

PRELIMINARY NUMERICAL INVESTIGATION OF THE DYNAMIC CHARACTERISTICS OF HISTORIC MONUMENTS

G. C. MANOS¹, V. J. SOULIS², A. DIAGOUMA³

SUMMARY

This paper includes the most important findings of a series of linear dynamics analyses focusing on the dynamic characteristics of historic monuments. The historic monuments which were selected for this investigation are: a) The church of the Assumption of The Mother of God (Koimiseos tis Theotokou). b) The church of Agia Triada, at the District of Karditsa, Greece. The church of the Assumption of The Mother of God (Koimiseos tis Theotokou) is located at Zervati in the district of Argyrocastro in Albania, a district where numerous churches and monasteries are located dating from the 12th century A.D. This study was conducted as part of the current restoration effort that has been recently related to Christian monuments in this district. This church dates from the 16th century and is a three-nave basilica with a central dome and a transverse nave. For the church of the Assumption of The Mother of God a numerical investigation for the behaviour of the structure prior and after the intervention process is studied under gravitational and earthquake loading. The church of Agia Triada at the village of Drakotrypa in the prefecture of Karditsa in Greece dates from the 1742 and is a single-nave basilica with a central dome and a transverse nave together with 3 apses at the sides of the church. This church belongs to the post-Byzantine type of construction. The numerical simulation for the church of Agia Triada is utilized by applying vertical and earthquake loads and by trying to predict the initiation of failure at various structural elements.

INTRODUCTION

A variety of simple or complex numerical tools are available in order to investigate numerically the behavior of historical monuments of various forms. Among some of the main difficulties that are encountered, in numerical simulation approximations, apart from modeling the complex architectural features are the following ([1], [2], [3]):

- Understanding the importance of the load transfer mechanisms, for both the gravitational and seismic forces. In this way certain simplifications can be introduced in representing the structural system by the numerical model.

¹ Professor, Department of Civil Eng, Aristotle University of Thessaloniki, GR 54006, Greece email gcmanos@civil.auth.gr

² Research Assistant, Department of Civil Eng, Aristotle University of Thessaloniki, GR 54006,Greece

³ Research Assistant, Department of Civil Eng, Aristotle University of Thessaloniki, GR 54006, Greece

- The difficulty in adopting values for important mechanical properties or failure criteria, as will be discussed in this paper.

- Employing linear or non-linear numerical approximations combined with the difficulty in assigning values to important physical properties.

The current numerical study included linear dynamic analyses for the determination of the fundamental eigen-modes and frequencies of three different historical monuments. For the church of Agia Triada gravitational and earthquake loads were applied in order to identify certain areas of the load-bearing system that exhibit stress concentration that reaches critical tensile and shear strength values for the stone masonry. For this church, the influence of particular structural elements as tympana, openings, apses, etc was also investigated.

1. NUMERICAL ANALYSIS OF THE CHURCH OF THE ASSUMPTION OF GOD AT ZERVATI PRIOR TO ANY INTERVENTION

This church dates from the 16th century and is a three-nave basilica with a central dome and a transverse nave. All the load-bearing elements are rather regular lime-stone masonry constructed with lime-mortar ([3], Papas 2001). Table 1 lists values, which were assumed to be valid for the critical mechanical properties for the masonry segments belonging to the three distinct construction phases prior to any intervention, as the current examination did not include any experimental investigation, either at the laboratory or in-situ, for determining such properties.



Photo 1. View of the Dome and the Apse from the East

1.1. Assumptions of the performed elastic analysis.

A linear elastic numerical simulation was adopted utilizing various one-, two- and three-dimensional elements that are provided in the SAP2000 software package. This numerical simulation represents the

church as it stands prior to any intervention for its restoration. For the numerical simulation 4200 triangular and rectangular shell finite elements were employed together with 34 linear beam elements. Two types of load combinations were examined. The first included 1.35G + 1.5Q and the second $G + 0.3Q \mp E$ where G the permanent load, Q the snow and live load and E the earthquake load. The dead load for the masonry components was assumed to be equal to 20KNt/m3. Added to this value was a load of 18KNt/m3 to account for the roof weight.

Phase	Young's	Poisson's	Design Compressive		Design Tensile	
	Modulus	Ratio	Strength (Nt/mm2)		Strength (Nt/mm2)	
	(Nt/mm2)					
	Prior and after		Prior*	After**	Prior*	After**
1 st	2956	0.2	0.986	1.413	0.099	0.147
2^{nd}	3874	0.2	1.292	1.766	0.129	0.177
3 rd	3874	0.2	1.292	1.766	0.129	0.177

Table 1. Assumed properties of the stone masonry prior and after the proposed intervention

* assumed safety factor = 3 **assumed safety factor = 2.5

1.1.1. Assumed mechanical characteristics of the stone masonry.

Certain values were assumed to be valid for the critical mechanical properties of the masonry, as the current examination did not include any experimental investigation, either at the laboratory or in-situ, for determining such critical values. Table 1 lists the most significant mechanical properties assumed to be valid for the masonry segments belonging to the three distinct construction phases, prior to any intervention. Thus, the Young's modulus for the masonry was assumed to be in the range of 2956 to 3874 Nt/mm2, the characteristic compressive strength was assumed to be in the range from 0.99 to 1.29 Nt/mm2 and the characteristic tensile strength was assumed to be 10% of the corresponding compressive strength value (assumed safety factor equal to 3, see also table 1). This variation in the mechanical properties was introduced in order to reflect a weakening of the masonry at the initial construction phases as compared to that of the later phases. These values were as good approximation as possible based on studies of a similar nature conducted for masonry monuments in Northern Greece.

A variation in the mechanical properties was introduced in order to reflect a weakening of the masonry at the initial construction phases as compared to that of the later phases. It is recognized by the authors that verification by in-situ or laboratory tests of the values listed in table 1 is highly desirable as the approximation of the results and conclusions of the present study is linked with these assumed values. Such verification can be provided by in-situ and laboratory experimental means. This has been proposed but has not been possible so far due to lack of sufficient resources.

1.2. Obtained results.

1.2.1. Dynamic analysis results.

Table 2 lists the translational fundamental period values and the corresponding modal participation ratios for this church prior to any intervention, as they resulted from the performed numerical simulation.

Tuere 20 millioual Engen i ene as ana millioual i antenap million for any meet ender							
Mode No.	Period	Individual Modal Participating Ratio %					
	(sec)	Ux	Uy	Uz			
1	0.138	0.0004	52.858	0.0001			
4	0.095	39.399	0.0000	1.8020			
Cumulative after 10 th mode		66.366	67.2162	18.6614			

Table 2. Modal Eigen-Periods and Modal Particiapting Ratios Prior to any Intervention

1.2.2. Areas of stress concentration for the church prior to any intervention.

The numerical simulation was a three dimensional representation of the whole monument, with the vertical load bearing components assumed to have base fixity at the foundation level, 600mm below ground level. The numerical study included the determination of the fundamental eigen-modes and frequencies and then through the above load combinations to identify certain areas of the load-bearing system that exhibit stress concentration that reaches critical axial compressive or tensile strength values for the stone masonry. This numerical analysis identified three distinct areas of stress concentration that are above the assumed critical strength values (figure 1). These are the following:

- 1. The internal west vertical central pier.
- 2. One of the external piers between two window openings at the South peripheral wall.
- 3. The intrados of the top vault and arches of the main church at the mid-key location.

For all these cases the worst load combination was the one including the earthquake loads.



Figure 1. Stress concentration at masonry walls for the various loading combinations

1.3. Proposed partial structural intervention for the church of the Assumption of the Mother of God at Zervati

On the basis of the above results, certain intervention measures were considered that included, among other things, the partial strengthening of the masonry with mortar injections, the removal of the inclination of the vertical elements, the maintenance of the foundation and roof system and the introduction of metal ties between the top of the interior vertical columns of the central dome system. For this type of stone-masonry the strengthening of the structural elements with low-pressure mortar injections compatible to the old masonry has been applied in many cases recently. A numerical simulation, partly accommodating these interventions, was performed in order to check if they have the

desired effect. This is demonstrated by the comparison of the corresponding stress, as found from the numerical analysis, over the assumed strength ratio values before and after the intervention.

The following is a brief description of the proposed partial structural interventions:

1. The restoration of the original roofing system with a height that is lower than the one it has today. The current roofing system appears to present stability problems and partial collapse.

Thus, a replacement of the wooden trusses of the roofing system is proposed. Moreover, the proposed intervention includes a restoration of the masonry walls at the top and their connections with the roofing system.

2. Special fluid mortar injections for the external masonry walls that primarily resist the combined actions, as was demonstrated by the performed analysis and the observed damage.

3. A system of slightly pre-stressed metal cables is proposed which will be placed at the main vaulting system supporting the central dome.

4. Special fluid mortar injections for the vaulting systems where there are signs of structural cracks.

The proposed measures can be considered as rather mild structural interventions that are fully compatible with the existing structural elements ([4], Croci 1998). The placing of the pre-stressed metal cables will be implemented in such a way as to be fully reversible. The main consequences of the proposed structural interventions are:

a. Strengthening of the roofing system and its links with the masonry structural elements.

b. A considerable decrease in the mass of the roofing system, from its reconstruction and from the lowering of its height. This will result in a reduction of the amplitude of the seismic forces.

c. An increase in the bearing capacity of the masonry structural elements resulting from the injections.

d. An increase in the stability of the dome and vaulting system from the placement of the metal cables.



Figure 2a. Longitudinal translational mode T=0.095 seconds before the intervention T=0.061 seconds after the intervention



Figure 2b. Transverse translational mode T= 0.138 seconds before the intervention T= 0.090 seconds after the intervention

Toble 2 N	Indal aigan	nomiada ana	I model meas	nortioinoting	rotion	ftor the n	i honorad i	ntomiontion
Table 5. IV	loual eigen-	Demous and	i mouai mass	Darticidating	Tatios a	нег ше р	ODOSEU I	петенноп

Mode No.	Period	Individual Modal Participating Ratio %				
	(sec)	Ux	Uy	Uz		
1	0.090	0.0002	45.0293	0.0000		
4	0.061	56.1767	0.0001	0.5474		
Cumulative after 10 th mode	62.8084	69.2670	11.4509			

1.3.1. Study of the eigen-modes for the church after the proposed intervention.

Table 3 lists the fundamental eigen-modes of the structure used in the dynamic analysis with the corresponding modal mass-participation ratios (see also table 2). Figures 2a and 2b depict the first two translational eigen-modes together with the corresponding eigen-frequencies prior to and after the

proposed intervention. As can be seen from these values, the proposed intervention resulted in a noticeable increase in the structural stiffness as well as in a decrease of the mass of the roof system. Moreover, as can be seen in table 3, with the inclusion of the 10^{th} mode, the cumulative modal mass is 63% of the total mass in the x-x (longitudinal) direction and 69% of the total mass in the y-y (transverse) direction.



Figures 3a and 3b. The performance of the masonry (East Columns and vaulting) prior to any intervention.

1.4 Discussion of results for the church prior and after the proposed intervention.

The predicted performance, in terms of numerically obtained stress resultants are combined in an interaction diagram of the normal forces and bending moments that also includes an envelop curve to reflect a compression-flexure failure criterion. Predicted values plotted outside this envelop curve signify compression-flexure failure. Figures 3a and 3b are selected results of structural elements prior to any intervention whereas figures 4a and 4b are the corresponding results, as predicted by the performed numerical analysis, after the proposed intervention.

1.4.1. Prior to any intervention.

- The performed 3-D linear elastic analysis provided a numerical simulation of the stone masonry structure of the church of the Assumption of The Mother of God at Zervati that seems to be quite realistic, given the limitations of the performed analysis. Confidence in the obtained results is provided by the reasonably successful prediction of the observed failure locations of the vaulting system.

- The performed analysis does not predict the inclination of the two central circular columns supporting the dome or the extensive inclination of the columns of the South portico. This must be attributed to the accumulation of the effect of the soil settlement that is amplified due to the foundation of these slender structural elements.

- The performed comparison between numerically predicted and allowable stress values was based on assumed values for the limit mechanical characteristics of the analyzed structure. The validity of the performed analysis would be further enhanced if such allowable stress values were based on measured mechanical properties.

- Due to the absence of measured mechanical properties, the only failure criterion that was applied was that of the exceeding of the normal tensile stress. It is of interest to be able also to apply a realistic shear stress failure criterion.



Figure 4a and 4b. The performance of the masonry (East Columns and vaulting) after the proposed intervention.

1.4.2. After the proposed intervention

In figures 10 and 11are depicted the M-N interaction diagrams of the interior columns (figure 10) either at the springing or the crown of the vaults (figure) based on predicted demands as they resulted from the numerical analysis of the church after the proposed intervention, as discussed in sections 4.1. and 4.2. These values of the predicted demand are plotted as distinct points in these figures for the particular locations and load combinations. The assumed available strength is depicted in these figures by the M-N interaction curve. As can be seen by comparing the plotted demand values against the available strength (figures 3 and 4) whereas prior to any intervention this approach predicted failure in a number of critical locations the proposed intervention succeeded in changing the predicted performance of the masonry elements as no failure is predicted in any location. This is valid for both the interior columns as well as the vaulting system, where failure was predicted prior to any intervention.

2. NUMERICAL ANALYSIS OF AGIA TRIADA AT THE PREFECTURE OF KARDITSA.

In this section part of the numerical study of the behaviour of the church of the Agia Triada of the village of Drakotrypa of the prefecture of Karditsa in Greece will be presented (Photo 2). This church dates from the 1742 and is a single-nave basilica with a central dome and a transverse nave together with 3 apses at the sides of the church. This church belongs to the post-Byzantine type of construction. The most visible signs of damage are: a) Formation of cracks at the keys of the vaults and arches, b) East-West cracks of the vaulting system parallel to the longitudinal axis of the church.



Εικ. 3. Ναός Αγίας Τριάδος στην Δρακότρυπα. Ανατολική Όψη. Ιερό.

Photo 2. The Church of the Holy Trinity in Drakotrypa. East Elevation. Sanctuary.

The numerical simulation that was adopted utilized thick beam elements -, and three-dimensional shell elements that are provided in the Lusas 13.3 software package. Two types of load combinations were examined. The first included **1.35G + 1.5Q** and the second $G + 0.3Q \mp E$ where G the permanent load, Q the snow and live load and E the earthquake load. The value for the live load (Q) was assumed to be zero. The dead load for the limestone was assumed 24.5KNt/m3. The roof weight was assumed 1.55KNt/m2. The material properties adopted in this analysis are a) Young's Modulus 2120 Nt/mm2, b) Poisson's ratio 0.2, c) Stone Masonry Compressive Strength 2.12 Nt/mm2. d) Stone Masonry Tensile Strength 0.1 Nt/mm2.

2.1. Study of the eigen-modes- Gravitational and Earthquake load behaviour

The numerical study included the determination of the fundamental eigen-modes and frequencies for 5 different numerical simulations in order to determine the contribution of common structural elements (tympana, apses, openings) to the dynamic behaviour of this church (Table 4).

The fifth model (Model 5 in table 4), which includes the 3 apses and the openings of the peripheral walls and central dome, represents a closer resemblance to the real structure. This fifth model was utilized to predict the behaviour of this church under gravitational and earthquake loads.



Table 4. Modal eigen-periods and mass participating ratios for 5 numerical simulations

Model 3 with 1 apse	Transverse translational mode			Longitudinal translational			
without tympana	T=0.124 seconds mode $T=0.096$ second					nds	
	Individual Modal			Individual	Modal		
	Participating Ratio %			Participati	ng Ratio %)	
	Ux	Uy	Uz	Ux	Uy	Uz	
	0.0000	54.60	0.0000	32.41	0.0000	0.071	



2.1.1. Variation of eigen periods.

- The influence of tympana is studied, in the comparison of periods obtained from Model 1 and Model 2. The addition of the tympana resulted in a decrease of 8% and 7% in the corresponding eigen-period values for the two dominal modes.

- The influence of including an apse in one direction is demonstrated, by the comparison of Model 1 and Model 3. The translational mode though, in the longitudinal direction is only marginally affected by the inclusion of the apse. The small percentage of mass participation in this case is possibly a reason behind this observation. The mass participation is 51.21% for the case of model without apses and without tympanum while the mass participation in the same mode is 32.41% for the model with 1 apse without tympana

- By including apses in both directions as shown in Model 4 the eigen-periods obtained are 6.5% less than that predicted by the Model 1 for the transverse mode, but for the longitudinal mode the eigen-periods predicted by Model 4 are 6.5% higher than the ones predicted by Model 1.

- Comparing Model 4 with Model 3 a noticeable reduction in the eigen-period in the transverse direction can be seen. Whereas, the opposite is true for the eigen-period in the longitudinal direction.

- Finally, in Models 4 and 5 the influence of the openings is studied. The structure with the openings (Model 5) results, as expected, in higher values of eigen-periods in both directions as compared to the eigen-period values without the openings.

The inclusion of the 10^{th} mode, for model 5, gives the cumulative modal mass equal to 58.39% of the total mass in the x-x (longitudinal) direction and 57.43% of the total mass in the y-y (transverse) direction. In order to account for the total mass, without the inclusion of eigen-modes further than the 10^{th} , the dynamic analysis results for the earthquake loads in the x-x and y-y directions were amplified by the inverse of the cumulative modal mass participation ratios in the corresponding directions (fxx = 100/58.39 = 1.7126, fyy = 100/57.43 = 1.741).



2.2. Numerical predictions of the deformation and state of stress for the church of Agia Triada.

The design spectrum, as defined for seismic zone III of the Greek seismic code was employed in defining the earthquake loads. The ground design acceleration was taken equal to 0.24g; it was assumed that the local soil conditions belong to soil category B. An importance factor equal to 1.3 was adopted and a load reduction coefficient equal to 1.5 was also assumed (structural elements made of stone masonry). The deformed shape for the two load combinations, the one including only 1.35G + 1.5Q, and the other including the $G + 0.3Q \mp E$, in the x-x (longitudinal) direction can be depicted in figure 5a and 5b. 2.2.1. Obtained results.

The five different numerical models, presented before, were utilized to obtained results for the deformed shapes and the distribution of stresses produced by the in-plane and out-of plane resultant forces in all the structural members of the studied monument (Dome, Cylindrical Vaults, Walls and Apses). This was done for all load combinations and produced a large volume of numerical results. The presentation and discussion in this paper will be confined to the numerical results of model 5 (with apses and openings) and for the load combination that included the gravitational forces together with the earthquake loads. This is shown in terms of the deformation pattern (figure 5b) predicted by the performed elastic analysis together with the in-plane normal stress resultant distribution (figure 6, Ny - vertical direction) and the in-plane shear stress resultant distribution (figure 7, Nxy).



Figure 6. Normal Stress distribution for the combination of permanent and earthquake loading.



Figure 7. Shear Stress distribution for the combination of permanent and earthquake loading.

The main areas of stress concentration for the normal and shear in-plane stress resultants can be identified in these figures. Due to the fact that the most demanding load combination is the one that includes the earthquake loads, areas of stress concentration appear on the external walls and apses. As a next step in this investigation the amplitude of these peak stresses, as they were predicted from the numerical analysis, was examined against certain simplified failure criteria.

2.2.2. Predicted zones of overstress

The failure criteria that were assumed to be valid are: a) That of exceeding the allowable compressive and tensile strength of Masonry, 2.12 Nt/mm2 and 0.1 Nt/mm2 respectively. b) That of exceeding the shear strength of stone masonry. Following the proposed revisions to Eurocode-6 Nov 2002 [5] the shear failure criterion that was adopted is:

$$f_{vk} = f_{vko} + 0.4 \sigma_n$$
^[1]

where : f_{vko} is the shear stress of the stone masonry when the normal stress is zero; it was assumed to be equal to 0.1 Nt/mm2. σ_n is the normal stress

Figure 8 presents zones of stress concentration where a comparison between the numerically predicted and the allowable tensile and shear stresses was performed for the church of the Agia Triada according to the previously stated failure criteria (section 5). The gravitational and earthquake loads were applied simultaneously. As can be seen, in this figure the employed failure criteria predict overstress at zones 2 and 4.



Figure 8. Comparison of predicted tensile and shear stresses to allowable tensile and shear strengths

As already stated a tensile failure criterion was assumed together with a failure criterion in shear. Failure due to exceeding the allowable tensile stress is predicted when the level of tensile stress is greater than 0.1 Nt/mm2.

Areas where tensile failure is predicted are:

- At the crown of the western longitudinal vault, as shown by zone 4a in figure 8.
- At the mid-key location of the arches of the cross vault in the transverse direction (zone 4b).

- At the apse which is positioned in the longitudinal direction, below the lower window opening (zone 4c).

- At the central dome below the openings and above the transverse vault (zone 4d).

- At the base of the pier near the transverse apse, in the east part of the longitudinal peripheral wall (zone 4e).

- Areas where shear failure is predicted are the ones where the shear stress values are higher than the ones predicted by equation 1. Such area exists at the piers of the west and east part of the longitudinal peripheral wall (zone 2).

- At zone 3, the tensile stresses that are developed are less than the tensile strength of masonry. However, these are areas of possible failure formations due to exceeding of tensile strength.

3. CONCLUSIONS

1. Based on the results of the performed numerical simulations certain intervention measures have been proposed for the church of the Assumption of The Mother of God at Zervati, through an increase in the stiffness of the structure, areas of over-stress are avoided.

2. As expected, for the church of Agia Triada the tympana, apses, and window openings have a noticeable influence in the stiffness of the structure as was shown by the numerical investigation.

3. The failure criteria considered in the study for the church of Agia Triada predicted zones of over-stress that are in agreement with the observed failure locations only at the mid-keys of the arches as well as, at the longitudinal axis of the western vaulting system(zones 4).

4. This analysis also predicted for the church of Agia Triada tensile and shear failure at the peripheral wall which is not correlated by the observed damage (zones 4 and zones 2, respectively).

REFERENCES

1. Manos G.C. et.al. "Correlation of the observed earthquake performance of the church of St. Constantine in Kozani-Greece with corresponding numerical predictions", STREMA97, San Sebastian, Spain, pp. 309-320.

2. Manos G.C. et.al. Earthquake Performance of the Church of St. Constantinos in Kozani-Greece. Correlation of the Observed Damage with Numerical Predictions", 4th Intern. Symposium on computer methods in Structural Masonry, Sept.1997,Florence,Italy.

3. L. Papas and G.C. Manos "A Numerical Study of the Church of the Assumption of The Mother of God (Koimiseos tis Theotokou) at Zervati of the District of Argyrocastro", Computer Methods in Structural Masonry – 5, Rome Italy, 2001, pp. 198-205.

4. Croci, Giorgio "The Conservation and Structural Restoration of Architectural Heritage", Advances in Architectural Series, "Computational Mechanics Publication, ISBN 1 85312 4826, 1998.

5. Proposed Revisions to Eurocode-6 Nov 2002.