

REINFORCED CONCRETE BOX GIRDERS UNDER CYCLIC TORSION

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SUMMARY

This paper presents a theoretical model for reinforced concrete members under cyclic torsion. It is based on the softened truss model (STM) but has been extended to include the tension stiffened portion of the torsional response. The STM was derived to predict a member's response under a monotonically increasing torsion. It uses a softened stress-strain relationship for concrete in compression derived from shear panel tests and neglects the tensile strength of concrete. The model proposed in this paper uses a bilinear relationship for concrete in tension and additional compatibility equations for the tension stiffened region. It provides an improved prediction of both the pre-cracking and post-cracking torsional behavior. The model is compared to the envelope curve of a reinforced concrete (RC) girder tested under pure torsion using full-reversal cyclic loading. It is one girder tested as part of an experimental investigation aimed at studying the behavior of RC girders loaded in combined shear and torsion under seismic-like cyclic loading. The hollow box girder tested was 14.7 meter long and loaded under several full-reversal torsional cycles. The model is also calibrated to three other torsional members found in literature.

INTRODUCTION

There are many civil engineering structures where torsion could be a significant loading condition. The most noticeable are bridges, spandrel beams, and the RC core around the elevator shaft of buildings. In bridges, the torsion could be due to the geometric complexities of horizontally curved bridges and/or to large eccentric vertical loads. In an earthquake, if the center of mass in a slender building is eccentric, it will cause cyclic torsional loads on its RC core. In practice, torsion is typically combined with shear and bending action. However the behavior of pure torsion under cyclic loading is investigated in this paper.

The behavior of an RC member is modeled differently before and after cracking. Before first cracking, the concrete behaves as an elastic, isotropic material and the reinforcement can be ignored [1]. In members with a square cross section, the shear stress developed in the beam flows around the member and is a maximum at the midpoint of the outside surface. When the principle tensile stress reaches concrete's tensile strength, cracking occurs. In RC girders under to pure torsion, the stiffness of the uncracked member can be predicted by St. Venant's theory. After cracking, the member behaves as a composite

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member and the properties of the concrete, the reinforcing, and their interaction must be considered to accurately predict the member response to torsion.

Different models have been developed to predict the torsional behavior of RC members under monotonically increasing loads. Shuzhi *et al* [2] developed a model based on the skew bending theory. A tri-linear model based on the truss model theory described below, which considers the cracking, yielding, and ultimate torque points was developed by Mo [3]. Two models based on the truss analogy are the compression field theory model (CFT) [4] and the softened truss model (STM) [5]. Both are rational models capable of accurately predicting the peak torsional capacity of a RC girder. Also, both models can predict the torque-twist response before and after peak reasonably well. The models differ in their treatment of spalling of the concrete cover and the assumed stress-strain relationship for concrete. Both of these models are based on the smeared crack assumption and neither incorporates tension-stiffening effects.

RESEARCH SIGNIFICANCE

The purpose of this paper is to provide a theoretical model capable of predicting the pre-cracking and post-cracking torsional response of a member under monotonic and cyclic loading. The STM for torsion was modified to account for the tension stiffening effect of concrete, and the model can now provide an improved prediction of the torsional response envelope for moderately to highly reinforced members.

The model is used to predict the behavior of three members in literature as well as the primary envelope curve of a 14.7 m long hollow box girder tested in this project. The girder is part of an experimental investigation aimed at studying the behavior of RC girders loaded in combined shear and torsion under seismic-like cyclic loading. The hollow box girder tested was full-size and loaded under pure torsion. Considerable information about the seismic performance of hollow concrete bridge girders will be obtained from the tests that are part of this research

SOFTENED TRUSS MODEL FOR TORSION

The STM for torsion is a truss model based on satisfying equilibrium, compatibility, and the uniaxial stress-strain relationships of concrete and reinforcing steel. The truss model was first proposed by Ritter [6] and Morsch [7] for shear, and then extended to treat torsion by Rausch [8]. The truss is formed when the principal tensile stress causes diagonal cracks in the concrete perpendicular to the principal compressive stress. In the truss model for torsion, the internal compressive stresses are assumed to spiral around the beam at an angle α with the longitudinal axis [1] as shown in Figure 1. The resulting member is a space truss with tension acting in the longitudinal and hoop reinforcement which is counteracted the compression in the concrete strut formed by diagonal cracking. The basic equilibrium and compatibility equations for a girder in pure torsion are shown below [5].

Equilibrium:

$$\sigma_l = \sigma_d \cos^2 \alpha + \sigma_r \sin^2 \alpha + \rho_l f_l \tag{1}$$

$$\sigma_t = \sigma_d \sin^2 \alpha + \sigma_r \cos^2 \alpha + \rho_t f_t \tag{2}$$

$$\tau_{lt} = (\sigma_d - \sigma_r) \sin \alpha \cos \alpha \tag{3}$$

Compatibility:

$$\varepsilon_l = \varepsilon_d \cos^2 \alpha + \varepsilon_r \sin^2 \alpha \tag{4}$$

$$\varepsilon_t = \varepsilon_d \sin^2 \alpha + \varepsilon_r \cos^2 \alpha \tag{5}$$

$$\gamma_{lt} = 2(\varepsilon_d - \varepsilon_r) \sin \alpha \cos \alpha \tag{6}$$



Figure 1. Truss model. (adapted from [4])

Additional equilibrium equations use Bredl's thin tube analogy to calculate the internal torque. Bredl's theory assumes that shear flow is constant thru the wall thickness along all four edges. By defining the area enclosed by the centerline of the shear flow as A_o , the internal torque can be expressed by the following simple expression [1].

$$T = 2A_o t_d \tau_{lt} \tag{7}$$

In the STM, the RC is modeled as a continuous medium after cracking, and uses average values for stress and strain. The stress-strain relationships for concrete in compression consider the softening effect due to diagonal tension cracks. Concrete softening was first reported by Robinson and Demorieux [9], and later quantified by Vecchio and Collins [10] using tests on RC panels subjected to pure shear. The STM assumes that non-softened concrete has a parabolic stress strain distribution. The amount of softening is quantified by the softening coefficient ζ . The softened stress-strain relationship for concrete in compression assumes proportional stress and strain softening. Figure 2 shows the relationship for cracked and uncracked concrete. Tests on shear panels showed that ζ is highly dependent on the average principle tensile strain, ε_r , but is also a function of the average principle compressive strain, ε_d , the angle of the diagonal cracks, α , and the load path [11]. Based on shear panel testing, the University of Houston proposed the following expression for ζ as a function only of ε_r [12].

$$\zeta = \frac{0.9}{\sqrt{1 + 600\varepsilon_r}} \tag{8}$$

$$\psi = \theta \sin 2\alpha \tag{9}$$

$$\varepsilon_{ds} = t_d \psi \tag{10}$$

$$\theta = \frac{p_0}{2A_0} \gamma_{lt} \tag{11}$$

Warping in torsional members causes bending stresses in the compressive strut in addition to axial compression. The girder wall is assumed to deform into a hyperbolic paraboloid shape [1]. Equation (9) gives the relationship between the wall curvature, ψ , the angle of the diagonal cracks, α , and the girder twist per length, θ .



Figure 2. Truss model. (adapted from [11])

Tests have shown that the curvature causes a linear strain distribution, with the maximum compressive strain occurring at the surface, and tension occurring at a certain depth into the wall [1]. The depth over which the stress remains compressive is defined as the shear flow zone [5]. The STM assumes the resultant compressive stress acts at a depth of the half shear flow zone thickness. Equation (10) shows the relationship between the depth of the shear flow zone, t_d , the maximum compressive strain, ε_{ds} , and the wall curvature, ψ . Equation (11) can be derived from the compatibility condition of warping deformation and relates the member's twist to shear strain.

REINFORCED CONCRETE IN TENSION

Testing of shear panels has shown that the average tensile strength of concrete is not zero after cracking [10, 12]. The tensile strength at a discrete crack location can be assumed to be zero. However the average stress along a length of concrete includes the contribution of concrete that is still uncracked and will provide resistance to tension stress [11]. Before cracking, the tensile response is nearly linear, but after cracking the concrete strength drops quickly with increasing strain [13].

The stress-strain relationship of a steel reinforcing bar stressed in tension is different than the relationship for a bar embedded in concrete. The embedded bar is stiffened by the concrete between the cracks. So the average stress of an embedded bar will be different than the stresses at a discrete point in a bare bar [14]. The contribution of concrete to the reinforcing stiffness is known as tension stiffening [15].

Relationships for the tensile strength of concrete and tension stiffening are interrelated, and were developed for membrane elements under shear. For simplicity, the STM for torsion neglects the contribution of concrete in tension, and uses an elastic perfectly plastic model for plain reinforcing bars [11]. By neglecting the tensile stresses in concrete, the STM assumes that the member is initially fully cracked. In reality the first cracks form at a few discrete locations, and the concrete between the cracks increases the girder's torsional stiffness.

STRESS-STRAIN RELATIONSHIPS OF CONCRETE IN TENSION

Equation (12) is a simple expression for the stress-strain relationship of concrete in tension before cracking. It is based on the assumption of linear elastic behavior before cracking. After cracking, expressions for the descending branch were developed at University of Toronto and the University of Houston and are given by Equations (13) and (14) respectively. Vebo and Ghali [16] used a simple linear expression for the descending branch as given in Equation (15).

$$\sigma_r = E_c \varepsilon_r \tag{12}$$

$$\sigma_r = \frac{f_{cr}}{1 + \sqrt{200\varepsilon_r}} \tag{13}$$

$$\sigma_r = f_{cr} \left(\frac{\varepsilon_{cr}}{\varepsilon_r}\right)^{0.4} \tag{14}$$

$$\sigma_r = f_{cr} - \frac{E_c}{5} \left(\varepsilon_r - \varepsilon_{cr} \right) \tag{15}$$

MODIFICATION OF STM TO INCLUDE TENSION STIFFENING

In torsional members with low to moderate levels of reinforcement, the pre-peak behavior is dominated by the tension stiffening of concrete. Tension stiffening is also important for modeling the envelope curve of a member under cyclic torsional loading. Including the effect of tension stiffening in the STM should improve the modeling of the pre-peak torsional response.

Adding tension stiffening to the STM involves changing two aspects of the model. First an expression for the principal tension stress, σ_r , must be developed. Second, compatibility Equation (10) needs to be modified because Bredl's theory assumes the shear flows uniformly in a thin tube near the outer edge of the member. Although this is a good assumption in a fully cracked member, in an uncracked member St. Venant's theory is assumed to be valid. Several modifications to the STM are needed to account for the transition between these two different behaviors in a member stiffened by concrete.

Concrete Stress-Strain Relationship in Tension

The STM for membrane elements accounts for tension stiffening by including the contribution of concrete in tension before and after cracking and by using the smeared stress-strain curve of reinforcing bars embedded in concrete. The expressions for reinforcing bars embedded in concrete are complex, but are necessary when using Equation (14) for the tensile strength of concrete. Using an elastic perfectly plastic relationship of a bare bar for the reinforcing steel with equation (14) for concrete will overestimate the capacity of a membrane element.

In order to use the simple relationship for a bare bar an alternate expression for concrete is needed. Equation (16) gives the simple bilinear model used in this analysis, and Figure 3 compares Equations (13), (14) and (15). The variable β is the fraction of concrete modulus, E_c , used as the descending branch stiffness. A value of 0.016 is used for β in Equation (16b) in this paper.

$$\sigma_r = E_c \varepsilon_r \qquad \qquad \text{For } \varepsilon_r \le \varepsilon_{cr} \qquad (16a)$$

$$\sigma_{r} = f_{cr} - \beta E_{c} \left(\varepsilon_{r} - \varepsilon_{cr} \right) \quad \text{For } \varepsilon_{r} > \varepsilon_{cr}$$
(16b)



Figure 3. Concrete Models.

Stress and Strain in Concrete Strut

The transition between an uncracked torsional member to a fully cracked one requires modifying the STM compatibility Equation (17) relating the surface compressive strain to the thickness of the shear flow zone. Figure 4 shows the relationship given by Equation (10) from the STM and the modified relationship given by Equation (17). The length *a* is added to t_d provide a transition between the St. Venant's theory and the thin tube theory assumed in the STM. Collins suggested using equation (19b) to calculate t_d for an uncracked member [17]. Equation (19b) is also used by the American Concrete Institute in ACI318-02 to derive the equation for the cracking strength of members under pure torsion. When *a* is equal to zero, Equation (17) is identical to Equation (10).

$$\varepsilon_{ds} = R\psi \tag{17}$$

where:
$$R = t_d + a$$
 (18)
 $t_s = R$ For $a = 0$ (19a)

$$t_d = \frac{3}{4} \frac{A_c}{p_c} \qquad \text{For } a > 0 \qquad (19a)$$

Adding the distance *a* to t_d requires a new expression for k_1 , the ratio of the average stress to the peak stress. By similar triangles, the strain ε at a distance *x* from the neutral axis are related in Equation (20). From the STM, the resultant of the softened compression stress block has a magnitude of *C*, as shown in Equation (21). The softened stress-strain relationship for concrete in compression is given by Equation (22), where the strain at the peak stress ε_p is equal to $\zeta \varepsilon_0$.

$$x = \left(\frac{t_d + a}{\varepsilon_{ds}}\right)\varepsilon \quad \text{or} \quad dx = \left(\frac{t_d + a}{\varepsilon_{ds}}\right)d\varepsilon \tag{20}$$

$$C = \sigma_d t_d = k_1 \sigma_p t_d = \int_0^{t_d} \sigma(x) dx = \int_{\varepsilon_a}^{\varepsilon_{ds}} \sigma(\varepsilon) \left(\frac{t_d + a}{\varepsilon_{ds}}\right) d\varepsilon$$
(21)

$$\sigma(\varepsilon) = \sigma_p \left[2 \left(\frac{\varepsilon}{\varepsilon_p} \right) - \left(\frac{\varepsilon}{\varepsilon_p} \right)^2 \right]$$
(22)



Figure 4. Comparison to STM.

$$k_{1} = \left[\frac{\varepsilon_{ds}}{\varepsilon_{p}}\left(1 - \frac{1}{3}\frac{\varepsilon_{ds}}{\varepsilon_{p}}\right) - \frac{\varepsilon_{a}^{2}}{\varepsilon_{p}\varepsilon_{ds}}\left(1 - \frac{1}{3}\frac{\varepsilon_{a}}{\varepsilon_{p}}\right)\right]\left(\frac{t_{d} + a}{t_{d}}\right) \qquad \text{For } \varepsilon_{ds} \le \varepsilon_{p} \qquad (23a)$$

$$k_{1} = \left[\frac{1}{3}\frac{\varepsilon_{a}^{2}\left(3\varepsilon_{p} - \varepsilon_{a}\right)}{\varepsilon_{ds}\varepsilon_{p}^{2}} - \frac{1}{3}\frac{\left(\varepsilon_{ds} - \varepsilon_{p}\right)^{3}}{\left(\varepsilon_{0} - \varepsilon_{p}\right)^{2}} + \left(\varepsilon_{ds} - \varepsilon_{p}\right)\right]\left(\frac{t_{d} + a}{t_{d}}\right) \qquad \text{For } \varepsilon_{ds} > \varepsilon_{p} \qquad (23b)$$
where: $\varepsilon_{a} = \left(\frac{t_{d} + a}{a}\right)\varepsilon_{ds} \qquad (24)$

Method of Solution

A detailed explanation of the solution method to solve the system of equations that are part of the STM is given by Hsu [11]. The modifications to the STM described above only have a minor effect on the solution method. Equation (25) gives R in terms of the smeared strains, A_o and p_o . Equation (26), and (27) give ε_l and ε_i in terms of f_l and f_r . They also include the tensile stress in concrete σ_r , and R instead of t_d .

$$R = \frac{A_0}{p_0} \left[\frac{(-\varepsilon_{ds})(\varepsilon_r - \varepsilon_d)}{(\varepsilon_l - \varepsilon_d)(\varepsilon_l - \varepsilon_d)} \right]$$
(25)

$$\varepsilon_{l} = \varepsilon_{d} + \frac{A_{0}(-\varepsilon_{ds})(\sigma_{d} - \sigma_{r})}{2\left[-p_{0}R(\sigma_{l} - \sigma_{r}) + (A_{l}f_{l})\right]}$$
(26)

$$\varepsilon_{t} = \varepsilon_{d} + \frac{A_{0}s(-\varepsilon_{ds})(-\sigma_{d} + \sigma_{r})}{2p_{0}\left[-sR(\sigma_{t} - \sigma_{r}) + (A_{t}f_{t})\right]}$$
(27)

When the longitudinal reinforcing steel is not yielding, calculating the strain in the longitudinal direction requires solving Equation (26) as a second degree polynomials for ε_i . In the STM, one of the solutions

will typically be negative so selecting the correct solution is simple. However with the addition of tension stiffening to the STM, both solutions can be positive, and selecting the correct one requires more consideration. The correct solution should satisfy Equation (1) for equilibrium. Equations (28), (29), and (30) can be derived from compatibility Equations (4) and (5) and the trigonometric identity $\sin^2 \alpha + \cos^2 \alpha = 1$.

$$\sin^2 \alpha = \frac{\varepsilon_l - \varepsilon_d}{\varepsilon_r - \varepsilon_d} \tag{28}$$

$$\cos^2 \alpha = \frac{\varepsilon_t - \varepsilon_d}{\varepsilon_r - \varepsilon_d}$$
(29)

$$\varepsilon_r + \varepsilon_d = \varepsilon_l + \varepsilon_t \tag{30}$$

Substituting these three equations into Equation (1) results in Equation (31) which is calculated for ε_{l_1} and ε_{l_2} , the two possible solutions. The ε_{l_i} which results is in the smallest value of Equation (31) is then used. Equilibrium Equation (2) is used to derive Equation (32), a similar expression for calculating the strain in the transverse direction.

$$\sigma_d \left(\frac{\varepsilon_r - \varepsilon_{li}}{\varepsilon_r - \varepsilon_d} \right) + \sigma_r \left(\frac{\varepsilon_{li} - \varepsilon_r}{\varepsilon_r - \varepsilon_d} \right) + \rho_l f_{li}$$
(31)

where: f_{li} is the smaller of $E_s \varepsilon_{li}$ and f_{ly}

$$\sigma_d \left(\frac{\varepsilon_r - \varepsilon_{ii}}{\varepsilon_r - \varepsilon_d} \right) + \sigma_r \left(\frac{\varepsilon_{ii} - \varepsilon_r}{\varepsilon_r - \varepsilon_d} \right) + \rho_t f_{ti}$$
(32)

where: f_{ti} is the smaller of $E_s \varepsilon_{ti}$ and f_{ty}

EXPERIMENTAL PROGRAM

This experimental investigation will include three girders loaded under combined torsion and shear in addition to the pure torsion girder described in this paper. A review of the test setups described in literature previously used to test specimens under monotonic combined shear and torsion were not conducive to full-reversal cyclic tests. The test setup for this investigation was designed to create a region of constant shear and constant torsion in the test regions. To create a constant shear load the girder was made symmetric with one test region on each side of the centerline, and a point load will be applied midspan. Two frames are used at each end provide reactions for the upward and downward vertical loads into the strong-floor. This way an inflection point is created in the moment diagram near the middle of each test region.

The girder specimen for pure torsion was 14.6m long, with a 760mm by 760mm square cross-section as shown in Figure 5. Two test regions in the specimen were hollow with 150mm thick walls. The longitudinal reinforcement consisted of 16 #13 bars evenly distributed around the cross section. Closed hoop reinforcement was spaced every 130mm in the test regions.



Figure 5. Test specimen cross-section.

The torsional load was applied at the middle of the girder, and reacted at each end by two sets of frames as shown in Figure 6. The reaction frames were capable of resisting torsion and vertical loads, but had rollers to allow the specimen free longitudinal movement. Another two reaction frames at the middle of the beam prevented longitudinal and transverse movement, but permitted vertical and rotational movement. Two displacement-controlled hydraulic actuators applied two equal but opposite forces to create the torsion acting on the girder. One end of the actuators was connected to a frame that "clamshelled" around the girder, while the other end reacted to the lab's strong floor through a frame.

The torsional load was applied in ten groups of three full-reversal cycles at each twist increment. A cycle consisted of twisting the girder to a specified rotation angle, then reversing the direction of twist and stopping at the same specified rotation angle in the opposite direction, and finally returning to the initial position at the end of the cycle. The rotation angle of subsequent twist increments was increased. Figure 7 shows the twist applied at each cycle.

A computer controlled the actuator displacements during loading using an internal linear variable differential transformer (LVDT) inside each actuator. Elastic losses in the steel loading and reaction frames and inelastic losses in bearing pads did not allow the actuator's LVDT to directly measure the girder's response to the applied cyclic loads. In order to measure the girder's response, the twist was recording at four locations along each test region. The twist was calculated from the displacement measurements of two LVDT's on opposite sides of the cross section.





Figure 6. Test setup.



Figure 7. Applied twist.

TEST RESULTS

The girder's response to cyclic torsional loads was determined from this experiment. Figure 8 shows the applied torque versus twist as calculated from the relative rotations of the two innermost rotation measurements in one test specimen. Point "A" indicates the formation of the first diagonal cracks. The initiation of yielding in the longitudinal bars occurred at point "B", and point "C" indicates where all the measured hoop and longitudinal bars were yielding. Significant spalling occurred after point "D." The pinched loops typically associated with shear behavior are observed and become significant in the last few cycles.

Three full-reversal cycles of displacement-controlled rotation were applied at each twist increment. The primary envelope curve made of the peak torque and the associated twist of the first cycle of each twist increment is shown in Figure 9. The same figure shows the envelopes for the second and third cycle of each twist increment, respectively. The location of points A, B, C, and D are also indicated in Figure 9. The initial cycle created the first cracks in the specimen. As expected, the girder's rotational stiffness was considerably less after this point, as shown by the reduction in slope of the envelope curves. The girder's rotational stiffness was slightly reduced again at the onset of yielding. The peak torque values in the second and third cycle envelopes are less than the first cycle values, but are nearly equal to each other until point D. After point D, the spalling concrete cover caused a considerable reduction in the torsional stiffness of each subsequent cycle.



Figure 8. Cyclic torsional response.



Figure 9. Envelope curves.

ANALYTICAL MODEL RESULTS

The modified STM equations presented in this paper are used to predict the envelope torsional response for the girder tested. Figure 10 shows a comparison of the measured response, the response predicted by the STM, and the tension stiffened response predicted by the modified STM presented in this paper. The measured response shown in Figure 10 is the positive torque portion of the first cycle envelope shown in Figure 9. Figure 11 shows a similar comparison for three torsional girders found literature [4] [18] [19].

Although the STM is able to accurately predict the girder's torsional capacity, the modified equations under-predict the tension stiffened response as shown in Figure 10. The low percentage of reinforcement in the girder caused response to be dominated by tension stiffening. Figure 11a shows another member where tension stiffened region dominates the torsional response. However for girders with a moderate to high percentage of reinforcement, the tension stiffened response was modeled by the proposed model very well as shown in Figures 11b and 11c.



Figure 10. Experimental and Predicted Torsional Behavior of Tested Girder.





Figure 11. Experimental and Predicted Torsional Behavior of Members found in Literature.

CONCLUSIONS

The model presented in this paper provides an improved prediction of the tension stiffened region of the torsional response envelope for members with moderate to high levels of reinforcement. At the end of the tension stiffened region, the model reduces to the equations presented in the STM.

The girder tested experienced spalling after both the longitudinal and hoop reinforcing yielded. Before spalling, the peak torque of second and third cycles at each load increment were nearly equal. Spalling considerably reduced the torsional stiffness of each subsequent cycle and caused the peak torque in the third cycle to be significantly less than the second cycle.

NOTATION

| A_{c} | = area enclosed by the outside perimeter of concrete |
|---|---|
| A_{o} | = area enclosed by the centerline of the shear flow |
| <i>P</i> _c | = outside perimeter of the cross-section |
| p_o | = perimeter of the centerline of shear flow |
| С | = resultant of the concrete compressive force |
| E_c | = modulus of elasticity of concrete, taken as $47000\sqrt{f'_c}$ in psi or $3900\sqrt{f'_c}$ in MPa |
| f'_c | = cylinder concrete compressive strength, in psi or MPa |
| f_{cr} | = stress at concrete cracking, taken as $3.75\sqrt{f'_c}$ in psi or in $0.311\sqrt{f'_c}$ MPa |
| f_l , f_t | = stress in the longitudinal and hoop steel, respectively |
| f_{ly} , f_{ty} | = yield stress in the longitudinal and hoop steel, respectively |
| k_1 | = ratio of the average stress to the peak stress |
| S | = stirrup spacing |
| Т | = torque |
| t _d | = depth of the shear flow zone |
| α | = angle of inclination of the diagonal compression struts |
| eta | = fraction of the initial concrete stiffness in the descending branch |
| γ_{lt} | = shear strain in the l-t coordinate system |
| \mathcal{E}_{a} | = concrete strain at a distance t_d from the surface |
| \mathcal{E}_{cr} | = strain a concrete cracking, taken as 0.00008 mm/mm |
| \mathcal{E}_d , \mathcal{E}_r | = average principal compression and tension strain, respectively |
| \mathcal{E}_{ds} | = concrete strain at the surface |
| $\boldsymbol{\mathcal{E}}_l$, $\boldsymbol{\mathcal{E}}_t$ | = strain in the longitudinal and hoop steel, respectively |
| $oldsymbol{\mathcal{E}}_{li}$, $oldsymbol{\mathcal{E}}_{ti}$ | = one of the two possible solutions for ε_i or ε_i respectively |
| ${\cal E}_p$ | = concrete strain at the peak concrete stress |
| ζ | = softening coefficient |
| θ | = angle of girder twist per length |
| $ ho_l$, $ ho_t$ | = volume percentage of longitudinal and hoop steel, respectively |
| $\pmb{\sigma}_{_{d}}$, $\pmb{\sigma}_{_{r}}$ | = average principal compression and tension stress, respectively |
| $\sigma_{l}, \sigma_{t}, 	au_{lt}$ | = average normal and shear stresses in the l-t coordinate system |
| Ψ | = wall curvature of the diagonal concrete struts |
| | |

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