13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 936

SEISMIC RESPONSE OF STRENGTH AND STIFFNESS DEGRADING SINGLE DEGREE OF FREEDOM SYSTEMS

Hasan A. PEKOZ[1] and José A. PINCHEIRA[2]

SUMMARY

A parametric study was conducted to investigate the effect of strength and stiffness degradation on the seismic response of single-degree-of-freedom systems. Two force-deformation models that included the effects of pinching, strength and stiffness degradation with increasing deformation amplitude and upon reversal of loading cycles were considered. Displacement amplification factors were calculated for oscillators with several strength ratios and degradation levels using sixty ground motions. Based on statistical and regression analyses, a simple equation for estimating the displacement amplification factor is proposed. The data show that the proposed equation represents a reasonable upper bound to the displacement amplifications for bilinear and strength degrading systems.

INTRODUCTION

In older building construction, where significant strength and stiffness degradation can be expected during a strong earthquake, lateral displacements can be much larger than those of new buildings, composed of strong, ductile members. Past studies on inelastic displacement estimates have focused mainly on elasto-plastic, bilinear or stiffness-degrading systems. Recent examples of such studies include Riddell and Garcia [1], Cuesta et.al. [2], Shimazaki and Sozen [3], Qi and Moehle [4], Miranda [5], Riddell et.al. [6] and Lepage [7]. The influence of strength degradation, which is typical for older construction, has been included only in few studies (Gupta and Kunnath [8], Gupta and Krawinkler [9], Song and Pincheira [10] and Al-Sulaimani and Roesset [11]). Although simplified methods for estimating inelastic displacement demands of the degrading systems have been recently proposed in the U.S. (FEMA 356 [12]), data to reliably estimate spectral displacement demands of strength and stiffness degrading systems are very limited.

In this paper, the results of a parametric study to quantify the influence of strength and stiffness degradation on the seismic response of SDOF systems are presented. The work presented here builds up on an earlier study (Song and Pincheira [10]) and is expanded to include two different force-deformation models and a larger earthquake ground motion database. A total of sixty ground motions recorded around

[1] Graduate Research Assistant, Department of Civil and Environmental Engineering, University of Wisconsin, Madison, WI, USA.
[2] Associate Professor, Department of Civil and Environmental Engineering, University of Wisconsin, Madison, WI, USA.

the world on various soil types were used. Near-fault records with known forward directivity effects were not included in this part of the study. Oscillators with a wide range of structural periods, 5 percent damping, and six strength ratio levels were considered. Statistical and regression analyses of the data were used to identify common trends and to develop a simplified expression for the calculation of the displacement amplification factor for strength and stiffness degrading SDOF systems.

DESCRIPTION OF THE MODELS

Two degrading, force-deformation models (namely model 1 and model 2) were considered in this study. These models consisted of a multi-linear force-deformation relation that included stiffness and strength degradation with increasing deformation amplitude and with repeated cycles of load reversals. The backbone curve used for both models is shown in Figure 1. It consists of a bilinear ascending branch until the maximum strength, F_u, is reached. After this point, the system continues to carry additional deformation with decreasing strength at a constant rate until a residual capacity, F_r, is reached.

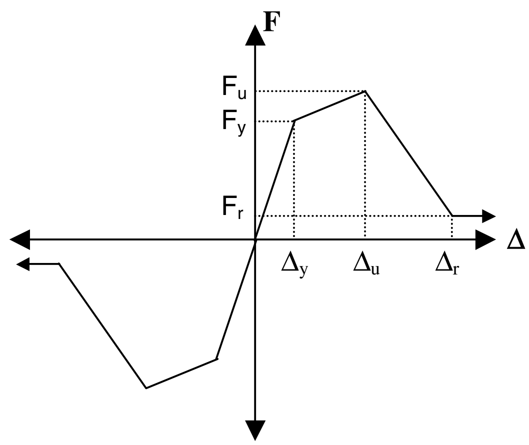


Figure 1. Force-deformation relationship of the degrading models

Figure 2 shows the main characteristics of the hysteretic laws for both degrading models. The laws account for the effects of pinching of the hysteresis loops, and strength and stiffness degradation with increasing deformation amplitude and upon reversal of load cycles. In model 1, however, it is assumed that once the system has reached its strength, F_u, in one direction, then it cannot be reached in the opposite direction. For example, the loading history depicted in Figure 3a shows that after unloading at A, reloading is directed toward A′, the symmetric of point A. Additional strength decay can be introduced in the model by reloading toward point A″ instead. Further details of the hysteretic laws can be found elsewhere (Song and Pincheira [10]). The reloading rule shown in Figure 3a may be considered too conservative, and may lead to inelastic displacement estimates larger than the actual displacements induced for some degrading systems. For this reason, it was decided to modify this rule so that the lateral resistance in one direction was independent from the loading history in the opposite direction. Accordingly, in model 2, reloading is directed toward the previous unloading point (point B′ in Figure 3b) instead. The actual behavior of older systems probably lies in between these two models.

Selection of Parameter Values

In this study, the strength of the oscillators was characterized by the strength ratio, η, defined as the yield strength of the system divided by the mass of the system multiplied by the peak ground acceleration (PGA) of the ground motion. This definition of strength ratio is related to the yield coefficient, C_y (yield strength divided by the weight of the system) as follows:

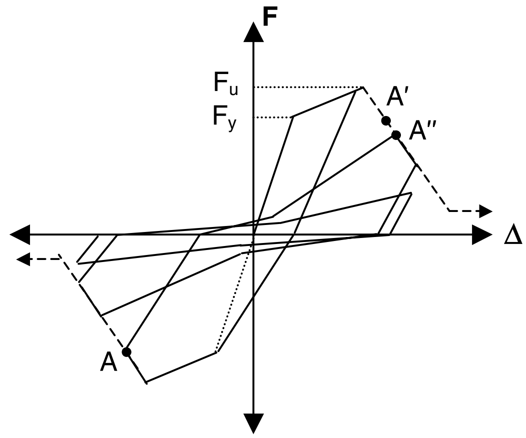


Figure 2. Main characteristics of the hysteretic laws

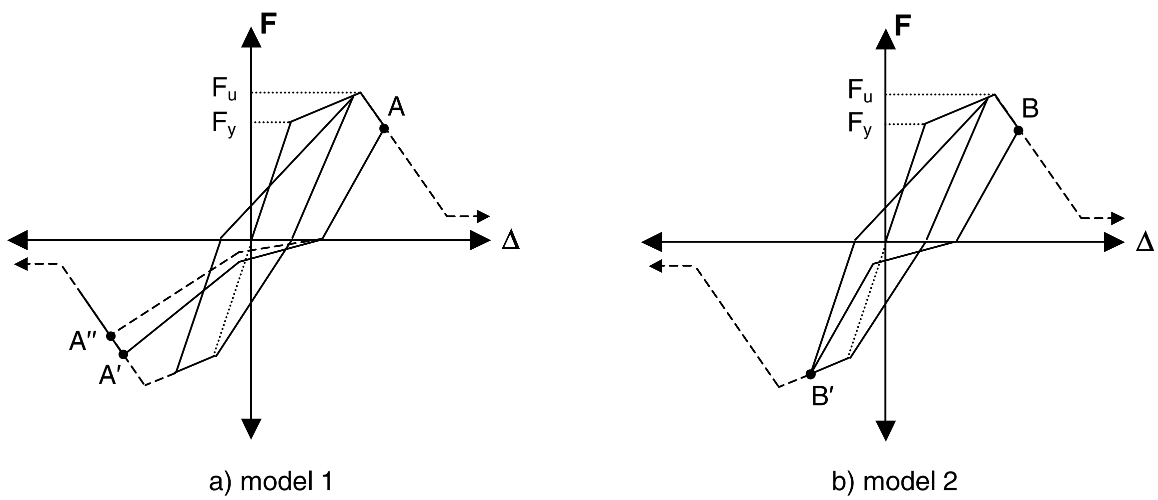


Figure 3. Reloading rules in models 1 and 2

$$C_y = \eta * PGA\ (g) \qquad (1)$$

In other words, for a constant η, the strength of the oscillator depends on the severity of the ground motion as measured by the peak ground acceleration. In the analyses, strength ratios between 0.1 and 0.6 using increments of 0.1 were considered for the oscillators. For a typical ground motion with a peak ground acceleration of 0.4g, these strength ratios correspond to yield strengths between 4 and 24% of the weight of the system, which is in the range of practical interest. The post-yield stiffness was taken to be 5% of the initial elastic stiffness, while the deformation corresponding to the peak strength, Δ_u, was assumed to be 25% larger than the yield displacement, Δ_y. A residual strength level, F_r, equal to 10% of the initial yield strength of the system was assumed.

The parameters that control the strength and stiffness degradation of the systems were varied to consider various decay levels which were labeled as low, moderate and severe. In addition, a bilinear system with no strength decay was included in the study for comparison purposes. Figure 4 illustrates the force-deformation response of all the systems considered here. These lateral load-deformation plots were

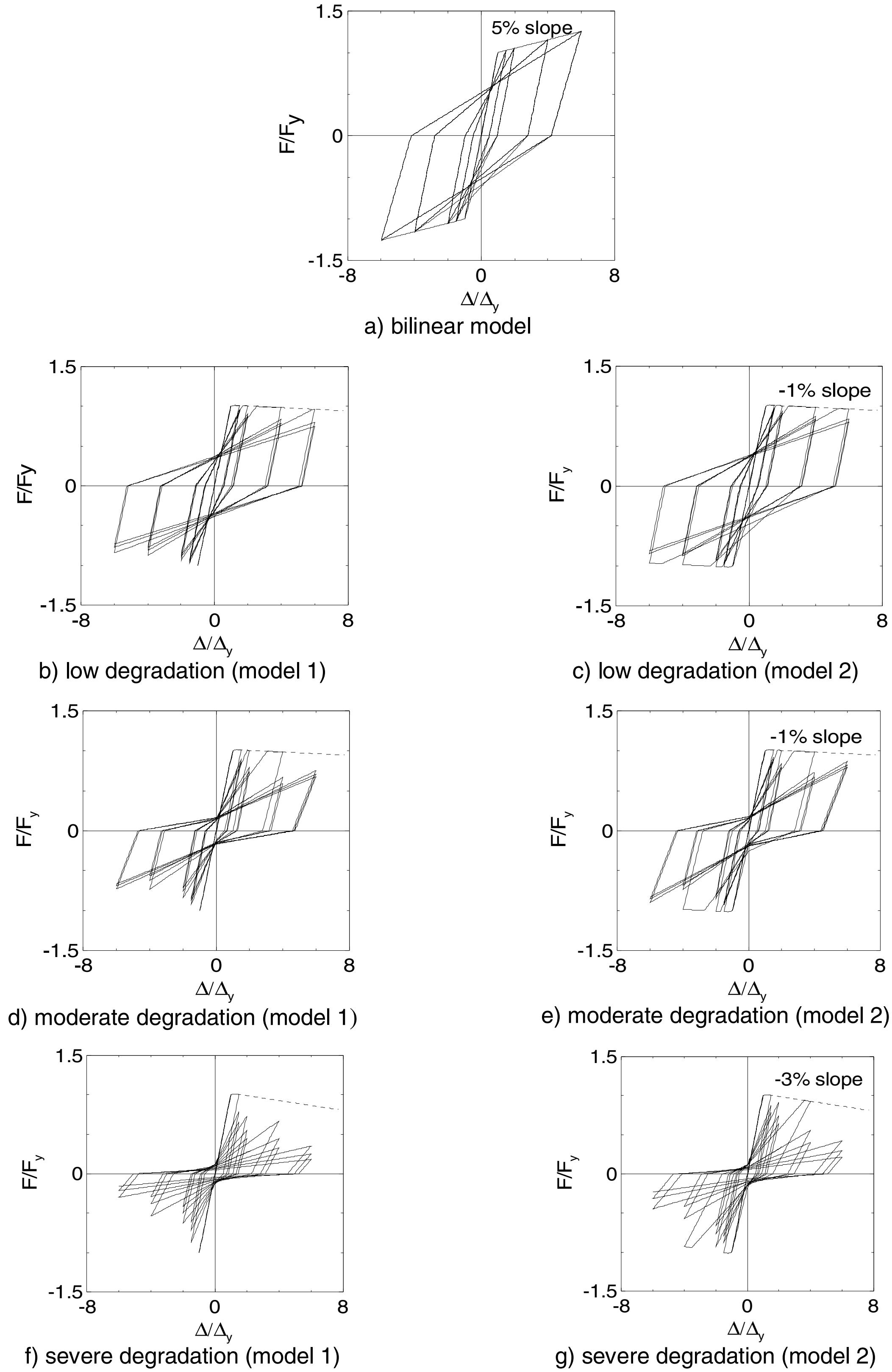


Figure 4. Response characteristics of oscillators without strength decay and with low, moderate and severe degradation levels using models 1 and 2

obtained using a reversed cyclic loading regime that has three cycles at deformation amplitudes of 1.5, 2, 4 and 6 times the yield displacement. The bilinear model considered here has a post-yield slope of 5% of the initial elastic stiffness and no pinching. The low and moderate degrading models have a post-peak stiffness of -1% of the initial elastic stiffness and about 22% strength decay at a displacement ductility of 6. The severe degrading model has, on the other hand, a post-peak slope of -3% and about 80% strength decay at the same displacement ductility. In addition, the shape of the hysteresis loops is different for each degrading model. As can be seen in Figure 4, the amount of pinching increases and the unloading stiffness decreases as the degradation level increases.

GROUND MOTIONS

A total of 60 ground motions recorded in the United States and in other parts of the world were used in this study. The ground motions were recorded on soil types B, C and D (20 records in each soil type) as defined by FEMA 356 [12]. Soil type B motions had peak ground accelerations varying between 0.11g and 0.69g with an average value of 0.28g, whereas motions recorded on soil type C had peak ground accelerations in the range of 0.21g and 0.94g with an average value of 0.48g. Peak ground accelerations for soil type D motions, on the other hand, varied between 0.23g and 0.84g, with an average value of 0.51g. The ground motions were obtained from different sources including the Pacific Earthquake Engineering Research Center (http://peer.berkeley.edu/smcat) and the Consortium of Organizations for Strong-Motion Observation Systems (COSMOS) Virtual Data Center (http://db.cosmos-eq.org/). Further details of the ground motions used in this study can be found elsewhere (http://www.cae.wisc.edu/~jpin/US_Japan/Index.htm).

In Figure 5, the mean value of the elastic response spectra of the ground motions recorded on soils B and D are compared with the BSE-2 (Basic Safety Earthquake 2) design spectrum of FEMA 356 [12] for a location in the San Francisco area. Also shown in the figure are the spectra corresponding to the mean plus- and minus-one-standard-deviation of the ground motions. In most areas of the United States, a BSE-2 hazard level has a 2% probability of exceedance in 50 years. However, in regions of high seismicity, the seismic hazard is generally due to large-magnitude events occurring on known faults. In such areas, the BSE-2 hazard level is calculated by a deterministic approach based on 150% of the median estimate of the expected ground motion. As can be seen from Figure 5, the elastic response spectra for soil types B and D compare well with the spectra corresponding to the BSE-2 hazard level for short periods. However, for longer periods, they are somewhat lower than that corresponding to the BSE-2 hazard level. The ground motions recorded on soil type C showed similar trends.

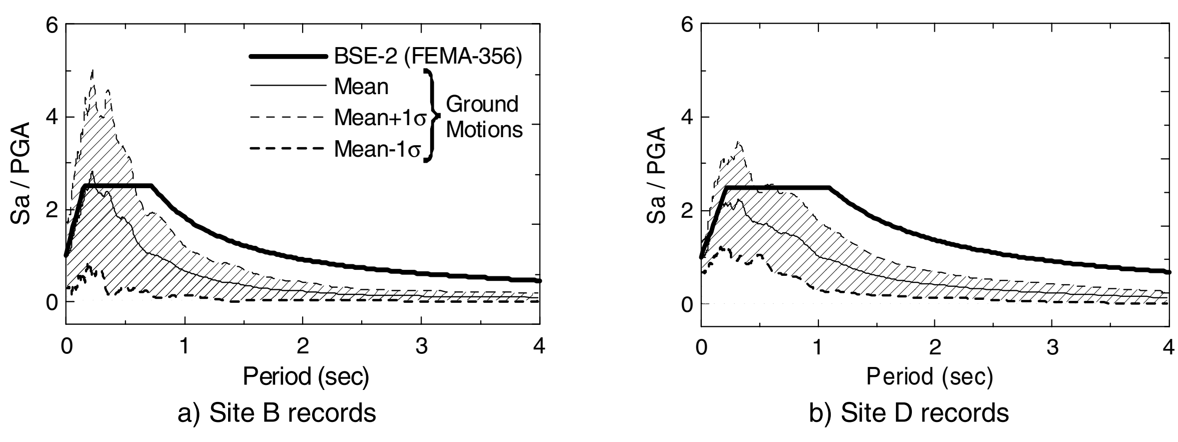


Figure 5. Comparison of mean elastic response spectra for motions on sites B and D with NEHRP BSE-2 design spectra for a location in the San Francisco area

Characteristic Period (T_g)

Past studies have shown that for a given ground motion, the maximum inelastic displacement of the system is bounded by the elastic displacement for vibration periods longer than a characteristic value. In contrast, the inelastic displacement of the system is larger than that of an elastic system for periods shorter than the characteristic value (Shimazaki and Sozen [3], Qi and Moehle [4]). While most researchers agree with this observation, different calculation procedures for the characteristic period exist (Shimazaki and Sozen [3], Qi and Moehle [4], Song and Pincheira [10] and Miranda [13]). It must be noted that most of these studies were conducted on non- or stiffness-degrading systems and thus it is uncertain whether these observations would also apply to degrading oscillators. Song and Pincheira [10] showed that the trends mentioned above were in general agreement for strength degrading systems as well, but their study considered a small number of records.

In this study, the characteristic period, T_g, was defined as the period corresponding to the maximum value of a 5% damped, elastic input energy spectrum. Figure 6 shows the calculated displacement amplification factor (DAF - ratio between the inelastic and elastic displacement) as a function of the ratio between the oscillator period, T, and the characteristic period, T_g, calculated in this manner. The figure shows the results for oscillators with a strength ratio, η, of 0.4 subjected to the motions on soil types B and D using force-deformation model 1 with two degradation levels (see Figure 4). It can be seen that nearly all the ground motions resulted in displacement amplifications less than or equal to 1.0 for $T/T_g \geq 1.0$ with only a few exceptions. This trend was typical and also observed for other degradation characteristics and the other records considered in this study. Therefore, it was concluded that the definition of T_g used here provided a good estimate of the characteristic period for SDOF systems with strength and stiffness degradation.

ANALYSIS OF RESULTS

Displacement response spectra and DAFs for oscillators with T/T_g values ranging between 0.06 and 3.0, and six different strength ratios between 0.1 and 0.6 were computed using the hysteresis models described earlier (see Figure 4) for all sixty earthquake records. A time integration step of 0.005 seconds and a viscous damping ratio of 5% were used in all of the analyses. Mean and mean-plus-one-standard-deviation DAFs were calculated for each soil type, degradation level and force-deformation model to identify trends and to develop an approximate equation.

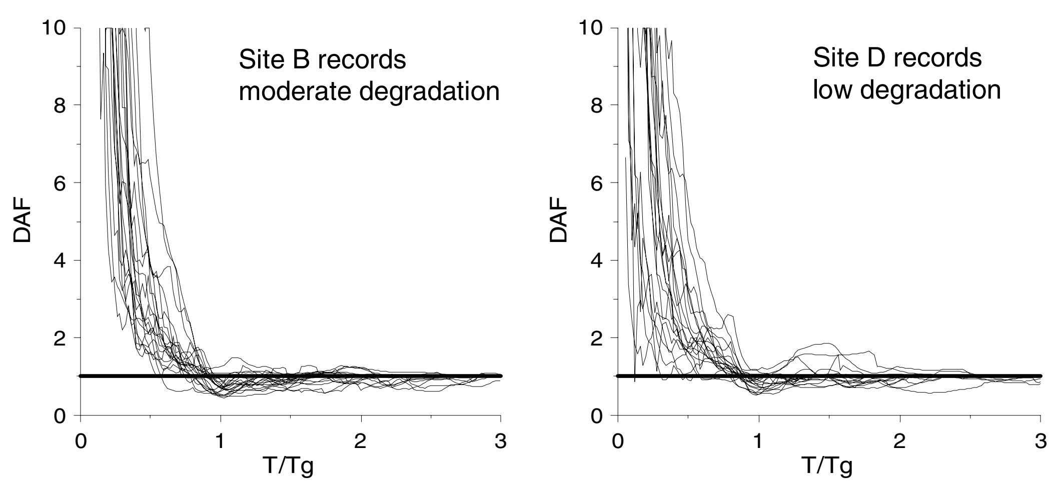


Figure 6. DAFs for systems with η of 0.4 using force-deformation model 1

Influence of Degrading Model Type

Figure 7 shows an example of the force-deformation response obtained with degrading models 1 and 2 under the N90E component of the Vannuys (Holiday Inn hotel) record measured during the 1994 Northridge earthquake. The oscillator had a period of vibration of 1.0 sec., a strength ratio of 0.3, and a moderate decay level (see Figs. 4d and 4e). It can be seen that the maximum ductility demand obtained with model 2 was almost 40% larger than that obtained from model 1. Previously, it was indicated that the reloading rule of model 1 represented a system with high decay level that could lead to spectral displacements larger than those with model 2. However, Figure 7 suggests that in some instances, model 2 may, in fact, yield larger displacements.

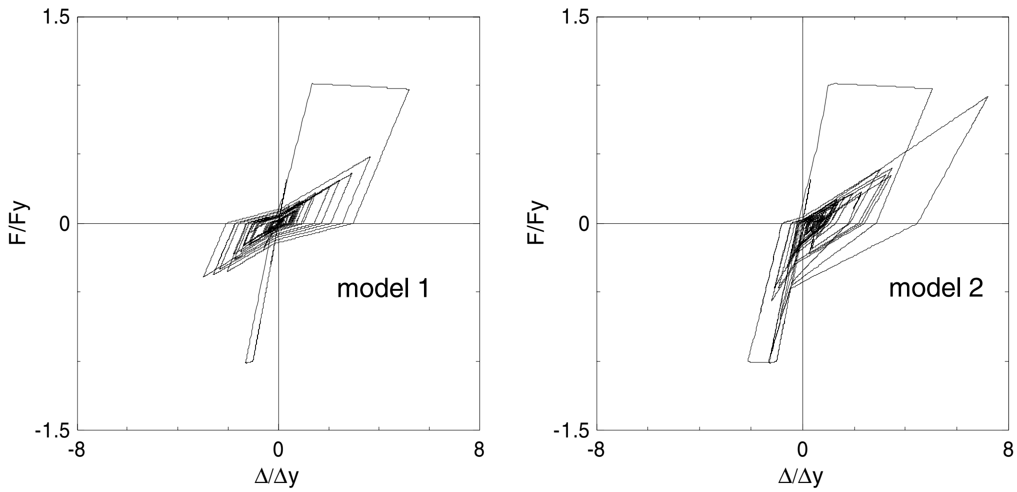


Figure 7. Force-deformation responses obtained with models 1 and 2 subjected to the Vannuys record (N90E component) (T = 1.0 sec, η = 0.3)

Figure 8 shows the DAFs obtained for oscillators with different periods of vibration and a constant strength ratio of 0.3 subjected to the N00E component of the Sylmar (County hospital parking lot) and the N90E component of the Vannuys (Holiday Inn hotel) motions, both recorded during the 1994 Northridge earthquake. Moderate degradation parameters were used for both degrading models (see Figures 4d and 4e). The displacement amplifications for bilinear systems with no strength decay (see Figure 4a) are also shown in the figure. For the Vannuys record, the DAFs tended to be larger for model 2 over most of the period range between $0 < T/T_g < 1$. For the Sylmar record, however, displacement amplifications were larger for model 1. The figure also shows that, on average, oscillators without strength decay yielded smaller DAFs than those obtained using either degrading model. These results were typical for other records, but they varied depending on the strength ratio considered.

Figure 9 shows the ratio between the DAFs computed with model 2 (DAF-m2) and those computed with model 1 (DAF-m1) obtained for the Sylmar record as a function of the period of the oscillator and the strength ratio, η. As above, moderate degradation parameters were used for both models. In the figure, a DAF ratio greater than 1.0 indicates that the inelastic displacement obtained with model 2 is greater than that obtained with model 1, and vice-versa. The data show that for low strength ratios, the inelastic displacements obtained with model 2 are lower than those obtained with model 1. For larger strength ratios, however, the opposite is observed. For very flexible systems and large strength ratios, the response is elastic for this record.

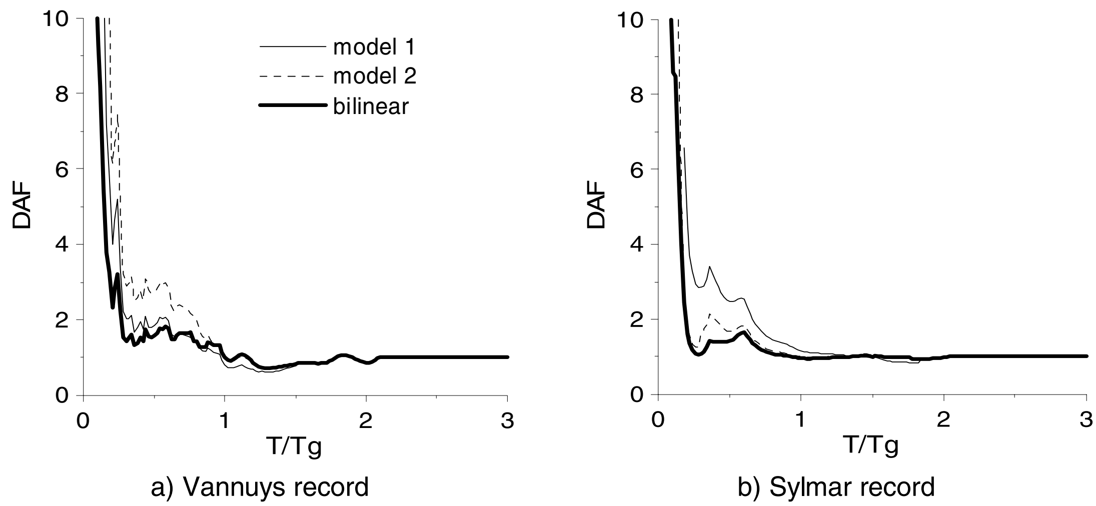


Figure 8. Comparison of DAFs obtained with models 1 and 2, moderate degradation, and η of 0.3 under Sylmar and Vannuys records

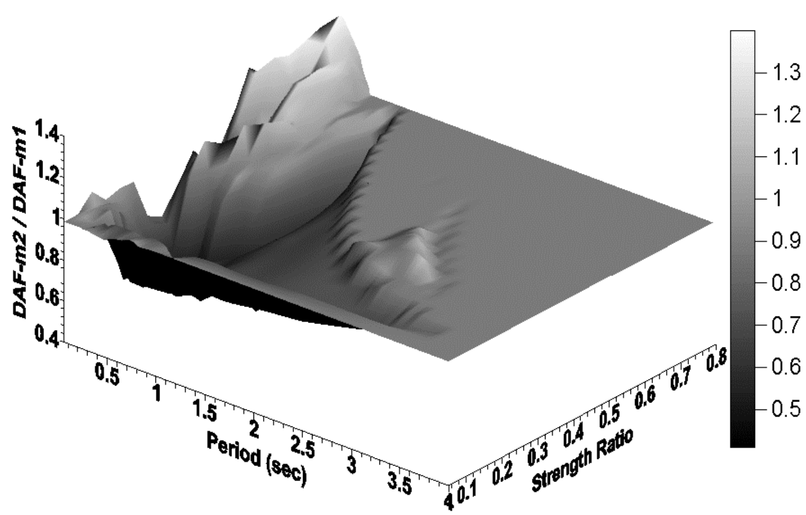


Figure 9. Ratio of DAFs obtained with models 1 and 2 for the N00E component of the Sylmar – County Hospital record

The data shown in Figures 8 and 9 were typical for the other records considered in this study and show that the choice of the force-deformation model can have a significant effect on the DAF, especially for short periods. These results also show that neither degradation model yielded consistently larger displacement amplification factors and suggest that both models should be considered for estimating the inelastic displacement of degrading systems.

Figure 10 shows the mean and mean-plus-one-standard-deviation DAFs obtained for oscillators with a strength ratio of 0.3 subjected to ground motions on soil types B and D, using both models with moderate degradation parameters. The data show that although the DAFs obtained for a given ground motion can vary significantly depending on the degrading model used (see Figures 8 and 9), the mean and mean-plus-one-standard-deviation values are, however, very close. The data obtained for other ground motions

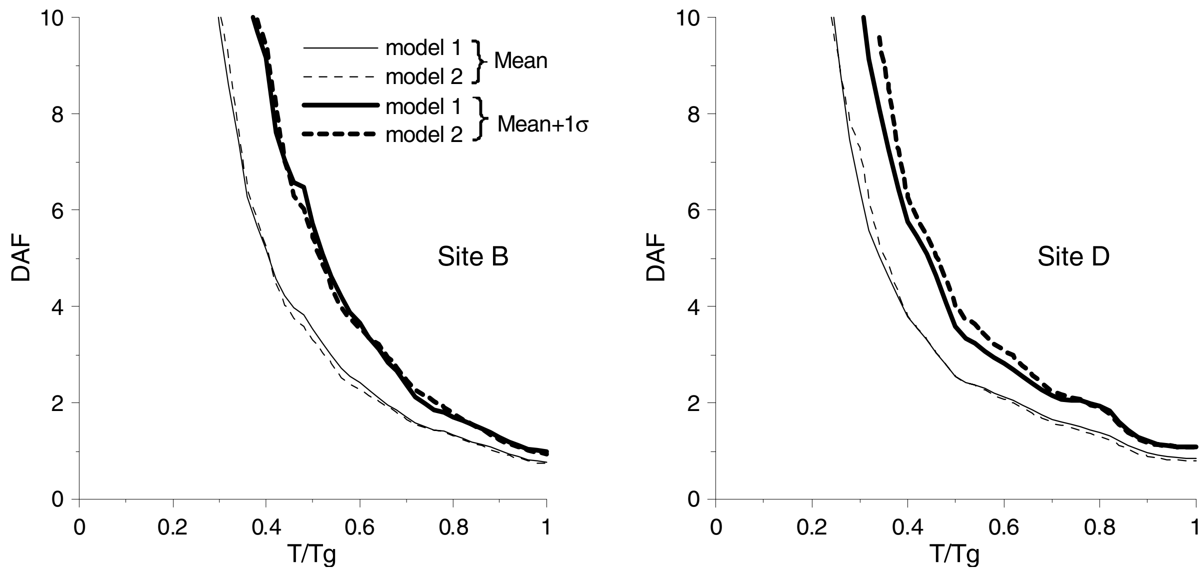


Figure 10. Comparison of the mean and mean-plus-one-standard-deviation DAFs for models 1 and 2 with moderate degradation level (soil types B and D, η= 0.3)

recorded on soil type C, other degradation levels and strength ratios yielded similar results. Overall, the mean and mean-plus-one-standard-deviation of the data were not highly sensitive to the degrading model considered.

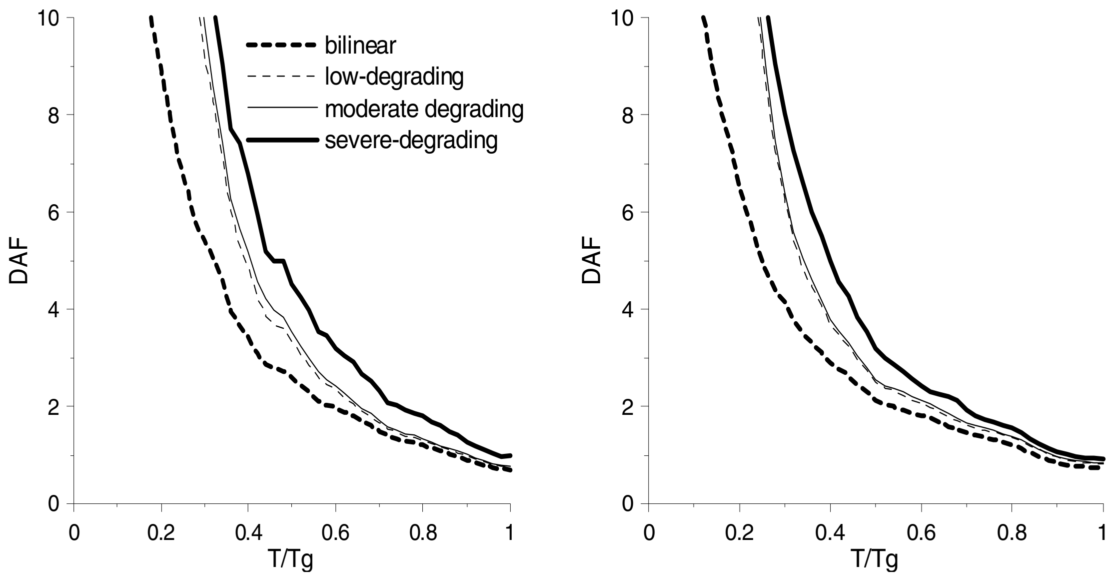
Influence of Strength Decay Level

Figure 11 shows the relationship between the displacement amplification factors and degradation level of the system. In this figure, the calculated mean for the DAFs obtained with force-deformation model 1 and a strength ratio of 0.3 are illustrated for oscillators with four degradation levels subjected to ground motions on soil types B and D. It can be seen that mean values are in accordance with the expectation that as the degradation level increases, the DAFs should also increase. The same trends were observed for other strength levels and for force-deformation model 2. It must be mentioned, however, that the DAFs for some records decreased with increasing levels of degradation over a portion of the period range.

Figure 11 also shows that the mean and mean-plus-one-standard-deviation DAFs obtained for low and moderate decay levels (Figures 4b, 4c, 4d and 4e) are nearly the same, while for a severe decay level (Figures 4f and 4g), they are consistently larger. This result suggests that the shape of the hysteresis loops does not have a significant effect, and that the post-peak slope (-1% for low and moderate degradation and -3% for severe degradation – see Figure 4) strongly influences the mean and mean-plus-one-standard-deviation DAFs.

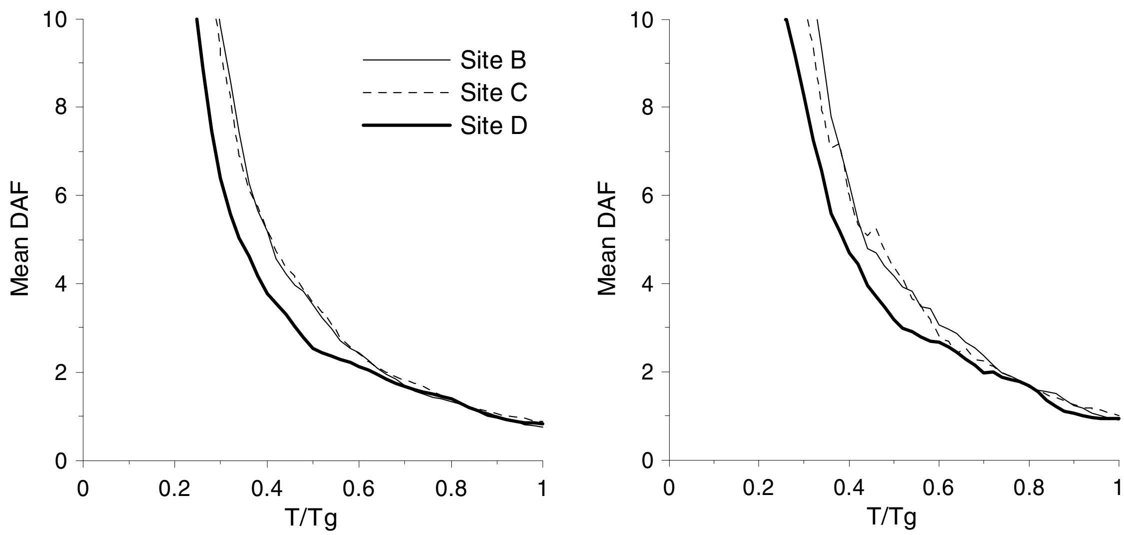
Influence of Soil Type

Figure 12 compares the mean values corresponding to the DAFs obtained for oscillators with a given strength and degradation level subjected to ground motions recorded on soil types B, C and D. It can be seen that the mean DAFs obtained for motions on soil type B and soil type C (40 ground motions in total) were practically the same. However, the mean curve for soil type D motions was significantly different for T/T_g ratios less than about 0.8. The same trends were observed for other decay parameters and strength levels, and for the mean-plus-one-standard-deviation of the data. These results suggest that the mean response for soil types B and C motions could be approximated by a single relationship. The response obtained for soil type D motions, on the other hand, would require a separate relation.



a) Site B, η = 0.3 b) Site D, η = 0.3

Figure 11. Comparison of mean DAFs for different degradation levels (model 1)

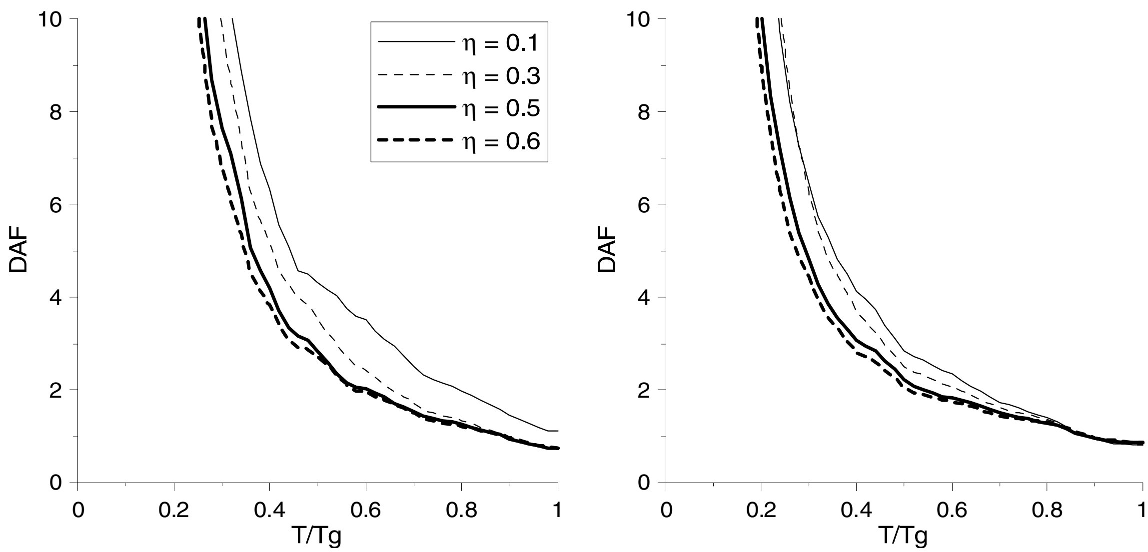


a) moderate degradation, η = 0.3 (model 1) b) severe degradation, η = 0.4 (model 2)

Figure 12. Comparison of mean DAFs for a given strength ratio, degradation level and force-deformation model obtained for motions on soil types B, C and D

Influence of Strength Ratio

The dependency between the displacement amplification factors and strength ratio of the system is illustrated in Figure 13. In this figure, the calculated mean for the DAFs obtained using force-deformation model 1 with strength ratios of 0.1, 0.3, 0.5 and 0.6 are shown for two different sets of decay parameters and ground motions recorded on soil types B and D. Although the DAFs at a given period for individual records showed that they can increase or decrease with strength, the DAFs corresponding to the mean or mean-plus-one-standard deviation shown in Figure 13 agree with the notion that as the oscillator strength increases, the DAF decreases.



a) moderate degrading, Site B records   b) low degrading, Site D records

Figure 13. Comparison of mean DAFs for different strength ratios (model 1)

Approximate formulation for DAFs

Based on the observed dependency of the DAF with respect to the decay level (Figure 11), soil type (Figure 12) and strength ratio (Figure 13), the following relationship is proposed:

$$DAF = c\left(\frac{T}{T_g}\right)^{-\frac{a}{\eta^{b}}} \qquad (2)$$

where a is a coefficient that accounts for the soil type and b is a factor that considers the decay level. The coefficient c corresponds to the displacement amplification factor for a T/T_g ratio of 1.

Given the uncertainty that often exists in estimating the response of existing structures and in the characteristics of the expected ground motions at the site, the use of mean-plus-one-standard-deviation rather than mean values seems prudent for seismic evaluation purposes. Based on a regression analysis of the data, the coefficients in Equation 2 were calculated using the mean-plus-one-standard-deviation curves for different soil types, degradation levels and force-deformation models. Since the mean-plus-one-standard-deviation DAFs were not very sensitive to the choice of degrading model (model 1 or 2 -- see Figure 10), the coefficients were computed to be independent from the type of degrading model.

Table 1 shows the recommended coefficients a, b, and c, as a function of soil type and degradation level. The latter is characterized by the value of the post-peak stiffness (degrading system) or the post-yield stiffness (bilinear system). Also, since the mean-plus-one-standard-deviation DAFs were nearly the same for soil types B and C (see Figure 12), the same coefficients are suggested for both soil types. In the table, the proposed displacement amplification factor at a T/T_g ratio of 1 (i.e., coefficient c) is 1.1 for bilinear systems and for oscillators with low and moderate degradation. A value of 1.5 is recommended for systems with severe degradation. These values reflect the inherent scatter of the data and the decision to allow a safety margin to the DAF for oscillators with periods greater than the characteristic value T_g.

Table 1. Recommended coefficients for Equation 2				
Soil Type		**Bilinear (5% strain hardening)**	**Low or Moderate (-1% post-peak stiffness)**	**Severe (-3% post-peak stiffness)**
B or C	a =	1.35	1.85	2.00
	b =	0.20	0.15	0.10
	c =	1.10	1.10	1.50
D	a =	1.10	1.45	1.60
	b =	0.20	0.15	0.10
	c =	1.10	1.10	1.50

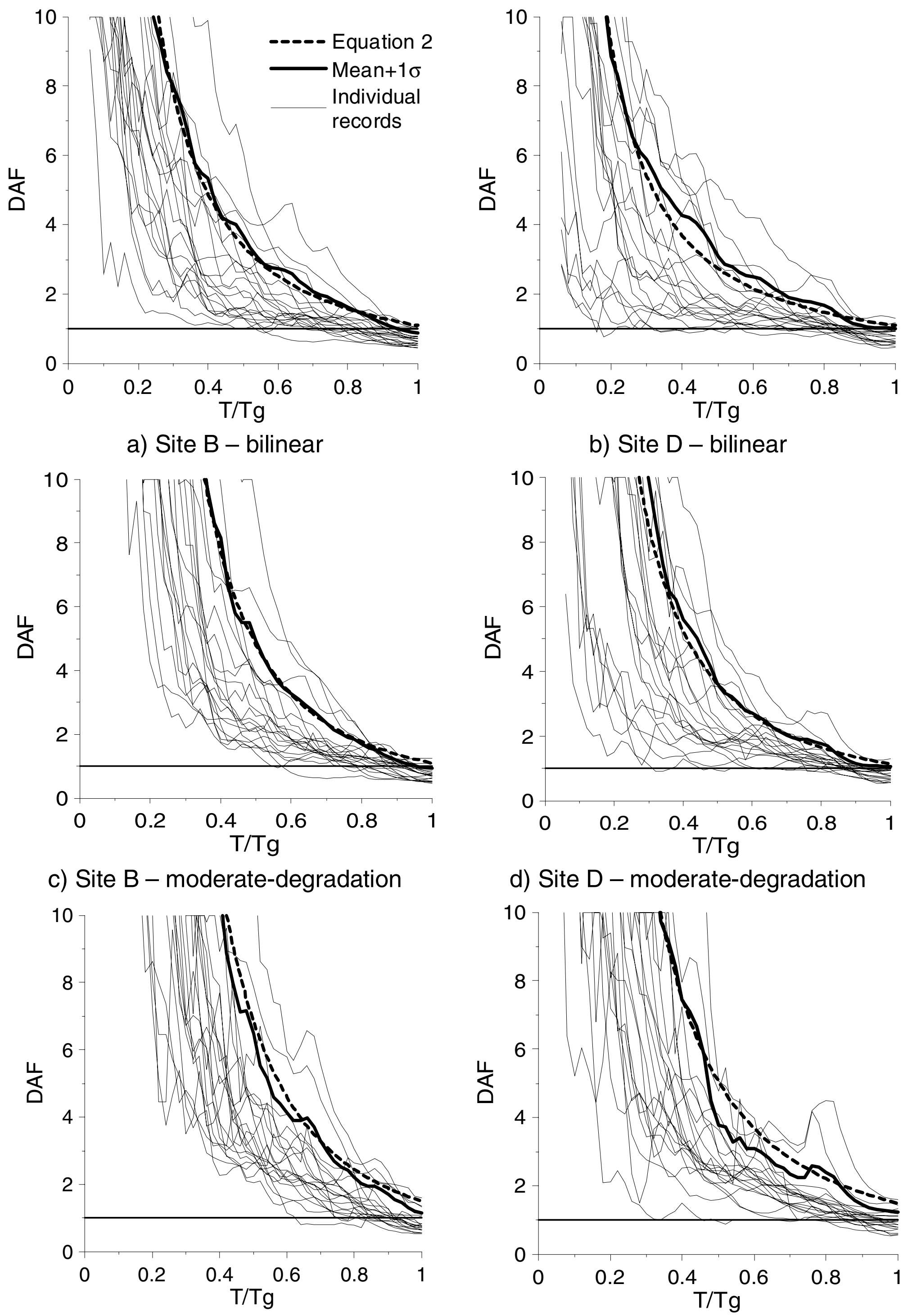
Figure 14 shows a comparison of the calculated DAFs using Equation 2 with those computed for the ground motions on soil type B and D and a constant strength ratio of 0.4 using model 1. Figure 15, on the other hand, shows a comparison of the DAFs for motions on soil type C with those obtained from Equation 2 for various strength ratios using model 2. The calculated mean-plus-one-standard deviation of the DAFs computed from the individual motions is also shown in the figures. The plots show that Equation 2 is a very good fit to the mean-plus-one-standard deviation of the data in all cases. Also, the proposed equation provides an upper bound for approximately 90% of the data and represents a reasonable estimate of the DAF for bilinear and strength degrading systems.

It was noted earlier that as the decay level increased, the mean values of the DAFs increased as well (see Figure 11). This trend was also observed for the majority of the ground motions considered in this study, although there were a few motions where DAFs were larger for lower decay levels. A similar observation was made earlier with respect to the strength ratio of the system, i.e. increasing the strength ratio did not always result in a lower displacement amplification factor when individual records were considered. The proposed formula, however, obeys the trends observed for the mean-plus-one-standard-deviation of the data and always results in higher amplifications for higher degradation levels and lower strength ratios.

It may also be noted that the estimated DAFs for motions on soil type D are generally lower than those for soil types B or C, irrespective of the strength ratio or the degradation level. Thus, if the coefficients in Equation 2 for soils B and C are used, they would result in a conservative estimate of the displacement amplification for the motions recorded on soil type D.

LIMITATIONS OF CURRENT STUDY AND FUTURE WORK

In this study, sixty ground motions recorded on soil types B, C and D were considered. Although the chosen records displayed a wide range of characteristics (peak ground acceleration, characteristic period), additional ground motions on the same soil types should be considered to corroborate the findings of this study and to improve the statistical reliability of the data. Also, ground motions recorded on soils classified as E and F and near-fault records with known forward directivity effects should be considered.



a) Site B – bilinear

b) Site D – bilinear

c) Site B – moderate-degradation

d) Site D – moderate-degradation

e) Site B – severe-degradation

f) Site D – severe-degradation

Figure 14. Comparison of DAFs with Equation 2 for different soil types and degradation levels (η = 0.4, model 1)

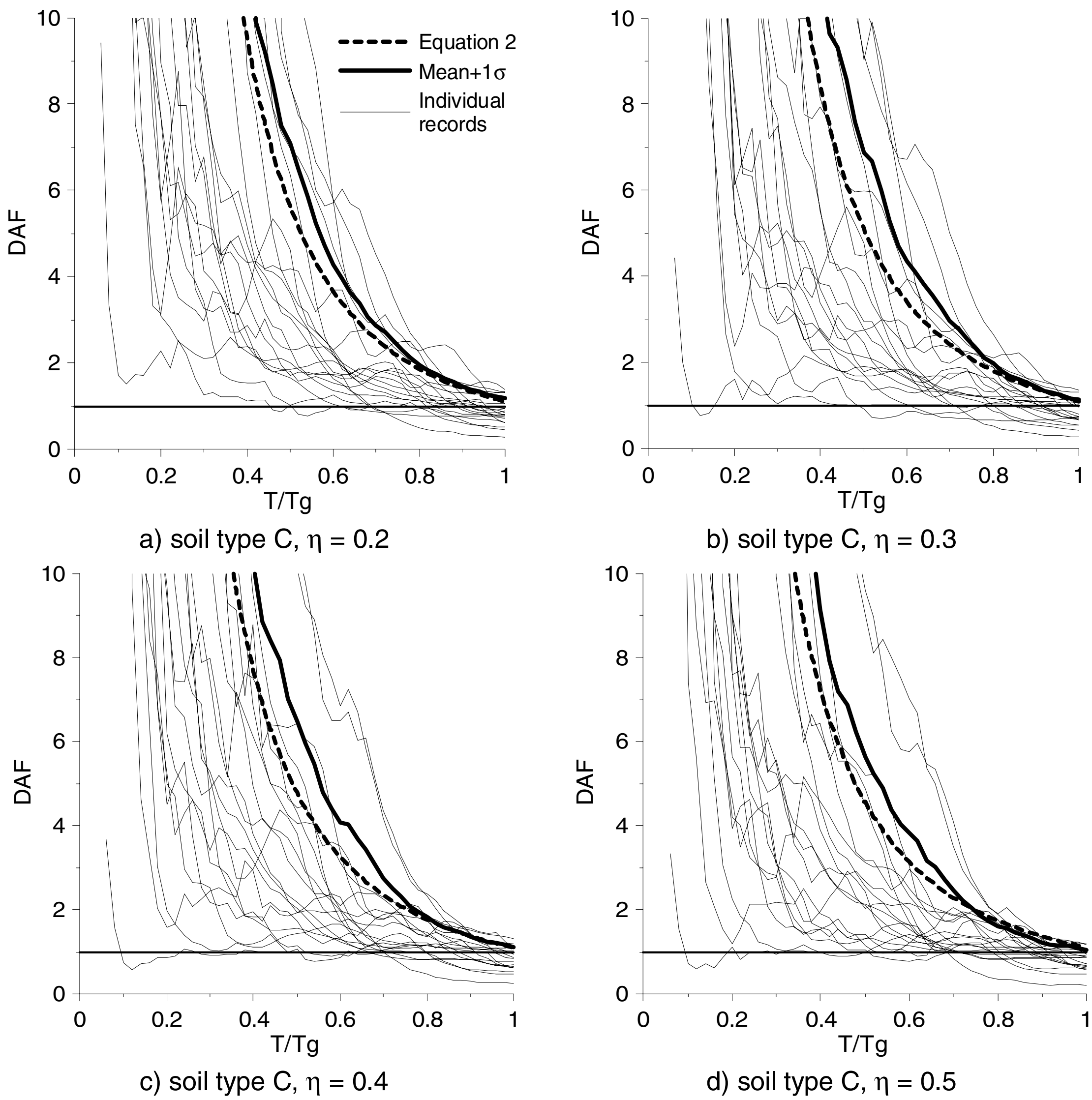


Figure 15. Comparison of DAFs with Equation 2 for various strength ratios (soil type C, moderate degradation, model 2)

CONCLUSIONS

Based on the analysis of the data obtained for the force-deformation models and the parameters considered in this study, the following main conclusions can be reached:

a) The displacement amplification factors of strength degrading systems can be significantly different for a given ground motion depending on the force-deformation model considered. The mean and mean-plus-one-standard-deviation of the data, on the other hand, were found to be nearly independent from the degrading model considered.

b) The general assumption that elastic spectrum provides an upper bound to the inelastic displacements for periods greater than T_g (calculated as the period corresponding to the largest peak in a 5% damped, elastic input energy spectrum) was found to be in good agreement with the displacement amplifications calculated for oscillators with strength and stiffness degradation.

c) A simple equation for estimating the displacement amplification factor of degrading systems is proposed. The equation is given as a function of the period ratio T/T_g and the strength ratio of the system. Three coefficients are used to account for different degradation levels and soil types. The data show that the equation provides a fair upper bound to the displacement amplifications calculated for sixty ground motions on soil types B, C and D and four strength degradation levels.

ACKNOWLEDGMENTS

The work presented in this paper has been sponsored by the National Science Foundation as a part of U.S.-Japan Cooperative Program under Grant No. CMS – 0200492. The authors would like to thank Professor Toshikatsu Ichinose and doctoral candidate Dinh Van Thuat at the Nagoya Institute of Technology in Japan for their assistance in the selection and classification of the Japanese ground motions used in this study. The opinions, findings, and conclusions expressed in this paper are solely those of the authors and do not necessarily represent the views of the sponsor or the individuals mentioned here.

REFERENCES

1. Riddell, R. and Garcia, J. E., 2001, "Hysteretic energy spectrum and damage control", Earthquake Engineering and Structural Dynamics 30, pp. 1791-1816.
2. Cuesta, I., Aschheim M. A. and Fajfar P., 2003, "Simplified R-Factor relationships for strong ground motions", Earthquake Spectra, Volume 19, No.1, pp. 25-45.
3. Shimazaki, K. and Sozen, M.A., 1985, "Seismic drift of reinforced concrete structures", Special Research Paper (Draft), Department of Civil Engineering, University of Illinois at Urbana-Champaign.
4. Qi, X. and Moehle, J.P., 1991, "Displacement design approach for reinforced concrete structures subjected to earthquakes", Report No. UCB/EERC-91/02, Earthquake Engineering Research Center, University of California at Berkeley, 186 pp.
5. Miranda, E., 2000, "Inelastic displacement ratios for structures on firm sites", Journal of Structural Engineering, Vol. 126, No. 10, pp. 1150-1159.
6. Riddell, R., Garcia, J.E. and Garces, E., 2001, "Inelastic deformation response of SDOF systems subjected to earthquakes", Earthquake Engineering and Structural Dynamics 31, pp. 515-538.
7. Lepage, A., 1997, "A method for drift-control in earthquake-resistant design of RC building structures", Ph.D. Thesis, Department of Civil Engineering, University of Illinois at Urbana-Champaign.
8. Gupta, B. and Kunnath, S.K., 1998, "Effect of hysteretic model parameters on inelastic seismic demands", Proceedings of 6th U.S. National Conference on Earthquake Engineering, Seattle, WA.
9. Gupta, B. and Krawinkler, H., 1998, "Effect of stiffness degradation on deformation demands of SDOF and MDOF structures", Proceedings of 6th U.S. National Conference on Earthquake Engineering, Seattle, WA.
10. Song, J.K and Pincheira, J.A., 2000, "Spectral Displacement Demands of Stiffness- and Strength-Degrading Systems", Earthquake Spectra, Volume 16, No.4, pp 817-851.
11. Al-Sulaimani, G.J. and Roesset J.M., 1985, "Design Spectra for Degrading Systems", Journal of Structural Engineering, ASCE, Vol.111, December, pp. 2611-2623.
12. Federal Emergency Management Agency, 2000, "Prestandard and Commentary for the Seismic Rehabilitation of Buildings" (FEMA 356), Washington, D.C.
13. Miranda, E., 1993, "Site-dependent strength reduction factors", Journal of Structural Engineering 119 (12), pp. 3503-3519.