,}^{2} & 6/L_{i,} & 12\varphi_{i,j}/L_{i,}^{2} & 6/L_{i,} \\ -6/L_{i,} & 3-\varphi'_{i,j} & 6/L_{i,} & 3+\varphi'_{i,j} \end{bmatrix} \quad (1)$$

where: EI is flexural rigidity; L is member length; $\varphi \equiv \alpha \cot \alpha$ and $\psi \equiv \frac{1}{3}\alpha^2 / (1-\varphi)$ are the symmetric and anti-symmetric bending stiffness coefficients; $\varphi' \equiv \varphi/\psi$; and $\alpha \equiv \frac{1}{2} L\sqrt{P/EI}$. The bending stiffness coefficients tend to unity as P tends to zero, and are defined as such. The corresponding vector of displacements is $\left[\, u_{i-1}\ \theta_{i-1,j}\ u_i\ \theta_{i,j}\ \right]^{\mathrm{T}}$, where u is storey ceiling-level displacement (relative to the ground) and θ is column upper end-rotation.

The elastic stiffness matrix for a beam (superscript B), neglecting shear in accordance with neglect of column axial DOFs, is given by

$$\mathbf{K}_{i,j}^{B} = \frac{2EI_{i,j}^{B}}{L_{,j}} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (2)$$

with corresponding vector of displacements $\left[\, \theta_{i,j,0}^{B}\ \theta_{i,j,1}^{B}\ \right]^{\mathrm{T}}$, where $\theta_{i,j,k}^{B}$ is the rotation at end k of beam (i,j).

Material Behavoiur

Inelastic behaviour is confined to the beam-to-column connections, which remain rigid while the applied moment is less than a critical magnitude. This critical magnitude, that is, the yield moment, M^{Y}, is considered to be a physical property of the connection. When the applied moment becomes equal in magnitude to M^{Y}, the connection yields perfectly plastically. An elastic beam with two such (identical) connections at its ends behaves, under anti-symmetric loading, as an elastic-perfectly-plastic, system (i.e. the connections act as plastic hinges).

In order to be able to study the progressive states of failure that lead to the collapse of a frame, this material model – the simplest of all inelastic models – is extended by introducing, as simply as possible, an ultimate failure state. This is defined in terms of the plastic deformation, θ^{P}, of a connection. While $|\theta^{P}|$ is less than its critical value, a connection behaves rigidly/plastically, as just described. This critical value, referred to as the ultimate deformation, θ^{X}, is, along with M^{Y}, considered to be a physical property of the connection. When $|\theta^{P}|$ becomes equal to θ^{X}, the connection fails, that is, it loses any capacity to transmit a moment; it becomes 'pinned'. The nett behaviour of the beam/connection assembly then becomes what may be called elastoplastic-pinned.

The objective here is to reproduce, not so much the observed hysteretic behaviour of real concrete or steel structural members, but rather the phenomena of degradation and collapse of framed structures; and the philosophy throughout is to keep the entire model as simple as possible. In contrast to the yield state, a connection cannot recover from failure. Through the failure state the model admits irrecoverable loss of strength and stiffness, corresponding to an extreme, idealized form of material degradation. This irreversibile behaviour is considered crucial for a meaningful study of collapse. (The more severe condition, where the beam becomes altogether detached from the column, has not been modelled.)

In the present series of experiments the ultimate deformations of individual connections are prescribed in terms of a rotational capacity (factor), μ_θ, as described below.

Ductility

Characterising the ductility of single degree of freedom systems can, by definition, be achieved with one parameter (either the ductility demand – a response variable which is a measure of the required ductility that a system must be able to achieve to survive; or the ductility factor – a material property which is a measure of the ductility of a given structure i.e $\Delta_{ult}/\Delta_{yield}$). Multi-degree of freedom system models do not lend themselves to simple characterisation by factors that describe the behaviour of the whole system. These systems may have different ductility characteristics for different elements or the loading regime may result in different elements with the same ductility characteristics having different ductility demands. Push-over analysis can determine the ductility demands of individual elements of a structure for a monotonic loading, but is incapable of dealing with any variation in ductility demand due to the variant nature of the loading.

As this paper is concerned with the analysis of structural frames up to the point of total collapse, it is not possible to describe the ductility of the structure in terms of a ductility factor as individual elements may have failed before the structure reaches the specified ductility factor.

The inelastic properties of the frame are defined, in the model presented here, in terms of the rotational capacities, μ_c, of individual connections. The ductility characteristics of individual building elements (represented by individual degrees of freedom in the model) then characterise the overall behaviour of the structure. The rotational capacity of the individual connections is defined as

$$\mu_c = \frac{\theta_{ult}}{\theta_y} \tag{3}$$

where θ_{ult} = the ultimate rotation of the connection and θ_y = the yield rotation of the connection. From a practical point of view defining ductility in this way has the advantage that designers can assign known ductilities to connections based on the structural detail adopted.

A symmetric portal frame, pinned at the base, and with two such identical beam-to-column connections, behaves as an elastoplastic system. One can easily show that the drift, Δ_{ult} at the point of first failure (of one of the connections), divided by the drift at yield, Δ_y, must satisfy

$$\frac{\Delta_{ult}}{\Delta_y} = 1 + \frac{(\mu_c - 1)}{1 + 2h/L}, \tag{4}$$

where *h/L* is the aspect ratio of the frame. The drift ratio defined in equation 4 is the so-called ductility (factor) that is well-known in the study of elastoplastic systems. Using this equation displacement ductilities of μ = 1,2,4,8 will result in rotational ductilities of μ_θ = 1,3,7,15.

Dynamics

The lumped-mass idealization is used, together with the constraint (reasonable for several types of floor system Chopra [15] that each floor diaphragm is rigid in its own plane but flexible in bending. The resulting mass matrix lumps the mass, m_i, of storey i onto the diagonal element corresponding to the deflection DOF for that storey:

$$\mathbf{M}_i = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{M}_i^{1,1} \end{bmatrix} \tag{5}$$

where $\mathbf{M}_{i\ (1,1)}^{1,1} = m_i$ and all other elements of $\mathbf{M}_i^{1,1}$ are zero. The mass matrix for the structure, $\mathbf{M}_o$, is formed from these sub-matrices.

The system of equations of motion of the structure, omitting damping forces, may now be written:

$$\mathbf{M}_o \ddot{\mathbf{D}}_o(t) + \mathbf{K}_o \mathbf{D}_o(t) = \mathbf{P}_o(t) \tag{6}$$

where $\mathbf{P}_o(t)$ is the vector of time-dependent external forces. As we are concerned here only with the dynamics induced by horizontal ground motion, the effective earthquake force $\mathbf{P}_o(t)$ is given by:

$$\mathbf{P}_o(t) = -\mathbf{M}_o \ddot{\mathbf{D}}_g(t)\,. \tag{7}$$

$\mathbf{D}_g(t)$ is the rigid-body displacement vector corresponding to the ground motion, $u_g(t)$: it is given by $\boldsymbol{\iota} u_g(t)$, where $\boldsymbol{\iota}$ is the influence vector, defined by $\boldsymbol{\iota}^u = \mathbf{1}$ and $\boldsymbol{\iota}^\theta = \mathbf{0}$.

The fact that all but m of the elements of $\mathbf{M}_o$ and of $\mathbf{P}_o$ are zero, allows the set of equations of motion to be conveniently separated into two coupled sets: one dynamic, but smaller and hence more efficiently solved than the full set; the other static, expressing rotational equilibrium (i.e. static condensation, such as that used by Chopra [15]) can be employed resulting in:

$$\mathbf{M}_o^{uu} \ddot{\mathbf{D}}_o^u + \mathbf{K}_o^{uu} \mathbf{D}_o^u + \mathbf{K}_o^{u\theta} \mathbf{D}_o^\theta = \mathbf{P}_o^u \tag{8}$$

$$\mathbf{K}_o^{\theta u} \mathbf{D}_o^u + \mathbf{K}_o^{\theta\theta} \mathbf{D}_o^\theta = \mathbf{0}\,, \tag{9}$$

Equation 9 gives the rotations as a linear combination of the translations, so that the former may be eliminated from equation 8 to give

$$\mathbf{M}_o^{uu} \ddot{\mathbf{D}}_o^u + \mathbf{K}_o^{*} \mathbf{D}_o^u = \mathbf{P}_o^u\,, \tag{10}$$

where the condensed stiffness matrix is

$$\mathbf{K}_o^{*} = \mathbf{K}_o^{uu} - \mathbf{K}_o^{u\theta} \left(\mathbf{K}_o^{\theta\theta}\right)^{-1} \mathbf{K}_o^{\theta u}\,. \tag{11}$$

EXAMPLES

The model has been validated in Wilkinson [13] where the results of analyses on simple plane frames were accurate to within 1% and the first three natural periods and modes shapes were correctly identified.

In this paper a number of generic frames have been analysed and their results are now presented. The examples chosen are generic single bay frames with differing number of storeys as shown in Figure 1.

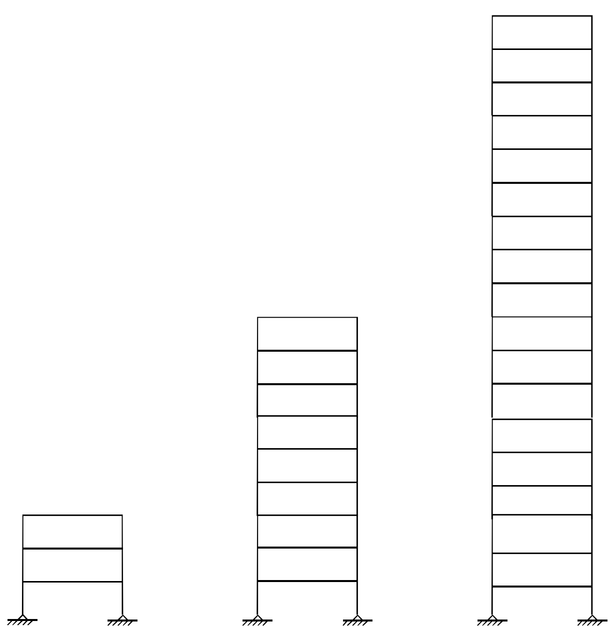


Figure 1 Multi-storey Examples

The key structural design parameters for a single bay moment resisting frame are: number of storeys, height-wise distribution of stiffness, beam-to-column stiffness ratio, height-wise distribution of mass; fundamental period, yield strength distribution, moment rotation relationship of the beam to column connections (ductility) and earthquake excitation. Other factors such as the total mass or the magnitude of the yield moment etc. are fixed indirectly by setting the key parameters (e.g. magnitude of yield strength is normalised by scaling the earthquake accelerations.

A number of generic structures have been analysed similar to those presented by Chintanapakdee [11]. Three test series have been analysed, each series consisting of a frame with a different number of storeys – namely 3, 9, 18. The column stiffness of each frame has been proportioned to achieve constant inter-storey drift when static loads are applied with a height-wise distribution as specified by the IBC. Beam stiffness was proportioned so that the beam to column stiffness ratio was equal to a quarter. In this paper the definition of beam/column stiffness ratio has been taken as $\rho = I_b h_c / 2 I_c L_b$. Where I_b is the second moment of area of the beam, I_c is the second moment of area of the column, h_c is the height of the column and L_b is the length of the beam.

A constant height-wise mass distribution was selected for all examples. The total mass was proportioned so that the fundamental period was equal to that calculated by EC8[16]; namely, $T = CH^{3/4}$ The value of C was 0.085 which represents a moment resisting frame of steel. Earthquake excitations have been selected from the European Strong-Motion Database; Ambraseys [17]. Seven excitations were chosen in an attempt to get a wide range of responses. The European seismic records were chosen using the following critera:

1. They were above a surface wave magnitude of 6.5. Surface wave magnitude was chosen over Richter magnitude as the Richter magnitude was not available for all records whereas the surface wave magnitude was. For the records where Richter magnitude was available, all values were over 6.
2. The horizontal component, of the direction with the maximum peak velocity from the station with the greatest intensity was chosen. For earthquakes where intensity information was unavailable, the station closest to the epicentre was chosen unless the peak ground velocity at a more distant station was significantly greater.
3. The seven records were chosen which gave the best combination of large spectral accelerations over a wide range of periods. This was done subjectively by looking at the response spectra.

The earthquakes records used in the analysis are given in Table 1 and the associated response spectra are given in Figure 2.

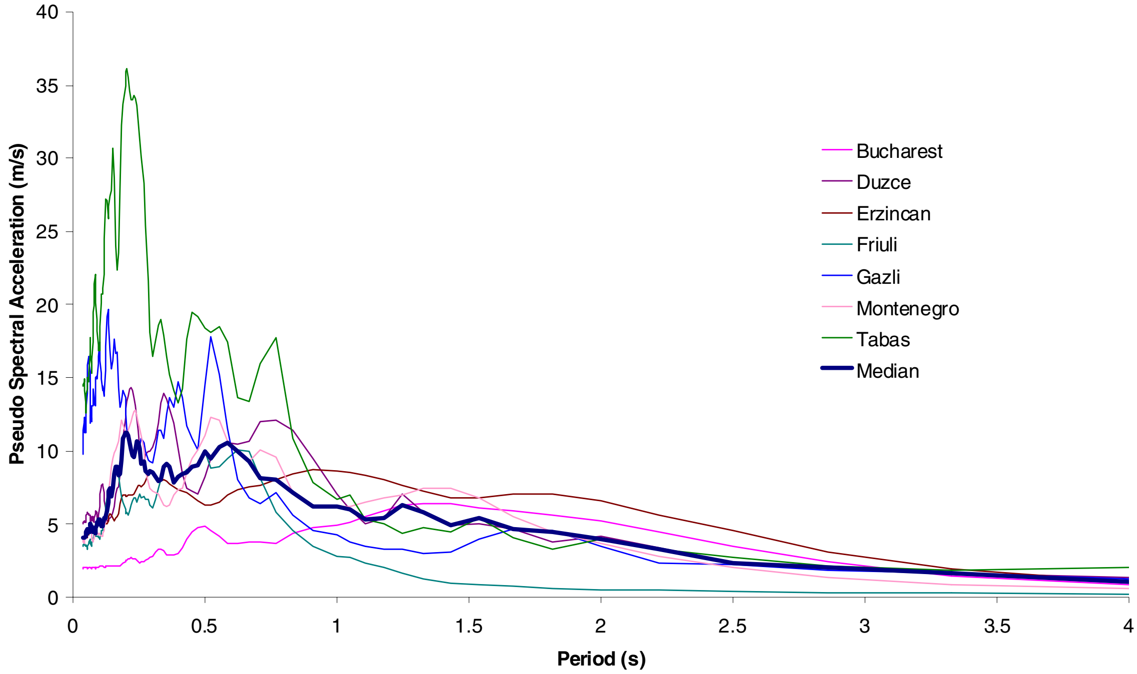


Figure 2 Response Spectra

Table 1 Earthquake Records

Name	Country	Date	Ms	Station Name & Component	Epicentral distance	Site intensity	Foundation category	Peak velocity cm/s
Bucharest	Romania	04/03/77	7.05	Bucharest-Building Research Inst, N-S	161 km	VIII	alluvium	73.13
Duzce	Turkey	12/11/99	7.3	Dulce-Meteoroloji Mudurlugu, W-E	9 km		unknown	63.53
Erzincan	Turkey	13/03/92	6.75	Erzincan-Meteorologij Mudurlugu, N-S	13 km		stiff soil	101.8
Friuli	Italy	06/05/76	6.5	Tolmezzo-Diga Ambiesta,E-W	27 km	VIII	rock	32.6
Gazli	Uzbekistan	17/05/79	7.04	Gazli, E-W	22 km	IX	V soft soil	62.7
Montenegro	Yugoslavia	15/04/79	7.04	Bar-Skupstina Opstine, E-W	16 km	IX	stiff soil	52.0
Tabas	Iran	16/09/78	7.33	Tabas, N74E	52 km	IX+	stiff soil	84.5

The yield moment was chosen so that the yielding would occur simultaneously in all storeys when the base shear equal to the weight of the structure was applied as static loads using a code-based height-wise distribution.

Load scaling

Response history analyses were performed for each combination of structural parameters, ductility, and ground motion. The seismic load is scaled systematically, so that the results are presented mostly in terms of *relative* performance.

It has been noted by Riddell [17], in the context of SDF damped oscillators, that when the ground acceleration and the resistance function (or restoring force) are both scaled by the same constant, the response ductility does not change.

In our experiments we are interested in certain limiting cases which are specified in terms of response ductility: they are the *plastic* limit (where $\mu<\mu_\theta$ for every connection) and the *collapse* limit (where $\mu\geq\mu_\theta$ for every connection, *i.e.* where every connection breaks). There is also the *elastic* limit, which is simply the plastic limit for the case $\mu_\theta=1$.

From a design perspective we may wish to know the (minimum) yield strength that would be required to ensure that the response ductility nowhere exceeds the ductility capacity (this would be the plastic limit case). The experiments answer the different, but equivalent question, namely: what is the (maximum) intensity of seismic load that would produce a plastic response?

Suppose that a certain frame is known to respond up to its plastic limit for some given earthquake. The scaling argument allows us to infer, for example, that, if we now modify the frame by doubling the yield strength, M^Y, (and, therefore, also the yield rotation) of every connection (without altering the ductility capacities, so that the ultimate rotations are also doubled), and then subject it to the same earthquake doubled in intensity, it too will respond up to its plastic limit. The response displacements will of course be doubled, but the response ductilities will remain the same.

The results of the various analyses are summarised in Table 2. The values in Table 2 are amplifications that would need to be applied to the earthquake excitation to reach the plastic and collapse limits. An alternative way of looking at these values is to consider them as factors that the yield moment could be reduced by before local failure (the plastic limit) or total collapse (the collapse limit) occurs. Since the yield moments of the beams were proportioned using the response spectrum method, the values presented in Table 2 are equivalent to the response modification factor (subject to the limitations of the modelling assumptions. Included in the table are the results of a single storey model (labeled ‘S’ in the ‘DOFs’ column). This has been included so that the results of the multi-storey examples (labeled ‘M’ in the ‘DOFs’ column) can be compared to the results that would be produced using the response spectra method - both elastic μ_θ=1 and inelastic μ_θ = 3, 7 and 15.

Table 2 R values

			Number of Storeys					
Earthquake	μ_θ	**DOFs**	**3**		**9**		**18**	
			Plastic	Collapse	Plastic	Collapse	Plastic	Collapse
Bucharest	1	S	1.00	1.05	1.00	1.06	1.00	1.33
		M	1.11	1.25	1.15	1.59	1.19	1.98
	3	S	1.93	1.93	1.99	2.42	3.14	3.56
		M	2.32	2.32	2.10	4.06	2.14	6.09
	7	S	2.34	2.35	3.41	3.55	5.23	5.61

		M	2.81	2.87	3.81	5.73	5.28	8.77
	15	S	2.76	2.76	8.33	8.61	8.78	9.15
		M	3.27	3.57	6.06	9.17	7.78	12.2
Duzce	1	S	1.00	1.01	1.00	1.13	1.00	1.63
		M	1.09	1.22	0.938	1.75	0.617	4.26
	3	S	1.84	1.84	1.95	1.97	1.99	2.14
		M	2.26	2.28	1.58	6.07	1.06	5.84
	7	S	2.59	2.63	5.19	5.20	3.17	3.17
		M	3.37	3.74	4.03	8.42	3.50	4.65
	15	S	5.04	5.05	6.44	6.44	4.85	4.86
		M	6.13	6.61	6.88	7.12	5.35	6.55
Erzincan	1	S	1.00	1.00	1.00	1.06	1.00	1.39
		M	1.08	1.18	1.16	1.79	1.09	4.47
	3	S	1.54	1.54	1.70	1.81	2.05	2.55
		M	1.83	1.88	2.10	3.15	2.32	6.87
	7	S	1.89	1.89	3.11	3.37	6.35	6.83
		M	2.23	2.37	3.88	6.75	5.31	9.68
	15	S	2.27	2.29	7.63	8.05	11.3	11.9
		M	2.89	4.18	11.1	12.1	8.92	14.6
Friuli	1	S	1.00	1.24	1.00	2.77	1.00	1.33
		M	1.09	1.75	0.520	3.33	0.142	1.29
	3	S	2.58	2.58	3.94	3.94	1.99	2.07
		M	3.77	4.75	0.783	5.51	0.245	2.43
	7	S	4.65	4.92	6.00	6.00	3.04	3.26
		M	5.27	7.95	1.45	10.7	0.390	3.39
	15	S	9.32	10.1	11.8	12.2	4.80	5.06
		M	8.95	28.8	10.2	11.5	3.15	3.82
Gazli	1	S	1.00	1.16	1.00	1.00	1.00	1.24
		M	1.03	3.29	0.443	1.25	0.518	6.09
	3	S	2.61	2.91	1.89	2.36	2.78	3.38
		M	3.73	3.81	0.838	2.24	0.884	11.0
	7	S	4.80	4.81	3.07	3.07	5.04	5.09
		M	5.59	7.16	2.82	5.07	1.53	20.1
	15	S	7.75	7.76	4.33	4.56	8.15	8.16
		M	9.68	10.8	6.84	9.19	7.75	31.4
Montenegro	1	S	1.00	1.04	1.00	1.39	1.00	1.31
		M	1.13	1.24	1.01	3.15	0.672	1.92
	3	S	2.46	2.47	2.11	2.32	1.83	3.57
		M	3.03	3.09	2.38	5.77	1.34	6.21
	7	S	4.14	4.14	5.40	5.40	6.35	7.37
		M	4.65	4.76	3.40	11.0	2.40	11.7
	15	S	5.73	5.85	8.55	9.40	12.0	12.6
		M	6.30	10.80	16.8	21.0	9.21	21.7
Tabas	1	S	1.00	1.18	1.00	1.00	1.00	1.30
		M	0.750	1.30	0.586	1.67	0.445	1.96
	3	S	2.28	2.29	2.07	2.15	2.79	3.07
		M	1.82	2.92	1.05	3.37	0.993	4.99

	7	S	3.36	3.36	2.70	2.91	4.56	4.84
		M	3.57	4.23	3.17	7.93	2.05	7.44
	15	S	4.07	4.07	4.20	4.36	7.82	8.15
		M	7.06	7.17	10.4	12.5	12.9	13.4

To compare the results obtained in Table 2 to static based methods, the frames were also subjected to a pushover analysis. Since pushover analysis uses a monotonically increasing load that has an invariant height-wise distribution, with respect to the loading, the results are only dependent on the magnitude of base shear (which is calculated using the different response spectra) and not from the individual characteristics of the earthquake record. As we are looking at the reserve capacity after the first yield, there will be only one value of μ for each building and not one for each earthquake. Furthermore, since the strengths of the beams were sized to produce simultaneous yielding at all levels when analysed using the response spectrum method, the building will immediately form a mechanism at the onset of yielding. Therefore the response modification factor is independent of the ductility factor and will be equal to unity for all cases.

DISCUSSION

Looking at Table 1 a number of observations can be made. Note that, for the special case of the elastic limit, which is simply the plastic limit when μ_θ=1 (*i.e.* when the connections are perfectly brittle), the value given in the table for all single degree of freedom systems is, by definition, 1.00. The systems studied here, however, have *two* static, rotational degrees of freedom (although there is only one dynamic degree of freedom). Furthermore, the model does not allow both ends of a single beam to fail simultaneously. Instead, one end is allowed, arbitrarily to 'break' first. This results in a reduction of the moment in the other end of the beam, which therefore continues to operate within its elastic range for at least a short time afterwards. A consequence of this local failure is that there is a reduction in stiffness of the system and therefore a corresponding lengthening of its period. As the spectral response generally gets smaller with increasing period, failure of one end of the beam will generally result in smaller inertia forces. The exceptions, where the spectral response for the half-broken system is *not* substantially lower than for the elastic system, are: Gazli, for the medium-period SDF system (corresponding to the 9-storey frame); and Erzincan, Duzce and Bucharest, for the short-period SDF system (corresponding to the 3-storey frame). In these four cases the collapse load, for the single-storey frames with μ_θ=1, is only slightly greater than the plastic load.

It can be seen that the magnitude of earthquake required to cause collapse generally increases with increasing number of storeys, while the magnitude of earthquake required to reach the plastic limit generally remains fairly constant (although the relationship is not as consistent). The reason for this is that a building with a greater number of storeys, can form more plastic hinges and therefore has the potential to dissipate more energy through hysteretic damping. On the other hand a building with a greater number of storeys can have a greater variation in ductility demand (although this is more greatly influenced by other parameters such as the characteristics of the earthquake) and therefore a connection is more likely to 'break' while other members are still only lightly loaded. An example where the collapse load appears not to increase with increasing number of storeys is for the Friuli earthquake. The reason for this can be explained by looking at the response spectrum of Friuli. In this earthquake the pseudo acceleration is large at the period corresponding to the three storey example and 'tails-off' quickly so that the pseudo acceleration is small at the fundamental period of the eighteen storey frame. In fact the (elastic) responses for the second and third natural frequencies are, respectively, over ten and almost twenty times that for the fundamental frequency. Consequently, the normalizing amplification factor for the 18-storey Friuli case is unduly large (with a value of 20.6), so that the values in Table 1 are misleadingly small.

By comparing the results of the SDOF cases to the MDOF cases we can get an indication of the effects of the higher modes and of inelastic behaviour on the seismic response of the frames. The response of each natural mode of a frame can be estimated, in theory, from an earthquake's response spectrum; and these responses can be combined (although the process is not straightforward; in the present paper we have simply used the fundamental mode for scaling purposes) to give an estimate of the *elastic* response of a multi-storey frame. This would correspond in our table to the 'plastic' limit cases with μ_θ=1. Similarly, the effects of plasticity/ductility can be observed in both MDOF and SDOF systems, and the similarities and differences can be interpreted in terms of the separate and combined effects of higher modes of vibration and hysteretic damping.

For example if we take the Bucharest earthquake we see that there are differences between the MDOF and SDOF results for μ_θ = 1 of 11%, 15% and 19% for the 3, 9, and 18 storey buildings respectively; however when we look at the case of μ_θ = 8, we see that the differences are 18.5%, -27% and -11% for the plastic case. Since the response of complex inelastic systems to complex loadings such as earthquakes can display such varied behaviour, the averages of the ratios between the MDOF and SDOF cases for the different earthquakes are presented in Table 3.

Finally some comments can be made about the reserve capacity of a structure after the onset of yielding. In this paper we have not looked at inter-storey drifts. Inter-storey drift ratio is the usual way of assessing potential damage to a structure. Due to lack of space, for this paper we define 'damage' as the onset of structural damage (i.e. yielding of a structural members) and we will define the reserve capacity of the structure as being the ratio of load to cause collapse, to the load to reach the plastic limit. It would also be possible to look at inter-storey drifts and this will be the subject of another paper.

Since the results obtained for different structures subjected to different earthquakes varies widely, it is useful to plot average values of the results from all earthquakes. Plotting averages allows the trends that are due to the structural characteristics of the building to be observed more readily. The average reserve capacity of the structure together with the average plastic limits (normalized by the elastic limit) and the average ratio of collapse limit to plastic limit for each building are presented in Table 3. These results are also plotted Figure 3.

Table 3 average reserve capacities

	Storeys								
	3			9			18		
μ_θ	P/E	C/E	C/P	P/E	C/E	C/P	P/E	C/E	C/P
1	1.00	1.57	1.57	1.00	2.86	2.86	1.00	5.82	5.82
3	2.58	2.94	1.13	1.84	5.69	3.23	1.90	11.38	6.19
7	3.83	4.62	1.17	4.13	10.89	2.89	4.12	17.23	5.02
15	6.26	9.88	1.53	5.94	15.85	1.22	14.75	26.31	1.87

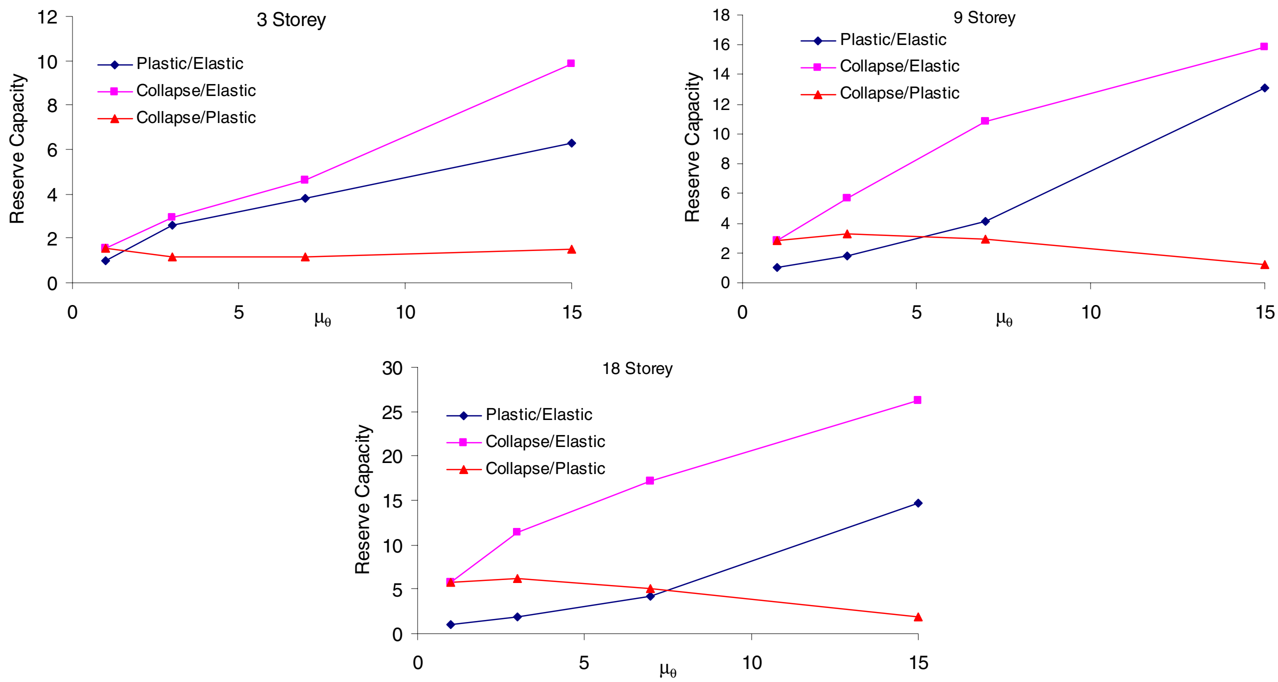
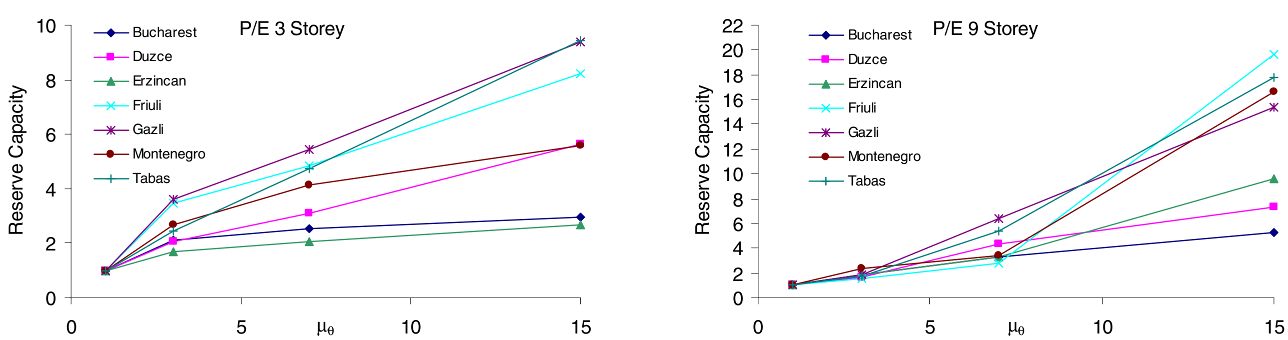


Figure 3 Average Plastic and Collapse Reserve capacities

From Figure 3 it can be seen that the average ratio of plastic limit to elastic limit varies from 1 when $\mu_\theta=1$ to 14.75 (for the 18 storey case $\mu_\theta=15$) and that both the plastic limit and the collapse limit increase with number of storeys and rotational capacity. For high-ductility frames, the influence of the number of storeys becomes less pronounced, although an increase in ductility always leads to an increase in both the plastic and collapse limits. The reserve capacity over the plastic limit remains fairly constant or reduces slightly as rotational capacity increases, but the absolute values increase (except for the highest ductility where the values converge).

While it is useful to plot averages, to observe trends, it is also important to plot the results of the individual earthquake so that the influence of these on the scatter of results can be observed. For this reason, the Plastic/Elastic reserve capacities for individual earthquakes have been presented in Figure 4. As would be expected, there is no scatter in the results when $\mu_\theta=1$ (i.e. no ductility) but the scatter in the results gets quite large as the rotational capacity increases (with maximum values of 9.4, 7.5 and 29 for the 3, 9 and 18 storey examples respectively). The 3 and 9 storey examples show that the scatter remains fairly constant over the range of rotational capacity, while the 18 storey example has significant scatter for $\mu_\theta = 15$



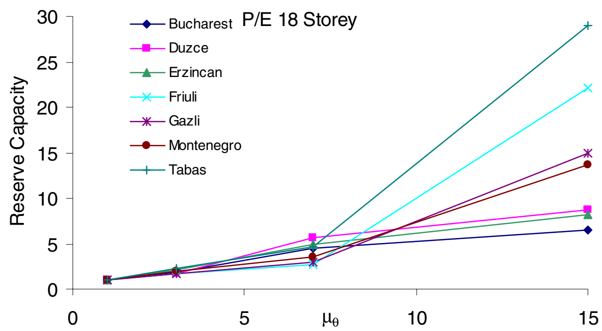


Figure 4 Plastic Reserve Capacities for Individual Earthquakes

CONCLUSIONS

Some reserve capacity will exist in redundant systems even if the individual members have no ductility with the provisos that, 1) both ends of the member do not fail simultaneously and 2) the beam will not become detached from the column at the ultimate rotation.

The plastic capacity of buildings in general remains fairly constant as the number of storeys increases, but this is fairly dependant on the earthquake excitation.

The collapse capacity of buildings increases with increasing numbers of storeys, although this may not be true if the strength distributions have been specified based on only the first mode response and there are significant contributions to the response from higher modes

The plastic limit increases with rotational capacity of the connections; however, the average reserve capacity (i.e. ratio of collapse limit to plastic limit) is relatively independent of this (although it is strongly affected by the choice of earthquake and the building configuration).

Simple pushover analysis is an extremely useful tool in assessing the potential seismic performance of a structure in a qualitative way, but is limited in terms of quantitative analysis. Furthermore in addition to the fundamental mode of vibration, other, higher modes should be assessed

Keeping in mind the limitations of the model and the relatively small number of examples and samples of earthquakes used, the following can be said about the reserve capacity of the structures presented in this paper. On average the extra capacity of a structure after first yield and before any connection 'breaks' varies from 1.00 to 14.75 (depending on the number of storeys and the rotational capacity). In addition to this, for buildings with ductile connections, on average, there is an extra capacity, of up to 87% before the building collapses. For structures with brittle connections the extra capacity over the plastic limit is even greater, although the ratio of plastic capacity to elastic capacity is much smaller.

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