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MODELS FOR CONCRETE COVER SPALLING AND REINFORCEMENT BUCKLING OF REINFORCED CONCRETE

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SUMMARY

To maintain seismic resistance of reinforced concrete bridge piers and main towers, their members are designed to fail in bending. Models to analyze concrete cover spalling and reinforcement buckling need to be developed for accurate evaluation of ductility in such members. New analytical models were developed for longitudinal reinforcement in a member that fails in bending. In these models, the following four factors that govern the member's buckling behavior are included: 1), the property of concrete material that surround the longitudinal reinforcing bars under tensile stress, 2), the confining force of transverse reinforcement (hoops and ties), 3), the bending stiffness of longitudinal reinforcement, and 4), the initial deformation according to member curvature. We introduced the following idea for determining the timing of concrete cover spalling. Spalling occurs when the width of a crack along the entire length of a buckled reinforcement exceeds the critical width where confinement force of concrete cover can not be provided. This idea assumes that the amount of buckling deformation equals the width of a crack along the longitudinal reinforcement. Moreover, the potential energy from deformation caused by spalling and buckling was included. These models, if installed into a deformation analysis, should enable the evaluation of ductility in concrete members with various reinforcement arrangements.

INTRODUCTION

Reinforced concrete (RC) piers and main towers are usually designed to fail in bending in order to assure seismic safety. For accurate evaluation of ductility of these RC members, it is necessary to develop analysis models for spalling of concrete cover and buckling of longitudinal reinforcement, both of which are observed after the maximum strength of the members. However, generalized analysis models for these post-peak behaviors of these members, in which effect of specific reinforcement arrangement and combined sectional force can be included, are not yet to be developed. Especially, studies on cases that concrete cover spalling and reinforcement buckling occur simultaneously are rare. Thus, analysis models for post-peak behaviors of these members that fail in bending is proposed in this paper. In this model, the effect of the following four determinant factors of concrete cover spalling and reinforcement buckling can be considered: 1), tensile characteristics of concrete confinig the longitudinal reinforcement; 2), confining effect of transverse reinforcement (hoops and intermediate ties); 3), bending stiffness of the longitudinal reinforcement including effects of plasticity; and 4), initial deformation of reinforcement relative to the curvature of the member.

ANALYTICAL STUDY ON CONCRETE COVER SPALLING

Here, it was assumed that cracks in concrete along longitudinal reinforcement develop due to minor buckling deformation of the reinforcement. Potential energy equilibrium of the member subjected to minor buckling deformation is considered. It was included into the model configuration that longitudinal reinforcement is subjected to compression stress with undertaking initial deformation induced by bending deflection of the member. On the other hand, it has been already known that buckling length ℓ of longitudinal reinforcement can

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be fairly accurately estimated by observing the changes in apparent modulus of elasticity E due to plasticization of the longitudinal reinforcement, and by setting the confining spring β of transverse reinforcement [Suda, Murayama, Ichinomiya, Shimbo, 1996]. Thus, the buckling model was developed assuming that the buckling length is known. The symbols and complicated equations for deriving those appearing in the paper are shown separately at the end of the paper.

$$\ell = 4.4\ \sqrt[4]{\frac{EI}{\beta}} \tag{1}$$

Modeling

As a model of a longitudinal reinforcement confined by concrete cover, a beam fixed at both ends having initial deflection and a linear spring (spring constant: kc) at the center of span and was assumed, as shown in Figure 1.

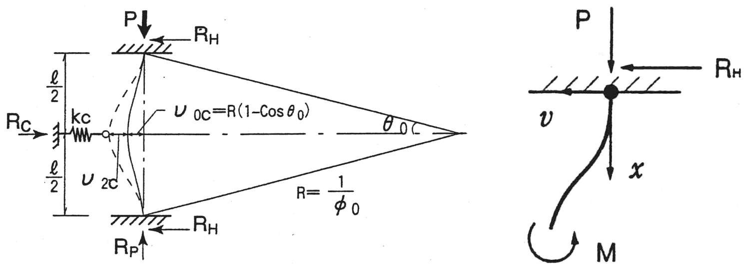


Figure 1: Simplified Model for study

Equilibrium equation of the system shown in Figure 1 is represented by Equation (2):

$$-EI\frac{d^2\upsilon_2}{dx^2} = P(\upsilon_0 + \upsilon_2) - \frac{k_c\upsilon_{2C}}{2}x \tag{2}$$

Referring to buckling shape of a fixed-ends beam, initial deflection is expressed by Equation (3),:

$$\upsilon_0 = \upsilon_{0C}\sin^2\frac{\pi x}{\ell} \tag{3}$$

By substituting Equation (3) into Equation (2) and solving it under boundary conditions, Equation of buckling deformation is derived as follows:

$$\upsilon_2 = \frac{\upsilon_{0C}\alpha}{2(1-\alpha)}\zeta\left(\frac{x}{\ell}\right) \tag{4}$$

Critical load of plasticized reinforcement

By substituting Equation (4) into Equation (5), bending moment of reinforcement is derived as Equation (6):

$$M = -EI\frac{d^2\upsilon_2}{dx^2} \tag{5}$$

$$M = -\frac{P_{E0}\upsilon_{0C}\alpha}{2(1-\alpha)}\left\{-\alpha Cot\frac{\pi\sqrt{\alpha}}{2}Sin\frac{2\pi\sqrt{\alpha}x}{\ell} + \alpha Cos\frac{2\pi\sqrt{\alpha}x}{\ell} - Cos\frac{2\pi x}{\ell}\right\} \tag{6}$$

Besides, suppose P becomes maximum when the bending moment of reinforcement reaches the total plastic (the whole part of the cross section become plastic) moment, the critical load $P_{m,cr2}$ restricted by material characteristics of reinforcement is obtained by Equation (7):

$$P_{m,cr2} \cong 4P_{E0} - \frac{42P_{E0}^2\upsilon_{0C}}{d^3\sigma_y + 12P_{E0}\upsilon_{0C}} \tag{7}$$

Confining force of concrete cover

In Figure 1, confining spring of concrete cover was assumed to be a linear spring. Here, relationship between the tensile stress σ_c and crack width ω is shown as Equation (8).

$$\sigma_c(\omega)= f_t\left(1-\frac{\omega}{\omega_u}\right) \tag{8}$$

First, the width of cracks of concrete cover along longitudinal reinforcement was assumed to equal the incremental displacement υ_2 (i.e., $\upsilon_2 \equiv \omega$). If the crack width is to reach the critical crack width ω_u when $x = x_0$, Equation (9) is derived based on Equation (6).

$$\omega_u = \frac{\upsilon_{0C}}{2}\frac{\alpha}{1-\alpha}\zeta\left(\frac{x_0}{\ell}\right) \tag{9}$$

By substituting Equations (4) and (9) into Equation (8) and then integrating the resultant function along the x-axis, reaction force R_c of the confining spring of concrete cover is obtained as shown in Equation (10):

$$R_c = 2\int_0^{\frac{x_0}{\ell}} \sigma_c\left(\frac{x}{\ell}\right)\cdot a\cdot\ell\cdot d\,\frac{x}{\ell} = 2 f_t a\ell \int_0^{\frac{x_0}{\ell}} \left\{1-\frac{\zeta\left(\frac{x}{\ell}\right)}{\zeta\left(\frac{x_0}{\ell}\right)}\right\} d\,\frac{x}{\ell} \qquad \therefore\quad \frac{R_c}{f_t a\ell} = 2\left\{\frac{x_0}{\ell} - \frac{1}{\zeta\left(\frac{x_0}{\ell}\right)}\int_0^{\frac{x_0}{\ell}} \zeta\left(\frac{x}{\ell}\right) d\,\frac{x}{\ell}\right\} \tag{10}$$

A unique relationship between x_0 and α is determined if ω_u and υ_{0C} are given from Equation (9). Furthermore, a unique relationship between R_c and ω is also determined, since Equation (10) correlates R_c with x_0. The above equations govern the relationship between the crack width and confining force. If loading and unloading are repeated as shown in Figure 2, then relationship shown in Figure 3 is derived. This is an example of a case subjected to cyclic axial compression load under the condition $\omega_u/\upsilon_{0C} = 0.1$.

Figure 3 is shown to help clarify the changes in confining force of concrete cover when cracks have already developed along longitudinal reinforcement. It is not intended to describe material deterioration of concrete under reversed loading. Therefore, the number of cycles has no significant meaning. For each of the cycles, the internal energy accumulated in the confining spring of concrete cover is obtained by multiplying the area (A_G) surrounded by the curves shown in Figure 3 by $f_t a\ell\cdot\upsilon_{0C}$. A_G is a function of only ω_u/υ_{0C} and $\upsilon_{2C}/\upsilon_{0C}$.

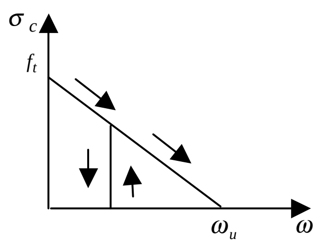


Figure 2: Relationship between σ_c and ω

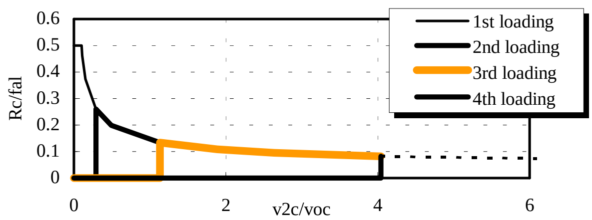


Figure 3: Confining force under reversed loading

Derivation of a stability equation against buckling

An equation for assessing stability/instability of a system against buckling was derived according to Timochenko's energy method.

In a stable state of equilibrium, it is known that the total potential energy• takes the minimum value. Thus, in a vicinity of this equilibrium, the change in the total potential energy $\Delta\pi$ is represented by Equation (11):

$\Delta\pi$ •ΔU + ΔV (11)

Here, ΔU is an increment in the internal energy accumulated by minute displacement in a close vicinity of the equilibrium, and ΔV is an increment of external potential energy in the same time interval. If •T is an increment of work done by the external force, then ΔV=-ΔT, and Equation (12) is derived:

$\Delta\pi$ •ΔU – ΔV (12)

A stable equilibrium is obtained when ΔU>ΔT ($\Delta\pi$ >0), and an unstable equilibrium is obtained when ΔU<•T ($\Delta\pi$ <0). The following two cases are assumed as equilibriums when the axial compression force P acts on the longitudinal reinforcement of the structural system shown in Figure 1, where buckling is involved:

(i) equilibrium with respect to initial deflection $\upsilon_0(x)$; and

(ii) equilibrium in extremely close vicinity of (i). (Deflection $\upsilon_2(x)$ was induced by buckling deformation.)

A minor increment in deflection is included for the reinforcement when the state described in (i) shifts to (ii). However, it was assumed that the magnitude of axial compression force P does not change and its direction remains constant. Internal energy ΔU accumulated in the non-linear spring of concrete cover due to bending deformation of the reinforcement while the state shifted from (i) to (ii) is expressed by Equation (13):

$$\Delta U = 2\left\{\frac{1}{2} EI \int_0^{\frac{\ell}{2}} \left(\frac{d^2\upsilon_2}{dx^2}\right)^2 dx\right\} + \int_0^{\upsilon_{2C}} R_c(\omega) d\omega \tag{13}$$

On the other hand, bending deformation of the reinforcement due to buckling causes axial compression force P to shift vertically downward for a distance e_2, through which the work ΔT of an external force is performed. ΔT is represented by Equation (14):

$$\Delta T = P \cdot e_2 = P \int_0^{\frac{\ell}{2}} \left\{\left(\frac{d\upsilon_2}{dx}\right)^2 + 2\left(\frac{d\upsilon_2}{dx}\right)\left(\frac{d\upsilon_0}{dx}\right)\right\} dx \tag{14}$$

The second term of Equation (13) can be represented as Equation (15), which shows an internal energy accumulated in the confining spring until the amount of shrinkage reaches:

$$\int_0^{\upsilon_{2C}} R_c(\omega) d\omega = f_t a \ell \upsilon_{0C} \cdot A_G\left(\frac{\omega_u}{\upsilon_{0C}}, \frac{\upsilon_{2C}}{\upsilon_{0C}}\right) \tag{15}$$

By substituting Equations (13) through (15) into Equation (12) and reorganizing the resultant equation, an stability equation against buckling is derived as Equation (16):

$$\frac{\Delta\pi}{P_{E0}\ell} = \frac{\upsilon_{0C}^2}{\ell^2} \frac{\alpha^4}{(1-\alpha)^2} \cdot g(\alpha) + \frac{\upsilon_{0C} f_t a}{P_{E0}} A_G\left(\frac{\omega_u}{\upsilon_{0C}}, \frac{\upsilon_{2C}}{\upsilon_{0C}}\right) \tag{16}$$

In Equation (16), the term $\Delta\pi$ is normalized with dividing it by $P_{E0} \cdot \ell$. The first term is a function of geometric configuration (the length of a span of reinforcement and its initial deflection) of the given system and the ratio α of axial compression load, while the second term is represented by the ratio between the spalling energy of concrete cover and buckling load of straight reinforcement fixed at both ends.

Stability of system

For further clarification of Equation (16), relationship between the first term and α are shown in Figures 4, relationship between the second term and α in Figure 5, and relationship between the dimensionless $\Delta\pi$ and α in Figure 6. The first term is a function of only the geometrical configuration (υ_{0C}/ℓ) of the system and α, and whether it is positive or negative is determined by only α. The first term becomes 0 when α =2.56, regardless of the value of υ_{0C}/ℓ .(See Figure 4.). In Figure 5, the values of the second term corresponding to that in the first and fourth cycles shown in Figure 3 are indicated. In the first cycle, the second term is much larger than the first term, and $\Delta\pi$ remains positive even if the first term became negative when α is larger than 2.56. On the other hand, $\Delta\pi$ turns negative in the fourth cycle as shown in Figure 6, since the second term remains 0 even if α is greater than 2.56, until buckling deformation becomes a certain degree. (See Figure 3.)

From the above, in case of the reinforcement embedded in concrete, it is easily imaginable that $\Delta\pi$ remains positive while confining force of concrete cover is not 0 even after cracks occur along axial reinforcement. This is because potential energy represented by the second term is larger than the first term by the order of two digits.

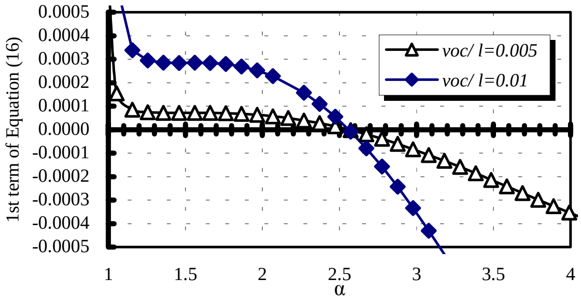


Figure 4: Relationship between α and the 1st term of Equation (16)

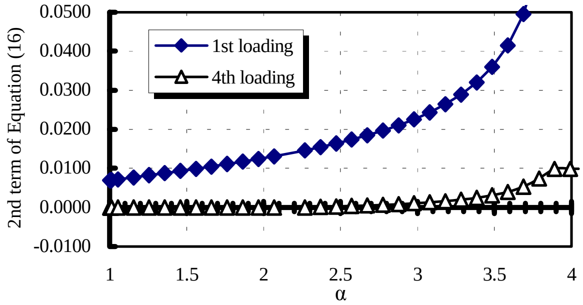


Figure 5: Relationship between α and the 2nd term of Equation (16)

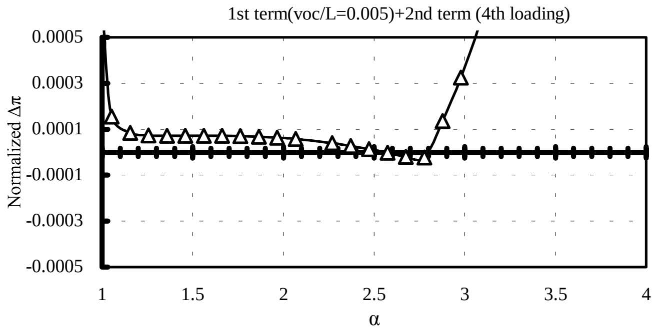


Figure 6: Relationship between α and normalized $\Delta\pi$

MODELS FOR CONCRETE COVER SPALLING AND REINFORCEMENT BUCKLING

Analytical study on reinforcement buckling after concrete cover spalling was also performed to derive models shown in Figure 7. The outline of processes to predict the timing of concrete cover spalling and reinforcement buckling is as follows:

(i) Compression force P acting on reinforcement, apparent modulus of elasticity E considering the plasticization of reinforcement, and curvature of the member ϕ_0 are read from the main routine;

(ii) Calculation of buckling length $\ell_{0,cr}$ is performed;

(iii) Calculation of initial deflection υ_{0C} is performed;

(iv) Predict whether concrete cover spalls or not; (If not, return to the main routine.)

(v) Predict whether reinforcement buckles or not; (If not, return to the main routine.)

(vi) Hysteresis rule for the stress-strain relationship of reinforcement after buckling is applied.

Prediction of concrete cover spalling

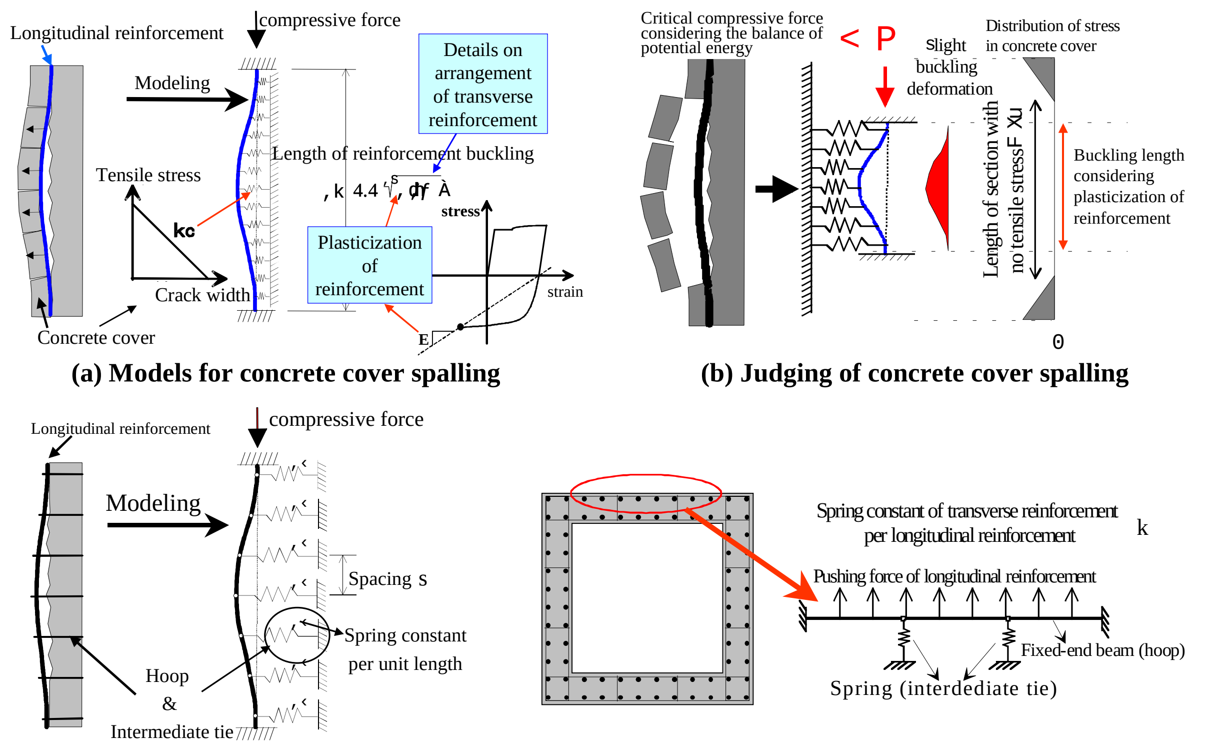
The following two were assumed as conditions for concrete cover spalling:

(a) Throughout the length $\ell_{0,cr}$, crack width has already become wider than the critical crack width; concrete cover can no longer provide the confining force because of the past reversed loading.

(b) The increment of work performed by external forces has exceeded the increment in internal energy of longitudinal reinforcement.

Condition (a) can be satisfied when the longitudinal reinforcement in concern is subjected to cyclic compression/ tensile loads, or when the plasticity of reinforcement proceeds under monotonic compression loading and $\ell_{0,cr}$ decreases. In other words, the condition (a) enabled comprehensive representation of the monotonic and cyclic loading conditions. In cases the direction of the compression stress and the longitudinal reinforcement is almost the same, potential energy induced by confining effect of concrete cover would be much larger than others. As such, it was assumed that the simplified modeling presented in this study would not affect in any significant way the prediction of concrete cover spalling. Condition (b) is satisfied when the ratio of axial compression load α equals 2.56 regardless of the initial deformation of longitudinal reinforcement, if confining effect of concrete cover is ignored. Thus, concrete cover spalls when both the conditions (a) and (b) are satisfied.

Since the confining spring of concrete cover has been simplified as one spring in the center of the span in the model, it does not represent the condition in which the concrete cover confines the reinforcement along the entire span. As a result, increment of deformation at the ends has been over-estimated. In the study, the upper limit of buckling length ($\ell_{0,cr,\max}$) was set to cope with this situation. Furthermore, the critical load $P_{m,cr2}$ determined by material characteristics of reinforcement, which is calculated based on the total plastic moment (bending moment of a bare bar when the whole section of the bar is plasticized), was set as the maximum compression force acting on it. The standard yield strength or the yield strength obtained in tensile tests would be used as σ_y used for calculation of $P_{m,cr2}$. However, this assumption may be inconsistent for setting a value of critical load for reinforcement that has already plasticized. For structures such as piers aimed at in this study, however, the effect of accuracy of the critical load $P_{m,cr2}$ on the prediction of concrete cover spalling is deemed negligible because the compression force borne by reinforcement that has buckled would be relatively small.



(a) Models for concrete cover spalling **(b) Judging of concrete cover spalling**

(c) Models for reinforcement buckling **(d) Modeling of transverse reinforcement**

Figure 7: Models for concrete cover spalling and reinforcement buckling

Prediction of reinforcement buckling

Whether longitudinal reinforcement buckles or not is judged based on the buckling load $P_{0,cr}$ derived using a buckling model of straight reinforcement which is confined only by transverse reinforcement. (See Figure 7 (c).)

Buckling is judged when the compression force acting on the reinforcement exceeds $P_{0,cr}$.

Thereafter, hysteresis rule after buckling [Suda, Murayama, Ichinomiya, Shimbo, 1996] was applied thereafter for the stress-strain relationship of the reinforcement element.

Strictly speaking, compression force which the reinforcement can bear under buckling deformation would differ according to initial deformation of the reinforcement. However, this effect was deemed insignificant for judging of buckling since the load equals $P_{0,cr}$ when buckling deformation is infinitely large. Another point to note is that for discussion of hysteretic absorption energy after buckling, it is necessary to base on the relationship between initial deformation and compression force. For structures such as piers, however, diameters of reinforcement is small compared to dimensions of the members. As such, compression force which reinforcement can bear after buckling would be relatively small compared to cases where reinforcement with large diameters has been used. Hence, it was deemed that minute changes in buckling load of the reinforcement would give only small effect on the strength, deformation and failure behavior of the members.

For the same reason as the prediction of concrete cover spalling, the critical load $P_{m,cr1}$ determined by the material characteristics of reinforcement, which is calculated based on the total plastic moment (bending moment of a bare bar when the whole section of the bar is plasticized), was also set as the maximum compression load exerted on it.

CONCLUSION

In this paper, new analysis models for concrete cover spalling and reinforcement buckling was proposed. Proposed models enable prediction of these phenomena based on only four values of the compressive force, stress-strain hysteresis, curvature (these three values are related with longitudinal reinforcements) and the critical crack width of concrete cover. Therefore models can be installed into existing analytical software easily.

In particular, proposed models have made it possible to include the effects of reinforcement arrangements directly. As a result, analytical studies on the effect of the difference in reinforcement arrangements on ductility and post-peak behavior of RC structures are made possible. This enables to evaluate the effectiveness of reinforcement arrangements more easily, and determine structural details of design code with clearer grounds.

SYMBOLS

$\Delta\pi$; changes in the total potential energy

$\alpha = \dfrac{P}{P_{E0}}$; ratio of axial compressive force

$P_{E0} = \dfrac{4\pi^2 EI}{\ell^2}$; buckling load of straight reinforcement fixed at both ends

P ; axial compressive force acting on longitudinal reinforcement

$P_{m,cr1} = \dfrac{d^3\sigma_y}{4.8\upsilon_{0C}} + 0.025P_{0,cr}$; the critical load determined by material characteristics without concrete cover

d ; diameter of the longitudinal reinforcement

$\upsilon_{0C} = \dfrac{\phi_0 \ell^2}{8}$; initial deflection at the center of the span of reinforcement

ϕ_0 ; average curvature of reinforcement just before buckling (i.e., the average curvature of the member)

$\ell \bullet \ell_{0,cr}$; buckling length of reinforcement

$$g(\alpha) = \frac{\left\{ -\frac{1}{2} + \left(\frac{1}{2} + \frac{1-\alpha}{4\alpha^2} \right) Sin^2 \frac{\pi\sqrt{\alpha}}{2} + \frac{Sin\left(2\pi\sqrt{\alpha}\right)}{2\pi\sqrt{\alpha}\,(1-\alpha)} + \frac{\sqrt{\alpha}\,Sin\left(\pi\sqrt{\alpha}\right)}{\pi(1-\alpha)} \right\}}{Sin^2 \frac{\pi\sqrt{\alpha}}{2}}$$; function of axial compression load

f_t ; tensile strength of concrete

a ; horizontal interval between longitudinal reinforcements

$A_G\left(\dfrac{\omega_u}{\upsilon_{0C}}, \dfrac{\upsilon_{2C}}{\upsilon_{0C}}\right)$; index of concrete cover spalling energy; normalized internal energy accumulated in concrete cover along the whole buckling length of reinforcement accompanying with the increments of crack width induced by buckling deformation

ω_u ; the critical crack width of concrete cover

x_u ; experienced maximum length that crack width exceeds ω_u

$\upsilon_{2C} = \dfrac{\pi \upsilon_{0C} \alpha \sqrt{\alpha}}{2(1-\alpha)} Cot \dfrac{\pi\sqrt{\alpha}}{2}$; increment in deflection at the center of the span due to buckling deformation of reinforcement (the width of cracks along the reinforcement at the center of the span)

$$\zeta\left(\frac{x}{\ell}\right) = Cot\frac{\pi\sqrt{\alpha}}{2} Sin 2\pi\sqrt{\alpha}\frac{x}{\ell} - Cos 2\pi\sqrt{\alpha}\frac{x}{\ell} + Cos 2\pi\frac{x}{\ell} - 2\pi\sqrt{\alpha}\frac{x}{\ell} Cot\frac{\pi\sqrt{\alpha}}{2}$$; deflection distribution mode

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