OPTIMAL STRATEGY FOR BUSINESS RECOVERY AFTER EARTHQUAKES

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SUMMARY

A methodology for choosing the best recovery strategy is presented for a company that owns multiple income-producing properties damaged in an earthquake. The basic idea is to choose recovery actions that maximize the net asset value of the properties owned by the company. This methodology helps to address the question of whether each property should be repaired or demolished. It allows the optimal expenditure rate and optimal time for repair or demolition to be determined. Furthermore, when the total cost of repairs is subject to a budget constraint, the methodology shows whether each property should be repaired immediately, delayed or never repaired.

INTRODUCTION

When an earthquake causes damage to multiple properties owned by a company, numerous decisions are required on how to make best use of the company's resources for recovery. Most of these decisions are financial and quantitative in nature, and the choices made are key to the effectiveness of recovery efforts. Despite the importance of this process, only a few researchers have studied quantitative tools to assist management in earthquake recovery [e.g. Cheng and Wang, 1996; Beck et al, 1999].

In this paper, we present a methodology for determining optimal recovery strategies for a company that owns income-producing buildings. The basic idea is to choose that recovery action among a set of possibilities that maximizes the net asset value of the properties owned by the company. The theory is applicable to many types of businesses that own properties subject to possible earthquake damage. However, the focus here is on a company that owns commercial property that it leases out.

The primary decisions following an earthquake are whether to repair or demolish each building that has been damaged and the speed at which repairs or demolition should be performed. For each building, the decision to repair or demolish depends on the present value of the building after accounting for repair costs when compared to the cost of demolition and either rebuilding the structure or selling the land. The speed of repairs or demolition may be chosen to maximize the present value of the property based on the work being completed.

In the following, we assume that the damage to structural and nonstructural components of multiple rental units owned by a company is known. A rental unit corresponds to a portion of a building that is under one lease, or is available for such a lease, before the earthquake. It may correspond to one or more floors, or a portion of a floor in a building. It is assumed that rental units may be repaired in parallel. The components of a rental unit (member connections, wall partitions, glazing, and so on), must be repaired before the rental unit can be occupied and rent received. It is assumed that all of the components of a unit must be repaired in series.

OPTIMAL EXPENDITURE RATE FOR REPAIRS

Here, we address the question of how rapidly repairs should be performed. To minimize lost income, the repairs should be done as rapidly as possible. On the other hand, this may require putting so many workers on the job that they get in each other's way and their efficiency is reduced. In addition, the present value of the cost of

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repairs is reduced if the repairs are done more slowly because of the discounting of future costs; conceptually, funds that are put aside at the present time can earn interest before they are spent on repairs at a later time. We first consider in some detail the case of one rental unit with one damaged component to be repaired. This case forms a basic element of the methodology. We then extend this to the case of multiple rental units.

One Rental Unit with One Damaged Component

Consider a single rental unit that has a rental income rate $R(t)$ and operating expense rate $E(t)$ at time t. The value of the rental unit at t=0 can be taken as the present value of its future net income stream

$$I = \int_0^\infty (R(t) - E(t))e^{-rt} dt \tag{1}$$

where r is the continuous discount rate. If an earthquake occurs at time t=0 and damages the rental unit so that it is not functional, the company receives no rental income until repairs are completed but it must still pay the operating expenses. If the time before repairs are completed is T, then the present value of the lost income and the cost of repairs for the rental unit are

$$L(T) = \int_0^T R(t)e^{-rt} dt \quad \text{and} \quad C(T) = \int_0^T x(t)e^{-rt} dt \tag{2}$$

where $x(t)$ is the rate of expenditure at time t to repair the damage. This is the control variable that will be used in seeking the optimal rate of repair. Although in reality $x(t)$ could not be changed continuously, the continuous-time cost-benefit analysis presented here gives insight into the recovery process.

Let D denote a *damage index* for the rental unit, that is, a scalar measure of the damage caused by the earthquake. For example, the Park-Ang index [Park and Ang, 1985] could be used or D may be defined as the "standard" cost of repair using a minimum size crew. (The actual cost of repairs can differ from the standard cost if multiple crews are used because this can alter the labor efficiency). Suppose that the rate at which the damage is repaired is related to the expenditure rate x through a *recovery function* $f(x)$. It is evident that

$$D = \int_0^T f(x(t))dt \tag{3}$$

The recovery function depends on the nature of the damage index and the required repairs, as well as the labor efficiency for the job. Clearly, $f(0)=0$. We further assume that for all x, $f'(x)>0$ and $f''(x)<0$. These last two properties model the "law of diminishing returns", that is, as the expenditure rate increases, the improvement in the rate of damage repair decreases.

The asset value of the rental unit after damage is repaired at time T is the present value of its original income stream less the present value of the lost income and repair costs

$$V(T) = I - L(T) - C(T) = I - \int_0^T \big(R(t) + x(t)\big)e^{-rt} dt \tag{4}$$

The optimal expenditure rate strategy can now be formulated as follows: find the repair time T and expenditure rate profile $x(t)$ for $t \in [0, T]$ that maximize the functional in Eq. (4) subject to the time-independent constraint that all the initial damage must be repaired, as given in Eq. (3). When the conditions for the recovery function are satisfied, the asset value $V(T)$ does have a maximum which can be found using the calculus of variations by introducing a Lagrange multiplier λ and then finding the unconstrained extremum of the modified functional

$$V^*(T) = I - \int_0^T \big(R(t) + x(t)\big)e^{-rt} dt + \lambda\Big(D - \int_0^T f(x(t))dt\Big) \tag{5}$$

The Euler equation for the optimal expenditure rate $x(t)$ for this variational problem is $e^{-rt} + \lambda\, df/dx = 0$ and so

$$f'(x(t)) = f'(x(0))e^{-rt} \qquad \text{or, by differentiation,} \qquad \dot{x}(t) = -r\, f'(x)/f''(x) \tag{6}$$

Taking into account the assumptions made on the recovery function, we can see that for positive discount rate r, the optimal expenditure rate increases over time.

To determine the optimal repair time $\hat{T}$, the condition $dV^*/dT = 0$ is imposed, which gives

$$R(\hat{T}) + x(\hat{T}) = f(x(\hat{T})) / f'(x(\hat{T})) \tag{7}$$

Let $x(t)=X(t; x_0)$ be the solution of Eq. (6) with initial value $x(0)=x_0 \geq 0$. Substituting this solution in the constraint Eq. (3), one finds the repair time $T = T(x_0)$ as a function of the initial rate of expenditure x_0. Then, we can deduce the optimal value $\hat{x}_0$ from Eq. (7) using the fact that the optimal repair time $\hat{T} = T(\hat{x}_0)$. The optimal rate of expenditure $X(t; \hat{x}_0)$ is then completely defined. The corresponding optimal asset value is

$$\hat{V}(\hat{T}) = I - \int_0^{\hat{T}} R(t)e^{-rt}dt - e^{-r\hat{T}}\, R(\hat{T})/r + \left[f(\hat{x}_0)/f'(\hat{x}_0) - \hat{x}_0\right]/r \tag{8}$$

The above analysis assumes that $\hat{x}_0 > 0$ and so the optimal x(t) is always positive and increasing. A more complete analysis can be done in which $x(t)$ is constrained to be non-negative by adding a term $\int_0^T \sigma(t)x(t)dt$ to $V^*(T)$ where $\sigma(t)$ is a Lagrange multiplier satisfying the Kuhn-Tucker conditions: $\sigma(t)>0$ if $x(t)=0$ and $\sigma(t)=0$ if $x(t)>0$. The Euler equation for the extremum of the revised V^* remains the same if $x(t)>0$ and so Eq. (6) still holds in this case. The Euler equation also shows that the optimal expenditure profile is always continuous. Therefore, if $x(t)$ is not always positive, then $x(t) = 0$ if $t \in [0, t_0]$ and $x(t) = X(t - t_0; 0)$ if $t \in [t_0, t_0 + T_0]$ where T_0 is the repair duration, that is, $D = \int_0^{T_0} f(X(t,0))dt$. Thus, it may be optimal to wait for a time t_0 before starting repairs. This possibility can be introduced in an analysis similar to the budget-constraint case presented later. This analysis shows that a necessary and sufficient condition for $X(t; \hat{x}_0)$ to be the optimal profile with $\hat{x}_0 > 0$ is

$$R(T_0) > f(X(T_0;0)) / f'(X(T_0;0)) - X(T_0;0) \tag{9}$$

This condition is equivalent to the present value of the rental income rate received after repairs, $R(T_0)e^{-rT_0}$, giving a return on the optimal cost of repairs, $\hat{C}(T_0)$, that exceeds the discount rate r when the expenditure rate on repairs is $X(t;0)$ for $t \in [0, T_0]$. If Eq. (9) is not satisfied, there are two important cases:

(a) $R(t)$ for $t > T_0$ is non-increasing: In this case, the optimal recovery strategy is to never start repairs, that is, $x(t)=0$ for $t \geq 0$. Conceptually, the rental income rate remains so low that the cost of repairs can never be compensated. In reality, local government regulations would most likely not allow this option and so demolition must be considered. The optimal asset value when the rental unit is never repaired is

$$V_0 = I - \int_0^{\infty} R(t)e^{-rt}dt = -\int_0^{\infty} E(t)e^{-rt}dt \tag{10}$$

(b) $R(t)$ for $t > T_0$ is non-decreasing: Suppose first that there exists $t_0 \geq 0$ such that

$$R(t_0 + T_0) = f(X(T_0;0)) / f'(X(T_0;0)) - X(T_0;0) \tag{11}$$

then the optimal recovery strategy is to wait for time t_0 (possibly zero) and then use $X(t-t_0;0)$ as the expenditure profile. In this case, repairs are delayed until rents have risen to a level which compensates for the cost of repairs. The optimal repair time is $\hat{T} = t_0 + T_0$ and the optimal asset value is

$$\hat{V}(\hat{T}) = I - \int_0^{t_0+T_0} R(t)e^{-rt}dt - e^{-r(t_0+T_0)}\left(f(X(T_0;0))/f'(X(T_0;0)) - X(T_0;0)\right)/r > V_0 \tag{12}$$

If Eq. (11) is not satisfied for any $t_0 \geq 0$, then the optimal recovery strategy is to never start repairs, as in (a).

If a limit on the expenditure rate is desirable, then an inequality constraint $x(t) \leq K$ may be imposed. In this case, a term $\int_0^T \rho(t)(K - x(t))dt$ is added to $V^*(T)$ in Eq. (5) where $\rho(t)$ is a Lagrange multiplier satisfying the Kuhn-Tucker conditions: $\rho(t)>0$ if $x(t)=K$ and $\rho(t)=0$ if $x(t)<K$. The Euler equation for the extremum of the revised V^* remains the same as before if $x(t)<K$. If $x_0=K$ then $x(t)=K$ for all t because an optimal solution never decreases. If $x(0)=x_0<K$ then Eq. (6) holds while $x(t)<K$. If $x(t)$ never reaches K, then the unconstrained optimal profile applies and the Euler equation for the optimal repair time $\hat{T}$ is the same as Eq. (7). If $x(t)$ reaches K, then the optimal rate of expenditure profile that maximizes the asset value is given by

$$\dot{x}(t) = -r\, f'(x)/f''(x) \quad \text{if } t \in [0,t_s]; \qquad x(t) = K \quad \text{if } t \in [t_s,T] \tag{13}$$

where t_s is the smallest t such that $x(t)=K$. By evaluating the Euler equation for $x(t)=K$, we get an equation for t_s:

$$e^{-rt_s} = f'(K)/f'(x_0) \tag{14}$$

Also, the Euler equation for the optimal repair time $\hat{T}$ becomes

$$(R(\hat{T})+K)e^{-r\hat{T}} = f(K)/f'(\hat{x}_0) \tag{15}$$

where $\hat{T}$ and $\hat{x}_0$ are related through the constraint Eq. (3) and the solution for $x(t)$ from Eq. (13) with $x(0)=\hat{x}_0$. Figure 1 shows some possible optimal profiles of the rate of expenditure over time.

As an illustrative example, consider a recovery function of the form $f(x) = \Omega x^{\alpha}$ with the constraint $0 \leq x(t) \leq K$. Let t_s be the smallest time such that $x(t) = K$ in the case where $x(t)$ reaches K. Then for $t< t_s$, the optimal expenditure rate is governed by Eq. (6), which implies that

$$x(t) = x_0 e^{rt/(1-\alpha)} = X(t;x_0) \tag{16}$$

Eq. (14) then shows that $t_s = [(1-\alpha)/r]\cdot\ln(K/x_0)$. The optimal initial rate of expenditure $\hat{x}_0$ and optimal repair time $\hat{T}$ follow from Eqs. (3) and (7) or (15)

$$D = \Omega\hat{x}_0^{\alpha}\left[e^{r\alpha\hat{T}/(1-\alpha)} - 1\right](1-\alpha)/r\alpha \quad \text{and} \quad \alpha R(\hat{T}) - (1-\alpha)\hat{x}_0 e^{r\hat{T}/(1-\alpha)} = 0 \quad \text{if } t_s \notin [0,\ \hat{T}\,] \tag{17}$$

$$D = \Omega\hat{x}_0^{\alpha}\left[e^{r\alpha t_s/(1-\alpha)} - 1\right](1-\alpha)/r\alpha + \left(\hat{T}\text{-}t_s\right)\Omega K^{\alpha} \quad \text{and} \quad \alpha\left(R(\hat{T}) + K\right)e^{-r\hat{T}} - \hat{x}_0\left(K/\hat{x}_0\right)^{\alpha} = 0 \quad \text{if } t_s \in [0,\ \hat{T}\,] \tag{18}$$

Consider a severely damaged rental unit characterized by the following data: undamaged present value, I = \$1,000,000; damage index, D = 1; discount rate, r = 10% per year; recovery function parameters, α = 0.8, $\Omega = 2\cdot 10e\text{-}5$; rental rate, $R(t)$=\$100,000 per year; maximum expenditure rate, K = \$140,000 per year. The corresponding optimal recovery results are shown in Table 1. The second and third columns are the optimal solutions for no upper limit on $x(t)$ and the upper limit K, respectively. The next column shows the results for an arbitrarily chosen constant profile $x(t) = k$ = \$100,000/yr and the last column gives the results for the optimal constant profile $x(t) = \hat{k}$, a special case which may also be addressed with the methodology presented herein. As one would expect, the asset value is highest for the unconstrained problem and lowest for the arbitrarily chosen non-optimal constant expenditure rate. Note that the difference in asset values between the optimal unconstrained rate and optimal constant rate cases is relatively small. This is a potentially useful observation, since approximating a constant expenditure rate may be more practical than following a specified curve.

Table 1: Example of one rental unit with one damaged component

$f(x) = \Omega x^{\alpha}$	$0 \le \hat{x}(t)$	$0 \le \hat{x}(t) \le K$	$x(t) = k$	$x(t) = \hat{k}$
$\hat{x}_0$ ($ per year)	103,944	95,341	100,000	238,122
$\hat{x}(\hat{T})$ ($ per year)	400,000	140,000	100,000	238,122
$\hat{T}$ (years)	2.70	3.93	5.00	2.50
Cost $\hat{C}(\hat{T})$ ($)	508,834	436,886	393,469	526,287
Asset Value $\hat{V}(\hat{T})$ ($)	259,861	238,353	213,061	252,697

Multiple Rental Units with Multiple Damaged Components

Consider the case where a company owns N rental units (R.U.) and M_i damaged components are to be repaired in unit i for i=1,…,N. The following notation is used:

D_{ij} = damage index for R.U. i and component j	I_i = present value of R.U. i before damage
$x_{ij}(t)$ = rate of expenditure to repair damage D_{ij} at time t	$f_{ij}(x_{ij})$ = recovery function for D_{ij}
T_{ij} = time at which the repair of damage D_{ij} is completed	$R_i(t)$ = rental income rate for R.U. i
L_i = present value of lost income from R.U. i during repairs	E_i(t) = operating expense rate for R.U. i
C_i = present value of the cost to repair rental unit i	r = discount rate
V = asset value of the assemblage of all N repaired rental units	

As in the single rental unit case,

$$I_i = \int_0^{\infty} (R_i(t) - E_i(t))e^{-rt}dt \qquad L_i(T_i) = \int_0^{T_i} R_i(t)e^{-rt}dt \qquad C_i(T_i) = \sum_{j=1}^{M_i} \int_0^{T_{ij}} x_{ij}(t)e^{-rt}dt \tag{19}$$

where $T_i = \max_j T_{ij}$ is the total repair time for rental unit i. The asset value and repair constraint equations become

$$V = \sum_{i=1}^{N} \left(I_i - L_i - C_i\right) \qquad D_{ij} = \int_0^{T_{ij}} f_{ij}(x_{ij}(t))dt \tag{20}$$

For the case with N rental units each having a single damaged component, the formulation and results are exactly analogous to the case of one unit with one component. With no constraint on expenditure rate, the optimal profile for each unit is obtained by solving

$$\dot{x}_i(t) = -r\ f_i'(x_i)/f_i''(x_i) \qquad R_i(\hat{T}_i) + x_i(\hat{T}_i) = f_i(x_i(\hat{T}_i))/f_i'(x_i(\hat{T}_i)) \qquad D_i = \int_0^{\hat{T}_i} f_i(x_i(t))dt \tag{21}$$

where the subscript j=1 in Eq. (19) and (20) has been eliminated. The condition for the optimal repair time in Eq. (21) holds if $\hat{x}_i(0) \ge 0$; otherwise, repairs should be delayed, as described earlier.

Consider now the case of one rental unit with M components to be repaired in series. The subscript i=1 in Eq. (19) and (20) can be eliminated and Eq. (19) becomes

$$I = \int_0^{\infty} (R(t) - E(t))e^{-rt}dt \qquad L = \int_0^{T_M} R(t)e^{-rt}dt \qquad C = \sum_{j=1}^{M} \int_{T_{j-1}}^{T_j} x_j(t)e^{-rt}dt \tag{22}$$

Let $\tau_j = T_j - T_{j-1}$ (so $T_j = \sum_{k=1}^{j} \tau_k$), and T_0=0. The problem is to find the extremum of

$$V^* = I - \int_0^{T_M} R(t)e^{-rt}dt - \sum_{j=1}^{M} \int_{T_{j-1}}^{T_j} [x_j(t)e^{-rt} + \lambda_j f_j(x_j(t))]dt + \sum_{j=1}^{M} \lambda_j D_j \tag{23}$$

where the Lagrange multipliers λ_j have been introduced in order to impose the constraints that all damage to each component must be repaired. Define the new variables

$$\bar{x}_j(\tau) = x_j(T_{j-1}+\tau) \qquad \bar{R}_j(\tau) = R(T_{j-1}+\tau) \qquad \text{for } \tau \in [0, \tau_j],$$
$$\bar{x}_j(\tau) = 0 \qquad \bar{R}_j(\tau) = 0 \qquad \text{otherwise} \tag{24}$$

Then Eq. (23) may be written

$$V^* = I - \sum_{j=1}^{M} e^{-rT_{j-1}} \int_0^{\tau_j} [\bar{R}_j(\tau) + \bar{x}_j(\tau)] e^{-r\tau} d\tau + \sum_{j=1}^{M} \lambda_j [D_j - \int_0^{\tau_j} f_j(\bar{x}_j(\tau)) d\tau] \tag{25}$$

The corresponding Euler equation with respect to $\bar{x}_j(\tau)$ is $e^{-rT_{j-1}} e^{-r\tau} + \lambda_j f_j'(\bar{x}_j(\tau)) = 0$ which leads to

$$\dot{\bar{x}}_j = -r\, f_j'(\bar{x}_j) / f_j''(\bar{x}_j) \tag{26}$$

The initial state $x_j(0)$ is such that the constraint $\int_0^{\tau_j} f_j(\bar{x}_j(\tau)) d\tau = D_j$ is satisfied. Repairs may need to be delayed if the rental income rate is sufficiently low, as described before. If not, then optimizing the functional V^* with respect to repair time τ_j, we have

$$\bar{R}_M(\tau_M) + \bar{x}_M(\tau_M) = f_M(\bar{x}_M(\tau_M)) / f_M'(\bar{x}_M(\tau_M)) \quad \text{for } j = M \tag{27}$$

$$\bar{R}_j(\tau_j) - re^{rT_j} \sum_{k=j+1}^{M} \bar{C}_k e^{-rT_{k-1}} + \bar{x}_j(\tau_j) = f_j(\bar{x}_j(\tau_j)) / f_j'(\bar{x}_j(\tau_j)) \quad \text{for } j < M \tag{28}$$

where $\bar{C}_k = \int_0^{\tau_k} \left[\bar{R}_k(\tau) + \bar{x}_k(\tau)\right] e^{-r\tau} d\tau$. Comparing with Eq. (7) governing the optimal repair time for a single component, we see that Eq. (28) corresponds to the solution maximizing the asset value of a fictitious rental unit with one damage component j and rental income rate at τ_j of

$$\tilde{R}_j(\tau_j) = \bar{R}(\tau_j) - re^{rT_j} \sum_{k=j+1}^{M} \bar{C}_k e^{-rT_{k-1}} \tag{29}$$

The fictitious asset values $V_j(\tau_j)$ may be calculated from

$$V_M(\tau_M) = I - \bar{C}_M \qquad \text{and} \qquad V_j(\tau_j) = V_{j+1}(\tau_{j+1}) - \tilde{C}_j \quad \text{for } j<M \tag{30}$$

where $\tilde{C}_j = \int_0^{\tau_k} \left[\tilde{R}_j(\tau) + \bar{x}_j(\tau)\right] e^{-r\tau} d\tau$ for j<M. The asset value of the actual rental unit is then $V = V_1$.

The optimization to repair one rental unit with M damaged components is therefore equivalent to optimizing the repair in reverse order of M fictitious rental units, each with one damaged component.

When budget considerations do not limit the repairs that can be made, the general case of N rental units with M_i damaged components for unit i can be treated using a combination of the methods presented in the foregoing. Each rental unit can be optimized as described for the case of N units with one component, and the components of each rental unit can be optimized using the backward induction strategy already developed. In this case, all repairs of rental units are carried out simultaneously to the extent that this is possible.

Binding Budget Constraint

Consider now the case of N rental units with M_i components for unit i where there is a binding budget constraint on the present value of the total repair cost. By "binding", we mean that if the repair of each rental unit is independently optimized, the present value of the total repair cost exceeds the budgeted amount B. The budgeted funds may come from internal company funds, earthquake insurance, loans and government disaster-assistance funds. To reduce the repair costs, some waiting time before starting repairs may be necessary for each rental unit.

Let t_i be the waiting time before repairs are started on rental unit i. Then the total asset value of all rental units is

$$V = \sum_{i=1}^{N} V_i(t_i + \hat{T}_i) = \sum_{i=1}^{N} \left[I_i - L_i(t_i + \hat{T}_i) - \hat{C}_i(\hat{T}_i) e^{-rt_i} \right] \tag{31}$$

where we use the notation in equations (19) and (20) and $\hat{T}_i = \max_j \hat{T}_{ij}$ is the optimal total duration of repairs for rental unit i. The optimal cost of repairs without waiting is

$$\hat{C}_i(\hat{T}_i) = \sum_{j=1}^{M_i} \int_{\hat{T}_{i,j-1}}^{\hat{T}_{ij}} \hat{x}_{ij}(t) e^{-rt} dt \tag{32}$$

where $\hat{x}_{ij}$ and $\hat{T}_{ij}$ are the optimal expenditure rate and optimal repair time for damage D_{ij} of component j of rental unit i. If repairs are delayed, then the budget constraint may be written $\sum_{i=1}^{N} \hat{C}_i(\hat{T}_i) e^{-rt_i} \le B$, so introducing a Lagrange multiplier μ where μ>0 if the budget constraint is active (otherwise μ=0), we get

$$V^* = V + \mu\left[B - \sum_{i=1}^{N} \hat{C}_i(\hat{T}_i) e^{-rt_i}\right] = \sum_{i=1}^{N} \left[I_i - L_i(t_i + \hat{T}_i) - (1+\mu)\hat{C}_i(\hat{T}_i) e^{-rt_i}\right] + \mu B \tag{33}$$

Differentiating V^* with respect to the waiting time variables t_i leads to

$$dV^*/dt_i = -\hat{C}_i(\hat{T}_i)\left[\eta_i(t_i) - (1+\mu)r\right] e^{-rt_i} \tag{34}$$

where $\eta_i(t_i) \equiv R_i(t_i + \hat{T}_i) e^{-r\hat{T}_i} / \hat{C}_i(\hat{T}_i)$ can be interpreted as the return on the optimal cost of repairs given by the present value of the rental income rate received at completion of repairs. Since we want to maximize V^*, the sign of dV^*/dt_i determines the optimal strategy for rental unit i:

(a) If $\eta_i(0) \le (1+\mu)r$, then there are two important cases. First, assume that $R_i(t)$ is non-increasing for $t > \hat{T}_i$, then rental unit i should not be repaired (i.e. optimal $t_i \to \infty$ because $dV^*/dt_i \ge 0$). On the other hand, if $R_i(t)$ is non-decreasing, then either there exists $t_i \ge 0$ such that $\eta_i(t_i) = (1+\mu)r$, which gives the optimal waiting time, or else $\eta_i(t_i) < (1+\mu)r$ for all $t_i \ge 0$, which implies that the rental unit should not be repaired.

(b) If $\eta_i(0) > (1+\mu)r$, then the repairs of rental unit i should be started immediately (i.e. optimal t_i =0).

Numerical iteration may be required to determine the Lagrange multiplier μ which satisfies the budget constraint on the total cost of repairs. In this process, μ is gradually increased from zero (which corresponds to neglecting the budget constraint) using a small increment at each iteration until the budget constraint is satisfied for all the rental units which are to be repaired according to the above optimal strategy.

REPAIR OR DEMOLISH?

In general, a building consists of an aggregate of rental units. Since it is usually not practical to demolish individual rental units within a building, and even abandonment of selected units may not be an option, it is necessary to combine all of the building's rental units together when investigating whether to repair or to demolish the building. If the present value of the building, when repaired according to the optimal strategy, exceeds the present value for optimal demolition, then the repair option should be chosen. Otherwise, demolition is appropriate. The present value for optimal demolition is the maximum value of

$$V_d = V_L e^{-rT_d} - \int_0^{T_d} E(t) e^{-rt} dt - C_d \tag{35}$$

where C_d and T_d are the present value of the cost and the time to demolish the building and V_L is the value of the land less the balance of any loans on the property after demolition is completed. The optimal demolition rate can be investigated in the same way as for repairs, except that the recovery function is replaced by a demolition function relating the rate of progress on each job to the rate of expenditure on that job.

Another possibility is to demolish and rebuild. To assess this properly, the present value of the rebuilt structure and the rebuilding cost must also be evaluated. This could be carried out using the methodology described for repairs, except that the recovery function would now be replaced by a construction function relating the rate of progress on each job to the rate of expenditure on that job.

CONCLUSIONS

A methodology has been presented for determining the optimal actions for recovering from earthquake damage to company-owned properties. The strategy is to choose the actions which will maximize the asset value of the properties. It is shown that for repairs on each damaged component, the optimal expenditure rate should increase with time, although if the net income from each property is sufficiently small, it may be optimal to delay the repairs. When the total cost of repairs is subject to a budget constraint, the optimal strategy shows which properties should be repaired immediately, which should be repaired after a certain time delay and which should never be repaired. In the latter case, local government regulations would most likely not allow abandonment of the property and so demolition must be considered.

In current work on the research project, we are developing the optimal strategy methodology for the case of uncertain damage to each component of each property. For example, company management may wish to investigate recovery strategies, before detailed engineering inspections of each property are done, by using only the earthquake magnitude and location or the ground motion at each site. In this case, a probability distribution for the damage is calculated based on a detailed fragility analysis that treats each property as an assemblage of components. Because of the uncertainties involved, the optimal strategy is now to choose the actions which will maximize the *expected* asset value of the properties. This methodology may also be used to calculate the expected lifetime earthquake losses for each property. Then the possibility of optimal seismic upgrading of each property can be investigated, either during the recovery process or as a mitigation action prior to an earthquake.

ACKNOWLEDGEMENT

This paper is part of a joint Caltech-Stanford project funded by California Universities for Research in Earthquake Engineering under the CUREe-Kajima Joint Research Program with J.L. Beck as team leader and A.S. Kiremidjian heading the effort at Stanford University. This financial support is gratefully acknowledged.

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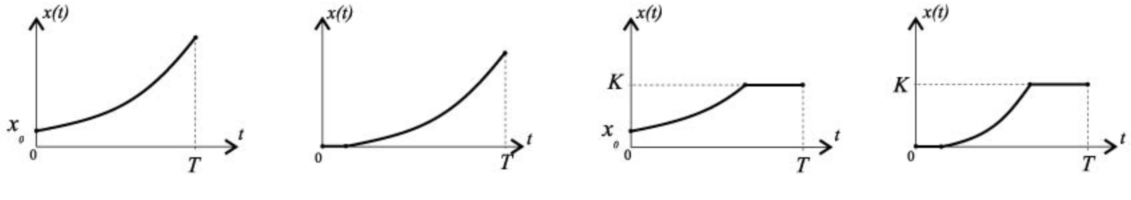

Figure 1: Possible Solutions for x(t)