

A STUDY ON HYSTERISIS MODEL FOR EARTHQUAKERESPONSE ANALYSISOF TIMBER STRUCTURES

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SUMMARY

This paper attempts to build the parameters of hysterisis models for resisting shear walls of timber str-uctures. Theses parameters enable engineers to make models easily by summation of the parameters. In or-der to establish these parameters, this paper focuses on the following three subjects. First : Proposition of a hysterisis model fit for timber structures. Second : Proposition of parameters of the resisting shear walls, based on about 100 tests. Third : Prediction of shakingtable tests from the models of summation of the walls.

In the first part, the combination model of bilinear model and slip model is adopted for its simple mechanism. In the second part, the database serves for determining parameters of five different types of walls. In the third part, three models of one full-scale wooden house are proposed. The earthquake responses obtained by the models correspond to the shaking table tests, and this result confirms the accuracy of the parameters.

Introduction

During the Hyogo-Ken Nanbu Earthquake in 1995, more than a hundred thousand wooden houses were collapsed and the collapse of houses caused the 90% of all casualties. After the earthquake, the safety of the wooden houses has been a serious problem. After that, through several full - scale shaking table tests in recent years, the seismic performances of wooden houses were confirmed to be safe. But, in order to understand those performances analytically, it's necessary to establish the hysterisis model and its parameters for an earthquake response analysis.

Although it's desirable to make hysterisis models of full-scale wooden house from the shaking table tests of full-scale house, it needs considerable costs and equipments. On the other hand, the resisting factors of wooden houses in Japan are generally "walls", and it's relatively easy to make shear-wall test. Consider theses yielding points, it is very rational if the dynamic performances of the houses are simulated by the summation of hysterisis models of walls. And it enables engineers to make models easily by summation of the parameters.

To establish these parameters, this paper focuses on the following three subjects.

- First : **Proposition of the way to make hysterisis model** fit for timber structures.
- Second : **Proposition of parameters of the resisting shear walls**, based on about 100 tests.
- Third : **Prediction of shaking table tests** from the models of summation of the walls.

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1. Proposition of the way to make hysterisis model

1.1 Hysterisis model

In this study, as the hysterisis model of walls, the combination model of bi-linear model and slip model is adopted because of its simple mechanism (Fig.1). Moreover, it can represent the "pinching" effects which are characteristic to the hysterisis curve of timber structures as well.

1.2 Process to determine the model

The skeleton curve of the shear wall tests is determined by the force deformation curve of the test (Fig. 2-1, SC_{test}-1), and from these curves, the Tri-linear skeleton curve model is determined(Fig.2-2, SC_{model}-2). The 1st yielding point (Deformation Angle : γ_1 Lord : P₁) and 3rd yielding point (γ_3 , P₃) of the SC_{model}is equal to on the yielding point of SC_{test}. In this study, γ_1 is supposed as 1/500rad. The reason is that the force deformation curve of wooden structures don't have any particular yielding point, it needs to be supposed properly. γ_3 is supposed depending on the kinds of the walls. After the model experiences max Load $(=P_3)$, the stiffness is fixed (= 0).

The 2^{nd} yielding point (γ_2 , P_2) is supposed between γ_1 and γ_3 , or [$\gamma_1 < \gamma_2 < \gamma_3$]. At this time, the energy of SC_{model} (Fig.2-2, E_{model} -1) is equal to that of SC_{test} (Fig.2-2, E_{test} -2). Each stiffness of SC_{model} , or 1st, 2nd and 3rd stiffness of the model, and that of bi-linear factor and slip fa-

ctor have following relationship:

1^{st} stiffness (of SC _{model}) = K _{b1} + K _{s1}	(1)
2^{nd} stiffness (of SC _{model}) = K _{b2} + K _{s1} or K _{b1} + K _{s2}	(2)
3^{rd} stiffness (of SC _{model}) = K _{b2} + K _{s2}	(3)

 $\begin{array}{l}K_{b1}, K_{b2} \ : \ 1^{st} \ and \ 2^{nd} \ stiffness \ of \ bi-liner \ factor \\ K_{s1}, K_{s2} \ : \ 1^{st} \ and \ 2^{nd} \ stiffness \ of \ slip \ factor \end{array}$



Fig.1 Combination model of Bi-linear model and Slip model and the relationship of each stiffness



From these relationship, the hysterisis model are categorized roughly into two group : the one is that 2nd stiffness is equal to $K_{b2} + K_{s1}$ (= "first Bi-liner yielding type"), the another is that 2^{nd} stiffness is equal to $K_{b1} + K_{s2}$ (="first Slip yielding type").

In the equation (1),(2)(3), all four stiffness ($= K_{b1}, K_{b2}, K_{s1}, K_{s2}$) are determined by supposing one of those stiffness. In this study, K_{b2} is supposed.

The index "fit rate" which represents how much the model is fit to the test, is defined following equation:

$$\mathbf{M}i = \frac{|Stest_i| - |dS_i|}{|Stest_i|} (\%) \quad (4)$$

In the equation(4), the index "i" means the loop number. " $|dS_i|$ " means the summation of the absolute value of the remainder of "force of test minus force of the model" for every deformation of unloading curve of the loop number "i", and "|Stesti|" means the absolute value of summation of the forces of unloadding curve of the loop number "i".

The combination of 2^{nd} yielding point and each stiffness in which ΣMi (= the summation of M of each loop) is maximum value determines the hysterisis model of the test.

The hysterisis model can make by determining each stiffness of SC_{model}, the 2nd yielding point, the 3rd yielding point, and the following ratio : $K_{b1} / (K_{b1} + K_{s1})$. So the following parameters are defined.

- 1) "*a*" means the ratio of the 1^{st} stiffness of bi-linear factor to the 1^{st} stiffness of SC_{model} : $a = K_{b1} / (K_{b1} + K_{s1})$
- 2) "b" means the ratio of the 2^{nd} stiffness of SC_{model} to the 1^{st} stiffness of SC_{model} : $b = (K_{b2} + K_{s1} \text{ or } K_{b1} + K_{s2}) / (K_{b1} + K_{s1})$ (6) 3) "c" means the ratio of the 2rd stiffness of SC_{model} to the 1st stiffness of SC_{model} :
- $c = (K_{b2} + K_{s2}) / (K_{b1} + K_{s1})$

These parameters represent the relationship of the combination of 2nd yielding point and each stiffness determined by the equation(4).

By the way, in the equation (4), the value of $|Stest_i|$ is not affected by the parameter a, b, c. On the other hand, as you understand from Fig.3, the value of $|dS_i|$ changes depending on the shape of the loop of the model. So the equation(4) is only affected the parameter a, b, c.

2. Proposition of parameters of hysterisis models

From data-base of shear-wall tests that we gathered, the parameters of hysterisis model are proposed. The following is the process to determine the parameters.

- (1) The hysterisis model is made from the data-base.
- (2) The parameters of hysterisis model are categorized by the kinds of the walls. In this study, about five kinds of walls are proposed parameters: Brace (= BR:5), Plywood board (= PW:15), Gypsum board (= GB:9), Brace + Gypsum board(= BG:10), Brace + Gypsum board + Siding board (=BGS:10) (the number means the number of the data).
- These five walls are well often used in Japanese wooden houses.
- (3) About the relationship of categorized parameters, the method of walls and condition of the test is studied. The factors that is possible to affect the parameters are (a)the gap of nails, (b)fix method between capital and base of column and beam or sill, (c) width of walls, (d) the shape of openings, (e)the way of tests(ex. tie rod exam, etc.), (f)the vertical load.
- (4) Considering (3), parameters are proposed.

For example, Chart 1 expresses the method and the parameters of Plywood wall. The following is the study on the relationship between the parameters, the method of the walls and the condition of the test.

- 1) About the value of *a*, *b*, *c*, there is no difference between the wall with HD irons and the ones with no HD irons.
- 2) Independently of the gap of nails, a is between 0.3 to 0.5, b is between 0.6 and 0.7, and about c, the gap of nails about @150, c is between 0.05 and 0.1, about less @100, c is 0.1.
- 3) About the 2nd yielding point, in the case of no openings and using HD irons is between 4.5 and 6.5 mrad.(=1/1000 rad.), in the case of no openings and no HD irons is about 10 mrad. In the case of the gap of nails, the 2nd yielding points about less @100 are between 4.5 and 6.5 mrad., the case of the gap of nails about @150 is about 10 mrad.

From these studies, the degree of fixation of capital and base of the columns and the methods of the nails is considered to affect the 2^{nd} yielding point.

Consider these studies, the parameter of Plywood is determined as Chart 2. Other four walls are also studied as Plywood. The determined parameters are on the Chart 2.

About the parameters, the 2^{nd} stiffness of skeleton curve models determined by equation(4) are all " K_{b2} + K_{s1} ", or "first Bi-liner yielding type". It suggests that "first Bi-liner yielding type" is needed to represent the "pinching" curve which is characteristic to the hysterisis curve of timber structures

And about the relationship between damages of joint that causes drop of the strength and the 2^{nd} yielding point, almost all damages concentrate after 10mrad, but the 2^{nd} yielding points of the models concentrate around 5mrad. These studies suggest that in this model the damages of joints and the 2^{nd} yielding point had no relation to each other.

3. Prediction of the hysterisis model of full-scale house from the summation of models of walls.

By using proposed parameters of wall (= after this "factor models"), the parameters of hull-scale house (= after this "hull-scale models") is predicted from the summation of the parameters of factor models, or "1P models" and "4_8P models", those are made from 1P, 4P and 8P test-data. Assumed test specimens are shown in Fig.4, and the three kinds of models are following:

name	Width	Num.	Thickness (mm)	Openings	HD iron	L. Load	Gap of n Outside	ail(mm) / Inside	1 st Stiff* ¹	2 nd Stiff* ¹	3^{rd} Stiff $*^1$	а	b	с	2 nd Y. point* ²	3 rd Y. point* ²
PW-1-1	1P	One	9	Non	Ν	N	100	200	0.82	0.60	0.07	0.34	0.74	0.08	10.00	33
PW-1-2	1P	One	9	Non	Ν	Y	150	150	0.80	0.48	0.01	0.41	0.59	0.01	9.00	20
PW-2N-1	2P	One	12.5	Non	_	N	75	75	6.58	3.59	0.84	0.48	0.55	0.13	5.50	33
PW-2N-2	2P	One	12.5	Non	_	N	100	100	5.69	4.00	0.32	0.32	0.70	0.06	4.40	33
PW-2N-3	2P	One	12.5	Non	_	N	100	100	1.88	1.13	0.04	0.42	0.60	0.02	5.90	33
PW-2N-4	2P	Both	7.5	Non	_	N	100	200	4.56	2.98	0.52	0.38	0.65	0.11	6.20	33
PW-2N-5	2P	One	12.5	Non	_	N	50	50	7.12	4.87	0.62	0.34	0.68	0.09	5.90	33
PW-2N-6	2P	One	12.5	Non	_	N	60	60	7.70	5.33	0.56	0.34	0.69	0.07	5.30	33
PW-4C-1	4P	One	12.5	Both edge	_	N	150	150	0.81	0.57	0.06	0.32	0.71	0.07	4.10	33
PW-4A-1	4P	One	12.5	Center 2P	_	N	150	150	0.70	0.36	0.06	0.56	0.52	0.09	5.40	33
PW-4W-1	4P	One	12.5	window	_	N	150	150	1.24	0.88	0.06	0.31	0.71	0.05	5.80	33
PW-2W-1	2P	One	12.5	Window		N	75	75	6.76	3.76	0.90	0.49	0.56	0.13	7.10	33
PW-4W-2	4P	Both	7.5	Window	_	N	100	200	5.07	3.17	0.69	0.39	0.63	0.14	7.00	33
PW-4D-1	4P	One	12.5	Door	_	Ν	150	150	0.90	0.60	0.08	0.35	0.67	0.09	5.70	33
PW-2D-1	2P	One	12.5	Door	_	Ν	75	75	3.75	2.17	0.46	0.45	0.58	0.12	8.50	33

Chart1 the method and the parameters of Plywood wall

*1 1st - 3rd stiff . Funit kN/mrad./m, [/m] means per width of 1m *2 2rd Y. point_3rd Y. point : unit : mrad.

Chart2 the method and the parameters of each wall

	width	Type of Brace	Openings	HD iron	Α	b	с	2 nd Y. point (mrad.)	3 nd Y. point (mrad.)	1 st Stiff.** (kN/mrad./m)
BR+GB*	4P (8P)	_	Non Door Window	Y	0.35	0.7	0.15	4.0 (5.0) 5.0 (6.0) 4.5 (5.5)	30	4.5* 3.2** 3.7**
	1P	Single	Non	Y	0.48	0.62	0.10	5.5	30	1.4
	1P	Double	Non	Y	0.45	0.65	0.15	7.5	30	1.6
			1							
BR+GB +SD*	4P 8P	_	Non Door Window	Y	0.4	0.65	0.15	5 6 5.5	30	6** 4** 5**
BR+GB +SD*	4P 8P 1P	— Single	Non Door Window Non	Y Y	0.4	0.65	0.15	5 6 5.5 5.5	30 30	6** 4** 5** 2.0
BR+GB +SD*	4P 8P 1P 1P	— Single Double	Non Door Window Non Non	Y Y Y	0.4 0.35 0.30	0.65 0.70 0.70	0.15 0.10 0.15	5 6 5.5 5.5 6.0	30 30 30	6** 4** 5** 2.0 2.1

*1P Brace exist only both edge of the wall (Per 4P walls) . **1st stiff. of 4P,8Pmodel unit: (kN/mrad.)

	Num.	Width	Gap of n Outside	ail(mm) / inside	HDiron	а	b	С	2 nd y.Point (mrad.)	3 nd y.Point (mrad.)	1 st stiff. (kN/mrad./m)
	One	1P	100	100	Y	0.37	0.65	0.05	5.5	33	2.2
	Both	1P	100	200	Y						2.5
PW	One	1P	75	75	Y	0.37	0.65	0.1	5.5	33	2.7
	One	1P	50	50	Y						2.9
		1P	100	200	Y	0.4	0.65	0.08	6		0.9
	One	1P	100	150	Y				3.5		1
		1P	150	150	Y	0.4	0.65	0.08	3.5		0.9
GB		1P	100	200	Y				3.5	20	1.1
		1P	100	150	Y				3.5		2
	Both	1P	150	150	Y	0.28	0.75	0.05	3.5		1.8
		1P	100	200	Y				3.5		2.1

	Type of Brace	Width	Edge of Brace	HDiron	а	b	С	2 nd y.Point (mrad.)	3 nd y.Point (mrad.)	1 st stiff. (kN/mrad./m)
BR	Single	1P	Plate	Y	0.35	0.7	0.05	7	15	0.8
	Double	1P	Plate	Y	0.45	0.7	0.15	8.5	15	1.4

(1) M-1 : "hull-scale models", or models made from static tests of hull-scale house

(2) M-2 : "4_8P models", or models made from static tests of 4P and 8P walls

(3) M-3 : "1P models", models made from static tests of 1P walls

4. Prediction of the parameters of walls with openings

When the parameters of 4_8Pmodels or Hull-scale models is predicted from the 1P models, the effects of openings and determination of the parameters is needed to be considered. For walls with no openings, the 1st stiffness of the 4_8P wall is able to be calculated from the summation of 1st stiffness of the 1P walls, but for walls with openings, it can not be calculated. The following is the way to assume the 1st stiffness of the walls with openings:

As the way of assuming force of the walls with openings of 2×4 construction houses (Fig.4), Mr. Sugiyama proposes following equations:

$$r = \left(\frac{1}{1 + (\alpha / \beta)}\right) \tag{8}$$

r : sheathing area ratio

$$\alpha : \frac{A_0}{H \cdot L} \tag{9} \qquad \beta : \sum L_i / L \tag{10}$$

 $\begin{array}{ll} A_0: \mbox{ Total area of openings } & \mbox{ H}: \mbox{ height of shear wall } \\ \mbox{ Li}: \mbox{ Length of walls of resisting factors } & \mbox{ L}: (\mbox{ Full }) \mbox{ Length of wall } \end{array}$

$$F(1/300) = \frac{3r}{5-2r} \tag{11}$$

Ratio of strength at 3.3 mrad. (=1/300 rad.) (= Walls with openings / walls with no openings)

When equation (6) is applied to (conventional) wooden structures (after this, "wooden structures"), the calculated force is rather smaller than test. About several wooden structures wall, the curve of $F(1/300) \times 1.35$ is match to test(Fig.5), so it is adopted for F(1/500). Fig.6 shows the example of assumed models.





Fig.4 the walls with openings supposed by equation (8), (9), (10) and (11)



Fig.6 Proposed hysterisis model

5. Prediction of the full-scale models by summation of factor models

When the full-scale model is predicted by the summation of the factor models, parameter "a" and 2nd yielding point is considered as many as the number of the summation. About *a*, the average is determined ($a_{maen} = \Sigma a_i / N$ $1 \le i \le N$, N: the number of summation).

And about 2^{nd} yielding point, the 2^{nd} yielding point of hull-scale model is determined by the way that the energy of hull-scale model (= E_{sum}) is equal to the summation of energy (= ΣE_i) of multiple factor models($E_{sum} = \Sigma E_i$).

Fig.7 shows the skeleton curve of full-scale test (= "S-test") and the curves of 3 kinds of models. And Chart5 expresses the values of parameters and stiffness of the models.

In Fig.3 and Chart5, the values of 1^{st} and 2^{nd} stiffness of M-2 is close to those of M-1, but because the 2^{nd} yielding point of M-2 is a little larger than that of M-1, the skeleton curve of M-2 is a little larger than S-test. On the other hand, the values of 1^{st} and 2^{nd} stiffness of M-3 is larger than those of M-1 and M-2, and because the value of 2^{nd} yielding point is smaller than that of M-1 and M-2, the skeleton curve of M-3 is a little larger than S-test.



The reason that 3rd stiffness of M-3 is larger than that of M-1 and M-2 is, for instance, that the effects of irons of base of columns overlap. And for 8P and hull-scale size test specimens, the effects of slip between members are larger than those of 1P and 4P size test specimens, so the stiffness of 8P and hull-scale size model are seemed to smaller than those of multiples of 1P models.

6. The earthquake responses analysis and its valuation

By comparing the earthquake responses analysis of three models(=M-1,M-2 and M-3) with vibration tests (= after this :"v-test"), the accuracy of the parameters is studied.

The input wave is "Kobe Kaiyoukishoudai" wave (= "Kobe NS", Max acceleration :818 gal). The earthquake responses is Newmark's $\beta(\beta = 1/4)$. Damping ratio is 5% of initial stiffness.

The input wave is "Kobe Kaiyoukishoudai" wave (= "Kobe NS", Max acceleration :818 gal). The earthquake responses is Newmark's β (β = 1/4). Damping ratio is 5% of initial stiffness. The analytical model is Mass-Spring model (Fig.8), Chart3 expresses its stiffness of every

story, and Chart4 expresses its mass. This model doesn't consider P- Δ effects.

Chart6 expresses the maximum response of three models and v-test, and Fig.9 shows the wave of the response of deformation of each model, and Fig.10 shows the force-deformation curve of v-test and M-3.

For maximum response of deformation, the value of v-test is 94.7mm, M-1 is 103.4mm, M-2 is 92.5mm, M-3 is 94.4mm. It's natural that the response of M-1 that's made from the hull-scale static test is close to the vibration test, but it's remarkable that the response of M-3 that is made from the multiple of 1P models is close to the vibration test.

Although these studies is based on the test, it's needed to be emphasized that these studies don't have enough generality because it's the study based on only one test result.

But the result of this study suggests that the possibility that the dynamic responses of hull-scale houses is predicted from the summation of parameters of factor models.

7. Conclusion

- First : The way to make hysterisis model fit for timber structures is proposed. And the data-base for tests is made. Then the parameters of the resisting shear walls based on about 100 tests are proposed.
- Second: The way to predict a hysterisis model of hull-scale houses from the summation of models of resisting factors.
- Third : Three models are proposed (predicted from 1P, from 4_8P and from hull-scale test), and the earthquake response is analyzed about these three models. The difference in responses of deformation between vibration test and three models is within 10%. Although it is a study based on only one test, the result of this study suggests the possibility that the dynamic responses of hull-scale houses is predicted from the summation of parameters of factor models by combining simple hysterisis models like bi-linier model and slip model.



Fig.8 Analytical model

Chart 4	Mass	and	Height
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	1F	2F
Mass (t)	m ₁ : 9.3	m ₂ :9
Height(mm)	h ₁ : 2,885	h ₂ : 2,930

Chart 5 Responses for each model

	Disp.(mm)	Vel.(kine)	Acc.(gal)
(a) V-test	94.7	80.6	1267
(b) M-1	103.4	77.5	1236
(c) M -2	92.5	60.9	1237
(d) M -3	94.4	67.6	1171
(b) / (a)	10.9	0.96	0.98
(c) / (a)	9.8	0.76	0.98
(d) / (a)	10.0	0.84	0.92



Fig.9 the wave of the response deformation (M-1,2,3,V-test)

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