EFFECT OF THE EXISTENCE OF UNDERLYING LAYER ON ANTI-PLANE DYNAMIC BEHAVIOUR

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SUMMARY

The response problem of the foundation supported on an elastic layer with finite thickness under action of dynamic anti-plane force is studied in the present paper. It consists of two parts. In the first part, the dynamic surface displacement is considered in the case that a point force and a strip load distributed within a strip of finite width exert on the surface of the elastic layer, respectively. Moreover, the effect of soil damping is considered. By use of the method of Laplace transform, the exact solution is obtained for the case of pulse load. The numerical calculation shows that under action of delta-pulse load the dynamic displacement of the ground surface will attenuate rapidly with time, as the damping of soil medium increases. When the ratio, H/x is equal about 10, the effect of the underlying rigid layer can be ignored. The second part is devoted to the dynamic anti-plane contact problem of the rigid foundation on the elastic layer under the action of the harmonic load. By use of Fourier transform, the problem is reduced to solve dual integral equations. By expanding the stress on the contact surface to a series of Jacobi polynomials, the analytic solution of the problem is obtained. The numerical calculation shows that when H/a is equal about 10 the effect of existence of the underlying rigid layer on the distribution figure of contact stress and the dynamic displacement of the foundation can be neglected.

INTRODUCTION

For the problem of soil dynamics, people usually focuses his attention on the plane and the axisymmetrical problems. On the one hand the reason is due to the need of calculation and design of the dynamic machine foundation, and on the other hand for the anti-seismic calculation of the structure the problems of the vertical vibration, the rocking vibration round the horizontal axis and the horizontal sliding motion of the structure under the action of surface waves can be treated as a plane or an axisymmetrical problem . But in the engineering practice there is a kind of structures, such as the shear wall of the house building, the pier of the bridge with long strip form, etc . Under the action of plane SH waves they behave as the feature of anti-plane motion, and must be treated as the anti-plane problem.

For the plane and the axisymmetrical problems, when the existence of the underlying rigid layer or the internal damping of the soil medium is considered, the contour integral with branch points can not be utilized or the contour integral becomes very complex. So far such a kind of problem has not studied fully. But for the anti-plane dynamic problem related to the plane SH waves, under the action of some simple type of surface loads the direct integral can be used, and the analysis of this problem can be obtained.

In the present paper, the authors investigate the anti-plane dynamic problem for the finite elastic layer (namely for the case being of existence of the underlying rigid layer). The soil medium is considered to be with the cohesive damping. The loads acting on the ground surface are the concentrative line force and the uniform forces distribute over a finite area. The dynamic contact problem between the rigid foundation and the groundbase is also studied. The contact between the soil layer and the underlying rigid rock is considered to be welding,

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namely there is no shear displacement on the contact surface. For comparison the case of the half space is also studied.

ELASTIC HALF SPACE WITH VISCOSITY

Considering that the soil medium is of viscous damping, the motion equation for the anti-plane of elastic dynamics can be written as

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=\frac{1}{c_s^2}\frac{\partial^2 w}{\partial t^2}+\frac{1}{\zeta^2}\frac{\partial w}{\partial t} \tag{1}$$

where, $c_s=(\frac{G}{\rho})^{\frac{1}{2}}$ is the shear velocity in the soil medium, ρ and G are, respectively, the mass density and the shear modulus. $\zeta=(\frac{G}{c})^{\frac{1}{2}}$, c is the damping coefficient of the soil medium. The relationship between the shear stress and the displacement in the soil is

$$\tau_{yz}=G\frac{\partial w}{\partial y},\quad \tau_{xz}=G\frac{\partial w}{\partial x} \tag{2}$$

Applying the Fourier cosine transform with respect to the variable x, the Laplace transform with respect to t in equation (1), and considering that the displacement and the stress approach zero as $y\rightarrow\infty$, one has

$$\left.\begin{aligned}\bar{w}&=\frac{2}{\pi}\int_0^\infty Ae^{-\upsilon y}\cos\xi x d\xi\\ \bar{\tau}_{yz}&=\frac{2G}{\pi}\int_0^\infty \upsilon Ae^{-\upsilon y}\cos\xi x d\xi\end{aligned}\right\} \tag{3}$$

where

$$\upsilon=(\xi^2+\alpha^2)^{\frac{1}{2}},\quad \alpha^2=\frac{p}{c_s^2}(p+\frac{c_s^2}{\zeta^2}) \tag{4}$$

i. Case of Line Load

Assuming the force acting on the surface of the elastic half space to be a line load as shown in Figure 1, the boundary condition is

$$\tau_{yz}\big|_{y=0}=-Q\delta(x)f(t) \tag{5}$$

Substituting (5) into the first of equation (3), one has

$$\bar{w}\big|_{y=0}=\frac{2Q}{\pi G}\int_0^\infty\frac{\bar{f}(p)}{\upsilon}\cos\xi x d\xi=\frac{2Q}{\pi G}\bar{f}(p)K_0\left[\frac{x}{c_s}(p)^{\frac{1}{2}}(p+\frac{c_s^2}{\zeta^2})^{\frac{1}{2}}\right] \tag{6}$$

where, $K_0(z)$ is the modified Bessel function of the second kind of order zero. When $f(t)$ is a delta pulse function, $f(t)=\delta(t), f(p)=1$. Inserting $f(p)=1$ into equation (6) and making the inverse Leplace transform, one has the displacement on the surface of the elastic half space as

$$\begin{aligned}w\big|_{y=0}&=\frac{2Q}{\pi G}\frac{1}{[t^2-(\frac{x}{c_s^2})^2]^{\frac{1}{2}}}e^{-\frac{c_s^2 t}{2\zeta^2}}ch\frac{c_s^2[t^2-(\frac{x}{c_s})^2]^{\frac{1}{2}}}{2\zeta^2},\quad t>\frac{x}{c_s}=0\\ &=0,\qquad t<\frac{x}{c_s}\end{aligned} \tag{7}$$

ii. Case of Uniform Load over a Finite Area

Assuming that the uniform forces with intensity $Qf(t)$ act over area $[-a, a]$ on the surface of the elastic half space, the boundary condition is

$$\left.\begin{aligned} \tau_{yz}\big|_{y=0} &= -Qf(t), \quad |x| \le a \\ &= 0, \qquad\quad\;\; |x| > a \end{aligned}\right\} \tag{8}$$

Inserting (8) into equation (3), one has

$$\overline{w}\big|_{y=0} = \frac{2Q}{\pi G}\int_0^\infty \frac{\overline{f}(p)\sin\xi a}{\xi\upsilon}\cos\xi x d\xi$$

$$= \frac{Q}{\pi G}\overline{f}(p)\sum_{i=1}^{2}\int_0^{x_i} K_0\left[\frac{x}{c_s}(p)^{\frac{1}{2}}(p+\frac{c_s^2}{\zeta^2})^{\frac{1}{2}}\right]dx \tag{9}$$

where

$$x_1 = x + a, \quad x_2 = x - a$$

For the case of delta pulse load, substituting $\overline{f}(p) = 1$ into equation (9) and making the inverse Laplace transform, one has the surface displacement

$$w\big|_{y=0} = \frac{Q}{\pi G}\sum_{i=1}^{2}\int_0^{x_i}\frac{1}{[t^2-(\frac{x}{c_s})^2]^{\frac{1}{2}}}e^{-\frac{c_s^2 t}{2\zeta^2}}ch\frac{c_s^2[t^2-(\frac{x}{c_s})^2]^{\frac{1}{2}}}{2\zeta^2}dx, \qquad t \quad \frac{x+a}{c_s}$$

$$= \frac{Q}{\pi G}\int_0^{x^2}\frac{1}{[t^2-(\frac{x}{c_s})^2]^{\frac{1}{2}}}e^{-\frac{c_s^2 t}{2\zeta^2}}ch\frac{c_s^2[t^2-(\frac{x}{c_s})^2]^{\frac{1}{2}}}{2\zeta^2}dx, \quad \frac{x+a}{c_s} \quad t \quad \frac{x-a}{c_s} \tag{10}$$

$$=0, \qquad \frac{x-a}{c_s} \quad t$$

ELASTIC LAYER WITH VISCOSITY

Assuming that there is a elastic layer with thickness H supported on an underlying rigid rock, and supposing that the contact between the elastic layer and the underlying rock is welding, namely there is no shear displacement on the contact surface, the boundary conditions are

$$w\big|_{y=H} = 0 \quad , \quad \tau_{yz}\big|_{y=0} = -\tau(x,t) \tag{11}$$

Under the boundary conditions (11), the Laplace transform solution of equation (1) is

$$\overline{w} = \frac{2}{\pi}\int_0^\infty A(e^{\upsilon y} - e^{\upsilon(y-2H)})\cos\xi x d\xi$$

where

$$A = \frac{\check{}}{G\upsilon(1+e^{-2\upsilon H})}$$

Thus, one has the Lapalace transform formula of the ground displacement as

$$\overline{w}\big|_{y=0} = \frac{2}{G\pi}\int_0^\infty \frac{\tilde{\overline{\tau}}(1-e^{-2\upsilon H})}{\upsilon(1+e^{-2\upsilon H})}\cos\xi x d\xi \tag{12}$$

For the case of Line load, inserting the boundary condition (5) into equation (12), one has

$$\overline{w}\big|_{y=0} = \frac{2Q\overline{f}(p)}{G\pi}\int_0^\infty \frac{1-e^{-2\upsilon H}}{\upsilon(1+e^{-2\upsilon H})}\cos\xi\, xd\xi$$

$$= \frac{2Q\overline{f}(p)}{G\pi}\int_0^\infty \frac{1}{\upsilon}\left[1+2\sum_{k\approx 1}(-1)^k e^{-2k\upsilon H}\right]\cos\xi\, xd\xi$$

Using equation (6) and

$$\int_0^\infty \frac{e^{-y(\xi^2+\gamma^2)^{\frac{1}{2}}}}{(\xi^2+\gamma^2)^{\frac{1}{2}}}\cos\xi\, xd\xi = K_0[\gamma(y_2+x^2)^{\frac{1}{2}}] \tag{13}$$

the foregoing formula can be integrated as

$$\overline{w}\big|_{y=0} = \frac{2Q\overline{f}(p)}{G\pi}\left[K_0(x\alpha^{\frac{1}{2}})+2\sum_{k=1}(-1)^k K_0(\alpha(4k^2H^2+x^2)^{\frac{1}{2}}\right] \tag{14}$$

For the pulse load, inserting $\overline{f}(p)=1$ into the foregoing formula and making the inverse Laplace transform, the displacement of the ground surface is obtained as

$$\left.\begin{aligned} w\big|_{y=0} &= \frac{2Q}{G\pi}\left\{\frac{e^{-\frac{c_s^2 t}{2\zeta^2}}}{\sqrt{t^2-(\frac{x}{c_s})^2}}ch\frac{c_s^2[t^2-(\frac{x}{c_s})^2]^{\frac{1}{2}}}{2\zeta^2}+2\sum_{k=1}^{N}(-1)^k\frac{e^{-\frac{c_s^2 t}{2\zeta^2}}}{\sqrt{t^2-(\frac{x_1'}{c_s})^2}}ch\frac{c_s^2\sqrt{t^2-(\frac{x_1'}{c_s})^2}}{2\zeta^2}\right\}, t>\frac{x_1'}{c_s} \\ &= \frac{2Q}{G\pi}\frac{e^{-\frac{c_s^2 t}{2\zeta^2}}}{\sqrt{t^2-(\frac{x}{c_s})^2}}ch\frac{c_s^2[t^2-(\frac{x}{c_s})^2]^{\frac{1}{2}}}{2\zeta^2},\quad \frac{(4H^2+x^2)^{\frac{1}{2}}}{c_s}>t>\frac{x}{c_s} \\ &= 0,\qquad \frac{x}{c_s}>t \end{aligned}\right\} \tag{15}$$

where

$$x_1' = (4R^2H^2+x^2)^{\frac{1}{2}},\quad N=\frac{1}{2H}(t^2c_s^2-x^2)^{\frac{1}{2}} \tag{16}$$

THE CONTACT PROBLEM ON ELASTIC LAYER

Let a infinite strip rigid foundation with the width $2a$ be on the elastic layer with the thickness H, and assuming the incident SH waves to be harmonic, the problem is of the following boundary conditions:

$$\begin{aligned} w\big|_{y=0} &= \Delta\ \bullet,\ |x|\bullet a \\ \tau_{yz}\big|_{y=0} &= 0\ ,\ |x|\bullet a \\ w\big|_{y=0} &= 0 \end{aligned} \tag{17}$$

In the case that the damping of the soil medium is ignored, the problem is reduced to solve the following dual integral equations:

$$\begin{aligned} &\frac{2}{\pi}\int_0^\infty \frac{\tilde{\tilde{\tau}}(e^{2\upsilon_1 H}+e^{-2\upsilon_1 H})}{\upsilon(e^{2\upsilon_1 H}-e^{-2\upsilon_1 H})}\cos\xi x d\xi = G\Delta', \quad 0<x<a \\ &\int_0^\infty \tilde{\tilde{\tau}}\cos\xi x d\xi = 0, \qquad x>a \end{aligned} \tag{18}$$

Here

$$\upsilon_1 = (\xi^2 - \frac{\omega^2}{c_s^2})^{\frac{1}{2}}$$

ω is the frequency of the incident SH wave. Neglecting the time factor $e^{\omega t}$, and assuming the distribution of the contact stress between the foundation and the ground to be

$$\tau(x_1,\omega) = \frac{a}{(1-x_1^2)^{\frac{1}{2}}}\sum_{k=o} A_k(\omega) G_k(0,\frac{1}{2},x_1^2), \quad |x_1|<1 \tag{19}$$

the Fourier cosinus transform is

$$\tilde{\tilde{\tau}}(\xi_1,\omega^*) = \frac{\pi a}{2}\sum_{k=0} A_k(\omega^*) J_{2k}(\xi_1) \tag{20}$$

in which $G_k(0,\frac{1}{2},x_1^2)$ is the Jacobi polynomials with order k,

$$x_1 = \frac{x}{a}, \quad \xi_1 = a\xi, \quad \omega^* = \frac{a\omega}{c_s}$$

Inserting (20) into (18) and considering

$$\int_0^\infty \mathrm{J}_{2k}(ax)\cos\beta x dx = \begin{cases} \cos(2k\arcsin\frac{\beta}{\alpha}), & \beta<\alpha \\ (\alpha^2-\beta^2)^{\frac{1}{2}}, & \beta>\alpha \end{cases}$$

one has

$$\sum_{k=0} A_k \int_0^\infty \frac{e^{2\upsilon' H'}+e^{-2\upsilon' H'})}{\upsilon'(e^{2\upsilon' H'}-e^{-2\upsilon' H'})}\mathrm{J}_{2k}(\xi_1)\cos\xi_1 x_1 d\xi_1 = G\Delta', \quad 0<x_1<1 \tag{21}$$

in which

$$H' = \frac{H}{a}, \quad \upsilon' = (\xi_1^2 - \omega^{*2})^{\frac{1}{2}}, \quad \Delta' = \frac{\Delta}{a}$$

Expanding $con\xi_1 x_1$ into a series of Jacobi polynomials in the interval $|x_1| \le 1$

$$con\xi_1 x_1 = 2\sum_{n=0} \mathrm{J}_{2n}(\xi_1) G_n(0,\frac{1}{2},x_1^2), \quad |x_1| \le 1$$

and substituting it into (21), comparing the coefficients of $G_n(0,\frac{1}{2},x_1^2)$, one has

$$\sum_{k=0} A_k\, D_{2k,0} = \frac{G\Delta'}{2}$$
$$\sum_{k=0} A_k\, D_{2k,2n} = 0, \qquad n = 1,2,3\cdots \tag{22}$$

Here,

$$D_{2k,2n} = \int_0^\infty \frac{e^{2\upsilon' H'} + e^{-2\upsilon' H'}}{\upsilon'(e^{2\upsilon' H'} - e^{-2\upsilon' H'})} \mathrm{J}_{2k}(\xi_1) J_{2n}(\xi_1) d\xi_1 \tag{23}$$

Assuming the mass of the foundation to be M_0, the force acting on the foundation to be $Qe^{i\omega t}$, the reactive force of the ground to the foundation to be $Pe^{i\omega t}$, thus the balance of the foundation is

$$-M_0\omega^2\Delta + p = Q \tag{24}$$

in which

$$P = 2a\int_0^a \frac{1}{(1-x_1^2)^{\frac{1}{2}}} \sum_{n=0} A_n(\omega^*) G_n(0,\frac{1}{2},x_1^2) dx_1 = \pi a A_0$$
$$\Delta' = \frac{\pi A_0 - \frac{Q}{a}}{G\gamma_1\omega^{*2}}, \quad \gamma_1 = \frac{M_0}{M_s}, \quad M_s = \rho\, a^2 \tag{25}$$

Substituting (25) into (22), one has

$$(2\gamma_1\omega^{*2}\, D_{0,0} - \pi)A_0 + 2\gamma_1\omega^{*2} \sum_{k=1} A_k D_{2k,0} = -\frac{Q}{2G}$$
$$\sum_{k=1} A_k D_{2k,2n} = 0, \qquad n = 1, 2, 3, \cdots \tag{26}$$

Solving the equations (26) from the known value Q and substituting the obtained A_k into (25) and (19), the displacement of the foundation and the distribution of the contact stress are reached.

NUMERICAL RESULTS

In Fig.5 the displacement time history curves are shown in the case that the elastic half space and the elastic layer with or without damping are suffered by a pulse line load . It can be seen from this figure that as the relative thickness H/x increases, the surface displacement of the elastic layer approaches to that of the elastic half space gradually. When the damping constant c_s/ζ increases, the attenuation of displacement becomes rapid. The distribution of the contact stress between the foundation and the ground is shown in Fig.6. It can be seen from this figure that, when the thickness of the elastic is about 5 time of the width of the foundation, there is no more difference to the stress distribution for the preceding two cases. The curves of the displacement to the frequency for the rigid foundation in the anti-plane contact problem are presented in Fig.7. It can be seen that for the case of higher frequency, there is small difference to the displacement for the elastic half space and the elastic layer. For the lower frequency case, there is much more difference for the preceding two cases. But when the thickness of the layer is about 2.5 time of the width of the foundation, the difference between the two cases much more decreases.

REFERENCES

[1] X. Zeng, A.S.Cakmak, 1984, “Dynamic structure-soil interaction under action of vertical ground motion, I: Axis symmetrical problem; II: The plane problem”, *Intern. J.Soil Dyn. and Earthq. Engng.*, Vol.3, No.4.

[2] X. Zeng, A.Askar and A.S.Cakmak, 1985, “Soil-structure interaction in coupled rocking and sliding vibration--The plane problem”, *Intern. J.Soil Dyn. and Earthq. Engng.*, Vol.4, No.3.

X. Zeng, 1987, “Anti-plane dynamic foundation-soil-foundation interaction for incident plane SH waves”, *3rd Symp. of Intern. Conf. on Soil Dyn. and Earthq. Engng., CML Publications*.

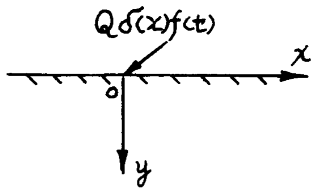


Fig. 1 Case of line load

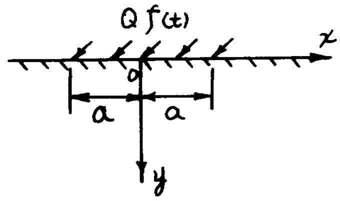


Fig. 2 Case of uniform load

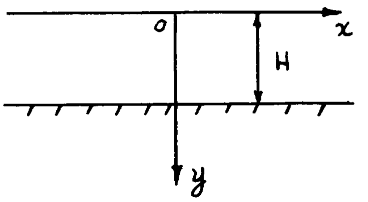


Fig. 3 Elastic layer with viscosity

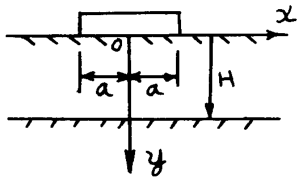
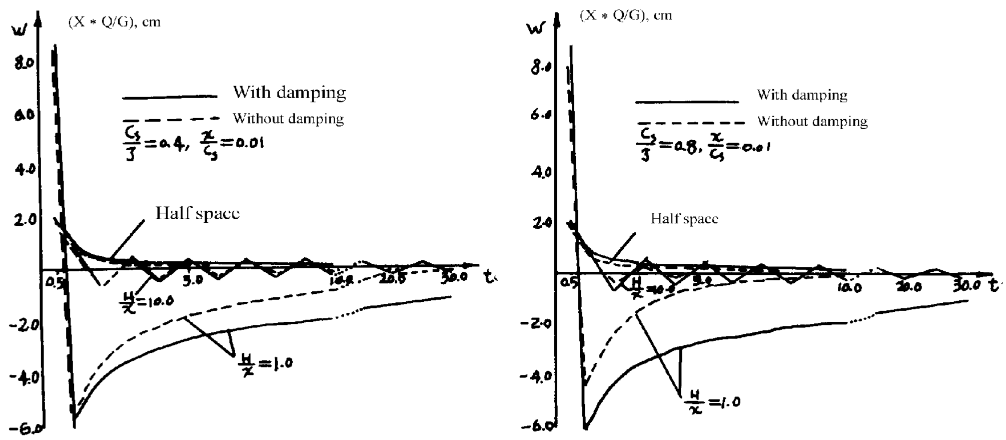


Fig. 4 The contact problem



(a) (b)

Fig. 5 The distribution of contact tress

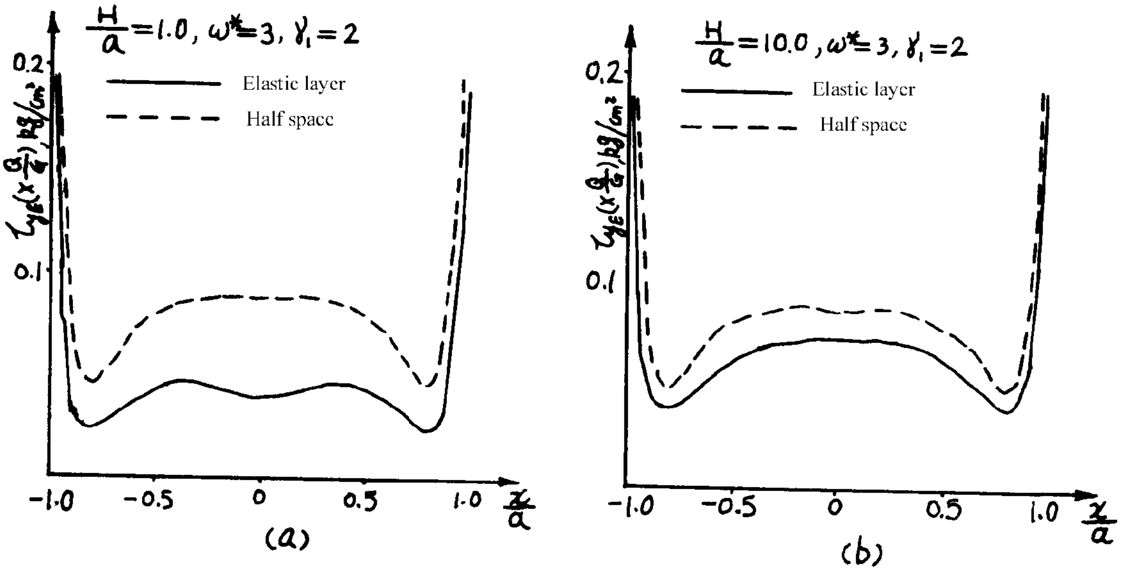


Fig. 6 The variation of contact stress with H/a

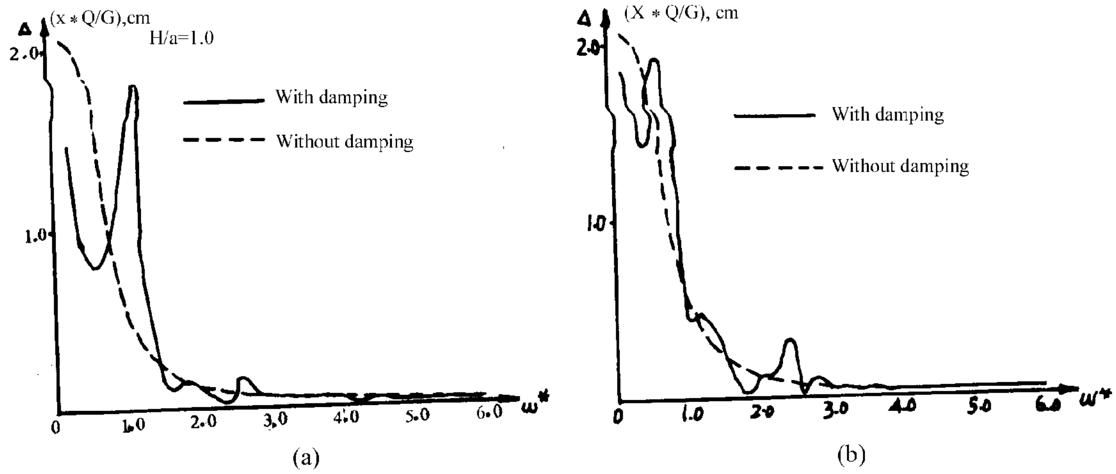


Fig. 7 The variation of displacement of rigid foundation with freguency