

# WHAT DO WE KNOW ABOUT THE PERFORMANCE-BASED DESIGN OF COLUMNS?

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## INTRODUCTION

Determination of the deformation capacity of reinforced concrete columns is of paramount importance in performance-based seismic design. The deformation capacity of a column can generally be expressed in several different ways: (1) curvature ductility, (2) displacement ductility, or (3) drift. Even though several performance-based confining reinforcement design procedures have been proposed, the relationship between various ductility factors is not clearly understood. The research reported herein summarizes the development of an analytical procedure that can be used to evaluate the deformation capacity of a reinforced concrete column. The analytical procedure employs various experimentally verified constitutive models for the rebar slip, inelastic buckling of rebars, and confinement of concrete. The  $P-\Delta$  effect is also taken into account. The effect of concrete strength, longitudinal reinforcement ratio, volumetric ratio of confining reinforcement, shear span-to-depth ratio, and axial load level on the relationship between various ductility factors is evaluated and discussed.

## BACKGROUND

Wehbe et al. [37] conducted tests on rectangular columns. Based on analytical and experimental results, they proposed a design expression relating the amount of confining reinforcement to attainable displacement ductility. Sheikh & Khoury [30] proposed a performance-based confining reinforcement design procedure. They employed curvature ductility as the performance criterion in their design equations. Saatcioglu & Razvi [23] suggested that there was a direct correlation between lateral drift and concrete confinement based on their analytical studies. Even though the aforementioned researchers used different ductility factors in their design expressions, the ultimate aims of their research was to quantify ductile performance and to calibrate design equations aimed at supplying desired levels of ductility.

## Wehbe, Saiidi, Sanders, and Douglas [37]

Wehbe et al. [37] conducted tests on rectangular columns. The columns tested in the experimental program contained 46% to 60% of the minimum lateral reinforcement required by the AASHTO provisions. The applied axial loads were 10% and 20% of  $A_g f_c'$ . The specimens were tested under

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constant axial loads and reversed cyclic lateral loads. The column specimens exhibited displacement ductilities,  $\mu_{\Delta}$ , ranging between 5 and 7. Based on analytical and experimental results, the following equation was proposed to relate the amount of confining reinforcement to attainable displacement ductility.

$$\frac{A_{sh}}{s_{t}h_{c}} = 0.1\mu_{\Delta}\sqrt{\frac{f_{c,n}}{f_{ce}'}} \left[ 0.12 \frac{f_{ce}'}{f_{ye}} \left( 0.5 + 1.25 \frac{P}{f_{ce}'A_{g}} \right) + 0.13 \left( \rho_{l} \frac{f_{y}}{f_{s,n}} - 0.01 \right) \right]$$
(1)

where,

 $S_t$ 

= spacing of transverse reinforcement along the axis of the member

- $h_c$  = cross-sectional dimension of column core measured center-to-center of confining reinforcement
- $f_c'$  = expected concrete strength
- $f_{ye}$  = expected yield strength of transverse reinforcement
- $f_y$  = expected yield strength of longitudinal reinforcement
- $\rho_l$  = longitudinal reinforcement ratio
- $f_{c,n} = 27.6 \text{ MPa} \text{ (or 4 ksi)}$

 $f_{s,n}$  = 414 MPa (or 60 ksi)

For the minimum amount of lateral steel in areas of high seismic risk, the use of  $\mu_{\Delta} = 10$  was recommended. The researchers also recommended that a displacement ductility value less than 10 could be used for moderate levels of ductility.

#### Sheikh and Khoury [30]

Sheikh & Khoury [30] proposed a performance-based confining reinforcement design procedure. The researchers employed curvature ductility,  $\mu_{\phi}$ , as the performance criterion in their design equations. The seismic performance of a column was classified to be in one of the following three categories: (1) highly ductile columns ( $\mu_{\phi} \ge 16$ ), (2) moderately ductile columns ( $16 > \mu_{\phi} \ge 8$ ), and (3) columns displaying low levels of ductility ( $\mu_{\phi} < 8$ ). The following equation was proposed and calibrated to relate the amount of lateral reinforcement in the potential plastic hinge regions of tied columns to axial load level and to curvature ductility.

$$A_{sh} = \alpha \cdot \left\{ 1 + 13 \left( \frac{P}{P_0} \right)^5 \right\} \frac{(\mu_{\phi})^{1.15}}{29} A_{sh,ACI}$$
(2)

The constant  $\alpha$  was used to take the arrangement of lateral and longitudinal steel into account. According to Sheikh & Khoury [30],  $\alpha$  may be equal to one for tightly knit lateral reinforcement configurations in which effective lateral support to longitudinal bars is provided. In the presence of less efficient lateral reinforcement configurations and higher axial load levels, greater  $\alpha$  values are recommended. They concluded that the ACI 318-02 requirements [2] for confining reinforcement may not be sufficient even in columns with efficient lateral reinforcement configurations to meet the high curvature ductility demands under moderate-to-high levels of axial loads. They recommended that at low axial load levels (P  $\leq 0.4P_0$ ) the code requirements may be relaxed. Bayrak & Sheikh [5] modified the above equation for high-strength concrete columns with concrete strengths ranging between 55 MPa and 115 MPa.

$$A_{sh} = \alpha \cdot \left\{ 1 + 13 \left( \frac{P}{P_0} \right)^5 \right\} \frac{(\mu_{\phi})^{0.82}}{8.12} A_{sh,ACI}$$
(3)

#### Saatcioglu and Razvi [23]

Saatcioglu & Razvi [23] suggested that there was a direct correlation between lateral drift and concrete confinement based on their analytical studies. They concluded that the shear span to depth ratio (L/h) did

not show a pronounced effect on drift capacity when the P- $\Delta$  effect was considered and that the amount of longitudinal reinforcement had a minor influence. In cases where the P- $\Delta$  effect was considered, the drift capacity increased by approximately 75%, when the L/h ratio was changed from 2.5 to 5.0. Based on these findings, they derived the following relation for a L/h ratio of 2.5 and a longitudinal reinforcement ratio of 2%.

$$\rho_c = 14 \frac{f_c'}{f_{yh}} \left[ \frac{A_g}{A_c} - 1 \right] \frac{1}{\sqrt{k_2}} \frac{P}{P_0} \delta$$
(4)

where,

$$k_2 = 0.15 \sqrt{\frac{b_c}{s} \cdot \frac{b_c}{s_l}}$$
(5)

$$\frac{P}{P_0} \ge 0.2 \text{ and } \frac{A_s}{A_c} - 1 \ge 0.3$$
 (6)

A lateral drift ratio of 2.5% was recommended to ensure ductile performance.

#### **Definitions of Ductility Parameters**

Since the behavior of reinforced concrete sections and members is not elastic-perfectly plastic, several definitions for ductility are available in the literature. In this study, the ductility parameters are defined using idealized backbone curves shown in Figure 1. In defining the yield curvature (or displacement), a straight line joining the origin and a point on the ascending branch (where  $M = 0.75M_{max}$  or  $V = 0.75V_{max}$ ) is used. This line passes through the moment-curvature (or the load-displacement) curve at 75% of maximum moment (or load) and reaches the maximum moment (or load) to define the idealized yield curvature  $\phi_1$  (or yield displacement  $\Delta_1$ ), as shown in Figure 1. Failure of the column is conventionally defined when the post-peak curvature  $\phi_2$  (or postpeak displacement  $\Delta_2$ ) reaches a point at which the remaining column strength has dropped to 80% of the maximum moment (or load). The curvature and displacement ductilities are defined as follows:

$$\mu_{\phi} = \frac{\phi_2}{\phi_1}; \ \mu_{\Delta} = \frac{\Delta_2}{\Delta_1}$$
(7)

Having defined the displacement and curvature ductility ratios, the drift capacity can be defined as the ratio of the maximum useful displacement capacity to the column height (L).



Figure 1. Definitions of Ductility Parameters

#### The Relationship between Various Ductility Factors

It was discussed earlier that different ductility factors were adopted by various researchers in the performance-based design of confining reinforcement. However, it is not clear how different ductility factors are related with each other under different conditions. It is likely that the level of axial load, concrete strength, amount, grade, spacing of longitudinal and lateral reinforcements, and shear span-to-depth ratio have different influences on various deformation parameters. The relationship between the curvature and displacement ductilities was previously investigated by Park & Paulay [15]. It should be noted that the  $P-\Delta$  effect, rebar slip and shear deformations were neglected in this equation.

$$\mu_{\Delta} = 1 + 3\left(\mu_{\phi} - 1\right) \frac{L_{p}}{L} \left(1 - 0.5 \frac{L_{p}}{L}\right)$$
(9)

Equation (9) indicates that the curvature and displacement ductilities have a linear relationship. These relationships are plotted in Figure 2 for various shear span-to-depth ratios (L/h) and equivalent plastic hinge lengths  $(L_p)$ . Figure 2 illustrates that the displacement ductility increases as the shear span-to-depth ratio (L/h) decreases and the plastic hinge length  $(L_p)$  increases. However, recent studies have found that the axial load has a very important role in the ductility, strength, stiffness, and energy dissipation characteristics of columns.



Figure 2. Relationship between curvature and displacement ductilities (Park & Pauley [15])



Figure 3. Relationship of Moment and Lateral Load Capacities

Figure 3 depicts a typical cantilever column subjected to an axial load, P, and a lateral load, V. The sectional behavior of this cantilever column under the given axial load is also shown in Figure 3. After reaching the peak lateral load ( $V_{max}$ ), the loss of lateral load capacity can be attributed to two factors: the loss of moment capacity and the P- $\Delta$  effect (Figure 3). The loss of lateral load due to the P- $\Delta$  effect indicates that even though a column has very large curvature ductility, the maximum attainable lateral displacement ( $\Delta_2$ ) of the column may be controlled by the P- $\Delta$  effect under high axial load.

#### **MODELING OF COLUMN BEHAVIOR**

Displacement components that contribute to the tip displacement of a reinforced concrete column can be assumed to be: (1) bending along the column length, (2) shearing along the column length, and (3) fixed end rotation resulting from the slip of the longitudinal reinforcement out of the joint. The degradation of flexural strength becomes severe under high axial compression loads combined with high lateral drifts and this produces secondary moments, known as the  $P-\Delta$  effect. This secondary moment may consume a significant portion of total flexural resistance. Therefore, the total displacement at the tip of a column can defined as follows.

$$\Delta = \Delta_{bending} + \Delta_{slip} + \Delta_{shear} + \Delta_{P-\Delta} \tag{10}$$

#### **Deformation due to Bending**

The bending displacement,  $\Delta_{bending}$ , is computed by integrating curvatures over the length of the column (*L*).

$$\Delta_{bending} = \int_{0}^{L} \phi(x) x dx \tag{11}$$

It should be noted that theoretical difficulties arise using a displacement-based incremental analysis using this procedure when the moment-curvature curve has a falling branch. This problem is handled by adopting an equivalent plastic hinge length method. It is assumed that the plastic hinge starts to develop when a section reaches its yield curvature. A full plastic hinge length  $(L_p)$  is assumed to develop when the curvature at the critical section reaches an inelastic curvature equal to three times the yield curvature. The full plastic hinge length is assumed to be equal to the column depth (h).

#### **Deformation due to Bar Slip**

The formation of a flexural crack at the interface of a column and a typical beam column joint (or foundation) strains the reinforcement crossing the crack. Widening of such a crack may produce inelastic strains in the reinforcement. This results in the penetration of yielding into the anchorage zone of the reinforcement, causing extension of reinforcement. Hence, additional rigid body deformations may also occur due to rebar slip, if longitudinal bars are not sufficiently anchored. Alsiwat & Saatcioglu [1] stated that the omission of bar slip in computing inelastic deformations might lead to erroneous results. This is particularly important when significant yielding of longitudinal tension reinforcement is expected under low levels of axial compression. Deformation due to bar slip is computed using the analytical model proposed by Alsiwat & Saatcioglu [1]. This model incorporates yield penetration and associated inelasticity in anchored reinforcement, as well as the possibility of slip. Once the bar slip at the end of a member is computed, the member end rotation and lateral displacement due to bar slip can be determined as follows:

$$\Theta_{slip} = \frac{u_{st}}{d-c} \tag{12}$$

$$\Delta_{iin} = \Theta_{slin} \cdot L \tag{13}$$

where  $u_{st}$ 

d

 $u_{st}$  = bar slip at extreme tensile bar

= distance from top fiber of concrete to center of extreme tensile bar

c = depth of neutral axis

#### **Deformation due to Shear**

For columns with low levels of shear, the shear deformation of uncracked reinforced concrete member from the principles of elasticity is adopted (Equation 14).

$$\Delta_{\nu} = \frac{VL}{AG}; \ G = \frac{E}{2(1+\mu)}$$
(14)

Based on this, the shear deformation along the column height is estimated by the following equation:

$$\Delta_{v} = \int_{L} \frac{V(x)dx}{G_{eff}(x)A_{eff}(x)} = V \int_{L} \frac{dx}{G_{eff}(x)A_{eff}(x)}$$
(15)

where  $G_{eff}(x) = \frac{E_c(x)}{2(1+\mu)}$ 

 $A_{eff}$  = effective shear area  $\approx$  area of concrete subjected to compressive strains  $E_{c,sec}(x)$  = secant Young's modulus of core concrete assessed from the corresponding normal compressive stress and strain at the ex-

corresponding normal compressive stress and strain at the extreme fiber of the concrete core.

v = Poisson's ratio ( $\approx 0.3$ )

#### Deformation due to P- $\Delta$ Effect

The axial compression load with high lateral drifts produces secondary moments. The distribution of the secondary moments is related to the deflected shape of a column. Combining with the deflections due to moments, curvatures along the column height can be obtained and the associated tip deflection can be estimated. Since the distribution of moments also depends on the deflected shape of a column, an iterative procedure is required.

#### **Concrete Confinement Models**

Several researchers proposed stress strain models for confined concrete: Scott et al. [28] proposed the modified Kent & Park model to include the strength and ductility enhancement due to confinement and strain rate. Sheikh & Uzumeri [31] incorporated the concept of effectively confined concrete within the concrete core. Sheikh & Yeh [32] later modified this model for flexural behavior of a column. Mander et al. [12] proposed a confined concrete model, which takes cyclic loading of concrete and the strain rate into account. By defining an effective lateral confining stress Mander et al. [12] modeled confinement due to various sources. The effective lateral confining stress was dependent on the configuration of the transverse and longitudinal reinforcements. All of the aforementioned concrete models were based on the column tests where normal strength concrete ( $f_c' \le 55$  MPa) was used.

Recently, Razvi & Saatcioglu [22] and Légeron & Paultre [11] proposed confined concrete models, which are applicable to both normal and high-strength concretes. In order to highlight the differences and similarities between various confinement models, a square concrete column made with 40 MPa concrete and detailed according to ACI 318-02 provisions is studied in Figure 4. This figure shows the effect of concrete models on confined concrete stress-strain curves. This figure clearly shows that there are large discrepancies between the predictions for confined stress-strain response of concrete. The confinement model proposed by Mander et al. [12] resulted in the most ductile predicted behavior; while the concrete model proposed by Légeron & Paultre [11] produced the most brittle predicted behavior (Figure 4).



Figure 4. Concrete Stress-Strain Relationship of Concrete Models

#### **Reinforcing Bar Buckling Model**

The inelastic buckling behavior of reinforcing bars has been generally ignored in analytical programs and rebar behavior in compression is simply assumed to be the same as tensile stress-strain response. Bayrak & Sheikh [7] showed the benefits of employing different stress-strain responses for rebar behavior in tension and compression. Bae, Mieses & Bayrak [4] proposed a rebar buckling model. In this model, the compressive axial strain is assumed to be the summation of axial strain due to axial stress and axial strain from transverse deformation of a bar due to inelastic buckling (Equation (16)).

$$\varepsilon_{com} = \varepsilon_s + \varepsilon_{tra} \tag{16}$$

where  $\varepsilon_{com}$ ,  $\varepsilon_s$ , and  $\varepsilon_{tra}$  are the compressive axial strain, axial strain due to axial stress and axial strain from transverse displacement, respectively. Axial strain due to axial stress,  $\varepsilon_s$ , can be obtained directly from the tensile stress-strain relationship of a bar.

For the calculation of the axial strain induced by the transverse displacement, the following two relationships were proposed by Bae, Mieses & Bayrak [4]:

- 1) Relationship of axial stress and transverse displacement at buckling.
- 2) Relationship of the axial strain and transverse displacement.

The proposed relationship for axial stress-transverse displacement was based on a large number of reinforcing bar tests and is shown in the following equations and Figure 5.

$$\frac{\Delta_{b}}{L_{b}} \le 0.04; \qquad \frac{f_{s}}{f_{y}} = 1 + (\frac{f_{s}}{f_{y}} - 1) \times \sqrt{1 - \left(\frac{\Delta_{tra}}{\Delta_{tra}} + 1\right)^{2}} \qquad \text{for } \frac{f_{s}}{f_{y}} \ge 1$$
(17)

$$\frac{f_s}{f_y} = \left(\frac{f_s}{f_y} - 1\right) \frac{\Delta_{tra}}{\Delta_{tra}} + 1 \qquad \text{for } \frac{f_s}{f_y} < 1 \qquad (18)$$

$$\frac{\Delta_{tra}}{L} > 0.04; \qquad \frac{f_s - f_s^*}{f_y} = A\left(\frac{\Delta_{tra} - \Delta_{tra}^*}{L}\right) \qquad \qquad \text{for } \frac{f_s}{f_y} \ge \frac{2}{3} \frac{f_s^*}{f_y} \tag{19}$$

$$\frac{f_s - 2/3f_s^*}{f_y} = \frac{A}{2} \left( \frac{\Delta_{tra}}{L} - x_1 \right) > 0.2 \qquad \text{for } \frac{f_s}{f_y} < \frac{2}{3} \frac{f_s^*}{f_y} \tag{20}$$

where;

$$\frac{\Delta_{tra}^{*}}{L} = 0.04; \ \frac{f_s^{*}}{f_y} = -0.45\xi^{1.5} \left[ \ln\left(\frac{L/d}{4}\right) \right] + \xi \le \xi$$
(21)

In which;

 $\xi = f_u / f_y$   $\Delta_{tra} = \text{transverse displacement of a bar at the mid-span}$  L = unsupported bar length d = nominal bar diameterA = initial slope of the descending branch (Figure 5)

The relationship of the axial strain and transverse displacement is illustrated in Equation (22) and Figure 6.

$$\varepsilon_{tra} = \operatorname{larger} \operatorname{of} \left\{ \left( \frac{0.035 \cos \theta + \theta}{\cos \theta - 0.035 \theta} \right) \frac{\Delta_{tra}}{d}, \frac{1}{\cos \theta - 0.07 \theta} \times \left( 0.07 \cos \theta + \theta \right) \left( \frac{\Delta_{tra}}{d} - 0.035 \right) \right\}$$
(22)  
$$\theta = \frac{6.9}{(1 + 1)^2} - 0.05$$

where;  $\theta = \frac{6.9}{(L/d)^2} - 0.0$ 

Comparisons with test results proved that inelastic buckling behavior of reinforcing bars could be predicted accurately through the use of this model. Detailed discussions and the derivation of this model can be found elsewhere (Bae, Mieses & Bayrak [4])



Figure 5. Relationship between transverse displacement and average axial stress



Figure 6. Relationship between transverse displacement and corresponding axial strain

#### **Verification of Proposed Analytical Procedure**

The proposed analytical method was verified with the experimental results. The displacement ductility and drift capacity of columns tested by various researchers was predicted using the analytical procedure described above. In predicting the response of confined concrete all confined concrete models discussed



Figure 7. Comparisons of Analytically- and Experimentally-Obtained Deformation Capacities ( $P \ge 0.2P_0$ )

earlier were used. However, the results presented in Figure 7 are from analyses where Razvi & Saatcioglu's confined concrete model was used. Figure 7 indicates that the proposed procedure provides good estimates for column deformation capacities.

#### DISCUSSIONS

To examine the relationships between various ductility factors, the behavior of a typical column section (300mm×300mm) was studied. The experimentally verified analytical procedure described earlier was used to conduct parameter studies. In all the analyses, longitudinal reinforcement was uniformly distributed along four faces and the center-to-center distance of extreme reinforcement layers was 240mm, i.e. ( $\gamma = 0.8$ ). The concrete strength was 40 MPa and the yield and ultimate strengths of the reinforcement were 415 and 620 MPa, respectively. The stirrup spacing was kept equal to six times the diameter of the longitudinal bars to avoid early bar buckling. It was assumed that enough anchorage length was provided to prevent excessive rebar slip. The height of the column was 1500 mm, resulting in a shear span-to-depth ratio of 5. The full plastic hinge length was assumed to be equal to the column depth (*h*).

#### Modeling the behavior of confined concrete

The influence of modeling the behavior of confined concrete on the relationship between various ductility parameters is illustrated in Figure 8(a). In all the analyses summarized in Figure 8(a) the axial load level was kept constant at  $0.3P_0$ . As can be observed in this figure all concrete models provide similar results within the practical deformation range. The confinement model proposed by Razvi & Saatcioglu is used in the subsequent parametric studies as this model provided somewhat more accurate estimations for the experimental ductility parameters considered in the model verification stage.

#### Level of axial load

The effect of the level of axial load  $(0.1P_0, 0.3P_0, and 0.5P_0)$  on the relationship between various ductility factors was investigated, as shown in Figure 8(b). At this stage the *P*- $\Delta$  effect was considered in some analyses and neglected in others. When the *P*- $\Delta$  effect was not considered, the relationships were linear, as suggested by Park & Pauley [15]. As can be observed in Figure 8(b) the inclusion of the *P*- $\Delta$  effect changed the nature of the relationships between various ductility factors. With increasing axial load, the *P*- $\Delta$  effect became more pronounced and the attainable displacement ductilities and drift capacities reduced considerably. For high axial load levels, drastic increases in curvature ductility resulted in considerably smaller increases in displacement ductility and drift capacity. For high axial loads (~0.5P<sub>0</sub>), the *P*- $\Delta$  effect had an important role in the loss of the lateral resistance. As the *P*- $\Delta$  effect is a function of the lateral displacement and the level of the axial load, improving the sectional behavior at high levels of axial load is not as effective as the low axial load levels.

## Concrete strength

The effect of concrete strength was studied using an axial load of  $0.3P_0$  and two different concrete strengths ( $f_c' = 40$  and 80 MPa). Figure 8(c) illustrates that an increase in the concrete strength resulted in reduced displacement ductility and drift capacities for a given curvature ductility. To achieve the same level of displacement ductility or drift capacity in a high strength concrete column, the use of a larger amount of confining reinforcement was required.

## Shear span-to-depth ratio

The effect of shear span-to-depth ratio (L/h) was studied by changing both the length of a column (L) and the depth of a column (h). The different depths of columns were modeled by keeping the same longitudinal reinforcement ratio and the same ratio of the center-to-center distance of extreme reinforcement layers to the column depth  $(\gamma)$ . The results for an axial load level of  $0.3P_0$  are shown in Figures 8(d) and (e). Figures 8(d) and 8(e) indicate that both approaches resulted in similar trends. As the shear span-to-depth ratio decreased, the displacement ductility and drift capacity increased for a given curvature ductility. To achieve the same level of displacement ductility or lateral drift capacity, different levels of sectional performance are needed for various shear span-to-depth ratios.

## Longitudinal reinforcement

The amount of longitudinal reinforcement was varied ( $\rho_l = 1, 2$  and 4%) to investigate its influence on the relationships between various ductility parameters. For the relationships, shown in Figure 8(f), the axial load was kept constant at 0.3P<sub>0</sub>. For a given curvature ductility, an increase in the amount of longitudinal reinforcement resulted in a decrease of displacement ductility, but in an increase of drift capacity. As the amount of longitudinal reinforcement capacity increased, the lateral load carrying capacity, the yield displacement and the ultimate displacement capacity increased. However, the increase in the yield displacement was more pronounced than the increase in the ultimate displacement capacity for a given curvature ductility as depicted in Figure 8(f).

## Equivalent plastic hinge length

A review of technical literature revealed the fact that several researchers suggested different equivalent plastic lengths (Table 1). The suggested plastic hinge lengths varied from 0.5h to h. Sakai & Sheikh [25] stated that the plastic hinge length could be affected by the amount of transverse reinforcement, axial load level, and the aspect ratio of a column section. The influence of equivalent plastic hinge length  $(L_p = 0.5h)$  and 1.0h) on the relationships between various ductility parameters was investigated (Figure 8(g)). A reduction in the plastic hinge length h to 0.5h resulted did not change the relationships between various ductility parameters significantly. Therefore, it can be concluded that the equivalent plastic hinge length is very important in predicting the individual column behavior, especially for columns displaying limited ductility, but it does not affect the relationships of various ductility factors considered in this study.



(c) Concrete Strength

Figure 8. Relationship between various ductility parameters



(d) Shear Span-to-Depth Ratio (L/h) - Column Length (L)



(e) Shear Span-to-Depth Ratio (L/h) - Column Depths (h)



(f) Amount of Longitudinal Reinforcement

Figure 8. (Cont.'d) Relationship between various ductility parameters



(g) Equivalent Plastic Hinge Length  $(L_p)$ 



Reseacher(s)	Proposed equation for the equivalent plastic hinge length $(L_p)$
Sawyer [27]	0.25D + 0.075L
Corley [8]	$0.5D + 0.2\sqrt{D}\frac{L}{D}$
Priestley & Park [18]	$0.08L + 6d_b \ (\approx 0.5h)$
Priestley, Seible, & Calvi [19]	$0.08L + 0.15f_yd_b$
Sheikh & Khoury [29] Bayrak & Sheikh [5]	1.0 <i>h</i>

Table 1. Equivalent Plastic Hinge Length

## CONCLUSIONS

The research reported herein summarizes the development of an analytical procedure that can be used to evaluate the deformation capacity of a reinforced concrete column. The analytical procedure employs various experimentally verified constitutive models for the rebar slip, inelastic buckling of rebars, and confinement of concrete. Based on the experimental research summarized in this paper the following primary conclusions can be drawn:

- The relationships between various ductility parameters (curvature ductility, displacement ductility and drift capacity) are affected by the level of axial load. As the axial load increases, the loss of lateral load carrying capacity becomes higher due to the  $P-\Delta$  effect. Because of this, the attainable drift capacity or displacement ductility can be limited at high axial load levels regardless of the amount of confinement. Conversely, because the  $P-\Delta$  effect is a function of the lateral displacement and the level of the axial load, improving the sectional performance at high levels of axial load may not increase the lateral deformation capacity of a column.
- An increase in the concrete strength results in reduced displacement ductility and drift capacities for a given curvature ductility. To achieve the same level of displacement ductility or drift capacity in a high strength concrete column, the use of a larger amount of confining reinforcement is required.

- As the shear span-to-depth ratio decreases, the displacement ductility and drift capacity increases for a given curvature ductility. To achieve the same level of displacement ductility or lateral drift capacity, different levels of sectional performance are needed for various shear span-to-depth ratios.
- It is extremely difficult, if not impossible, to generalize the relationships between various ductility parameters. The level of axial load, shear span-to-depth ratio, concrete strength, the amount of longitudinal and transverse reinforcement influence the deformation capacity of a column.

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