Evaluation of Sliding Displacement of Retaining Structure and Soil Slope regarding the Natural Frequency

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ABSTRACT :

This paper aims to show the significance of vibration properties in the evaluation of the sliding displacement of the structures such as retaining wall and soil slope. For the update of earthquake resistant design regarding the post-earthquake performance of structures, the evaluation of the sliding displacement during earthquake is important. In the seismic coefficient method widely used in earthquake resistant designs for various types of structures, only the margin for the failure of structure, but not the degree of damage or the deformation to performance of the structures is taken into consideration. The rigid mass-slider model originally proposed by Newmark is sometimes employed for the purpose, which makes it possible to calculate the sliding displacement of rigid mass on the frictional floor. In the framework of this method, the structure is modeled as a rigid mass and its natural frequency is disregarded. In the reality, however, the sliding displacement depends on the flexibility and natural frequency of structure. The evaluation method we introduce in this study employs a mathematical vibration-sliding model which consists of the mass, spring with dashpot, and slider to take account of the vibration properties of structure. A series of shaking table tests on a physical vibration-sliding model were conducted to verify the evaluation method. A good agreement was found between the sliding behaviors observed with the physical model and those calculated with the mathematical model.

KEYWORDS: Retaining structures, soil slope, spring-mass model, vibration, sliding, spectrum

1. INTRODUCTION

In the earthquake resistant design of retaining structure and soil slope, a performance-based method is lately preferably employed instead of the conventional seismic coefficient method. In the performance-based design, a certain degree of performance of the structure must be guaranteed even after the application of the earthquake with assumed intensity. In order to conduct the earthquake resistant design satisfactorily, the degree of the damage to the structure must be estimated quantitatively; the sliding displacement is a primary important factor as an index of the damage intensity for retaining structures and soil slopes. And the rigid mass-slider model originally proposed by Newmark (1965, 1974) is sometimes employed for the purpose, which makes it possible to calculate the sliding displacement of rigid mass on the frictional floor. In the framework of the method, the structure is modeled as a rigid mass and its natural frequency is disregarded. In the evaluation of sliding displacement, the effect of natural frequency must be evaluated adequately, as the response spectrum has been employed in order to evaluate impulse forces during earthquake. In this study, first, the vibration-sliding behavior calculated with the mathematical model is analyzed, and the significant of the structural vibration properties on the vibration-sliding behavior is presented. Next, the vibration-sliding behaviors observed in a series of shaking table tests are discussed, and the mechanism of the vibration-sliding behavior is investigated. Finally, the validity of the mathematical model is discussed in a comparison between the observed and the calculated behaviors.

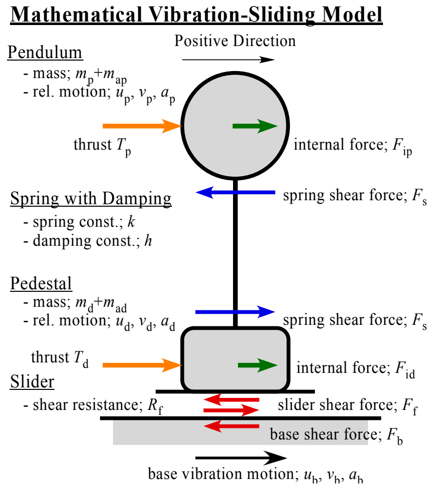

Figure 1 Mathematical Vibration-sliding model

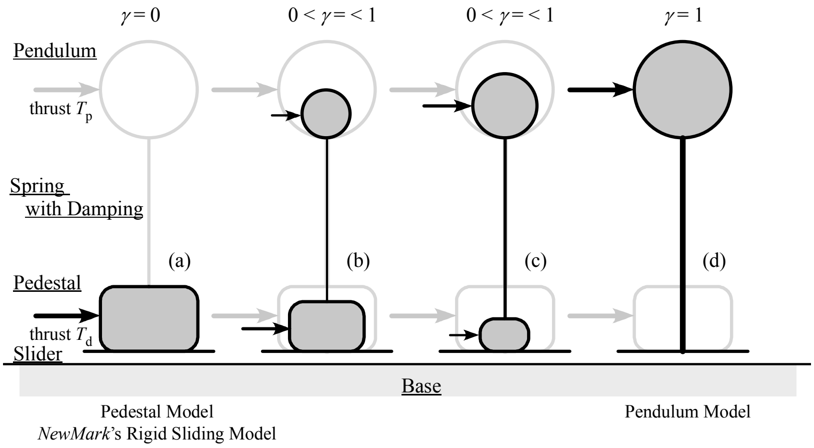

Figure 2 Ratio of mass γ

Mathematical Vibration-sliding model

$$\begin{cases} m_p a_p = T_p + F_{ip} - F_s \\ m_d a_d = T_d + F_{id} + F_s - F_f \end{cases}$$

$$m_p = \gamma m,\ m_d = (1-\gamma)m$$
$$T_p = \gamma T,\ T_d = (1-\gamma)T$$
$$F_{ip} = -m_p a_b,\ F_{id} = -m_d a_b$$
$$F_s = c v_s + k u_s$$
$$R_f = \mu m g$$

non-sliding case

$[v_d = 0, \text{and } F_f \le R_f]$

$$v_d = 0$$

sliding case

$[v_d > 0, \text{ or } F_f = R_f]$

$$F_f = R_f$$

2. MATHEMATICAL VIBRATION-SLIDING MODEL

2.1 Outline of Mathematical Vibration-Sliding model

The mathematical vibration-sliding model with double-degree of freedom is presented in Figure 1. The model consists of two masses, spring with dashpot, and frictional slider. Thus, the model is an inversed pendulum with a pedestal, and a frictional slider; the masses of pendulum and pedestal are m_p and m_d, respectively. The ratio of two masses of pendulum and pedestal is designated with the mass ratio γ ($0 \leqq \gamma \leqq 1$) which is varied according to the mechanical properties of the structure considered. In the case of γ=1 the model becomes a inversed pendulum of single-degree, as shown Figure 2 (d). In the case of γ=0, all the mass concentrates on the pedestal, and the model becomes equivalent to the rigid-sliding model without an inversed pendulum. The model is subjected to the constant thrust T corresponding to the inclination of sliding surface and/or earth pressures in ordinary condition with a certain margin against sliding, and is shaken with a base motion. The base motion and the relative displacements of the pendulum and pedestal are u_b, u_p and u_d respectively. The vibration property is designated with two masses m_p and m_d, spring constant k, and damping constant h. The slider possesses the frictional resistance R_f which can be evaluated with the Coulomnb's friction law as a function of the base roughness or artificially provided shear resistance. The sliding occurs in the slider, when the base shear force F_b reaches the frictional resistance R_f. The slider slides in the direction of the thrust T during the base shaking.

The mathematical vibration-sliding model in the case of γ=0 where the natural frequency is assumed to be infinity, becomes equivalent to the rigid model proposed by Newmark(1965); the mode was applied to the analysis of sliding deformation of fill embankment during earthquakes. On the other hand, Sawada et al. (1998) showed the possibility of modification of Newmark's method based on their study by means of Finite Element Method for the elasto-plastic continuum, and analyzed the permanent deformation of the fill embankment.

In this study the clear slip surface is defined on the structure base or through a soil mass, and its sliding displacement is analyzed. The equations of motion in the mathematical vibration-sliding model, and the conditional equations for sliding and non-sliding cases are expressed in the above boxed area. Where, the suffixes of p, d, and s correspond to pendulum, pedestal, and spring, respectively. As the equations of motion are

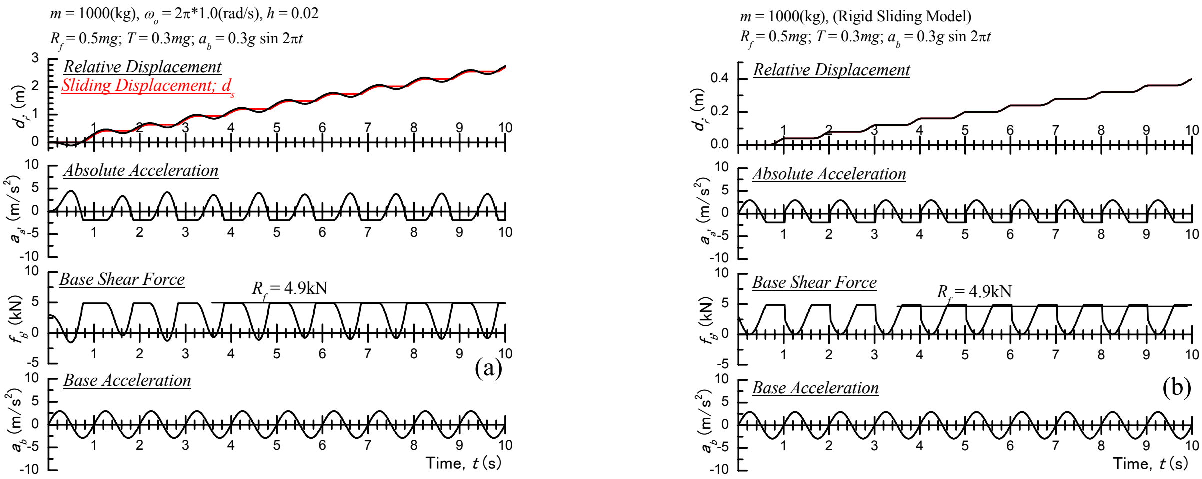

Figure 3 Time history of the vibration-sliding model under sinusoidal wave
(a) Vibration-sliding model (γ=1), (b)Rigid-sliding model (γ=0)

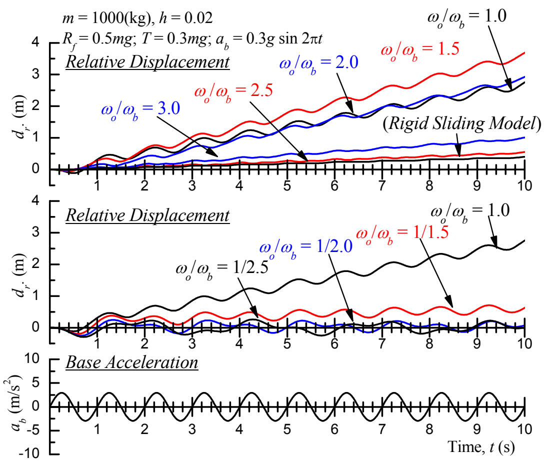

Figure 4 Effect of Natural Frequency

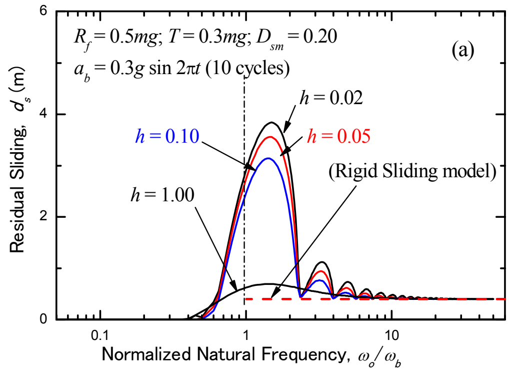

Figure 5 Sliding Response Spectrum

nonlinear simultaneous equations in time-domain, the equations were discretized, and the vibration-sliding behavior was calculated by the direct integration method.

2.2 Effect of Natural Frequency in Mathematical Vibration-Sliding model

The comparison of response time histories of the inversed pendulum model (γ=1) and of the rigid-sliding model (γ=0) under the fundamental condition is shown in Figure 3. The base acceleration, base shear force, absolute acceleration, and relative displacement of mass are shown from bottom to top on the figure face. In the top graph of the Figure 3(a), the sliding displacement in slider is indicated by a red line. Close examination of the figures tells that the sliding displacement occurs and accumulates when the base shear force reaches the frictional resistance in response to the base motion in both the mathematical vibration-sliding model and the rigid-sliding model. It can be seen that the accumulated sliding displacement of the inversed pendulum model is more than 5 times that of the rigid-sliding model.

The time history of the vibration-sliding behavior of the model during the sinusoidal base shaking of ten cycles is shown in Figure 4. The figure is explained with the ratio of the structural natural frequency to base shaking frequency, ω_o/ω_b. The calculated value of the rigid-sliding model of γ=0 is also presented in Figure 4. The amount of sliding displacement tends to become large around the frequency ratio ω_o/ω_b of unity, which corresponds to the condition of the resonance of vibration behavior. When the model becomes slightly harder than the frequency ratio ω_o/ω_b of unity, it can be seen that the amount of sliding displacement tends to become maximal. On the other hand, if the model becomes flexible and the natural frequency is one-half of the base shaking frequency, around ω_o/ω_b=1/2, the sliding displacement becomes negligibly small. It is interesting that the amount of sliding displacement is underestimated if the rigid-sliding model where the model's vibration properties is not taken into consideration. The calculated sliding displacement plotted against the ration of the structural natural frequency to the base shaking frequency, ω_o/ω_b with parameters such as damping constant h is shown in Figure 5. It can be seen that the response sliding displacement becomes a maximum value in range from ω_o/ω_b=1.5 to 2.0. In the figure the sliding displacement calculated with the rigid-sliding model of γ=0 is also presented by a horizontal broken line on right hand side, the value is fairly small compared with the maximum value calculated with the vibration-sliding model. The sliding displacement asymptotes to the value calculated with the rigid-sliding model (γ=0), if the model becomes harder and the frequency ratio ω_o/ω_b becomes larger in the right-hand side of the figure.

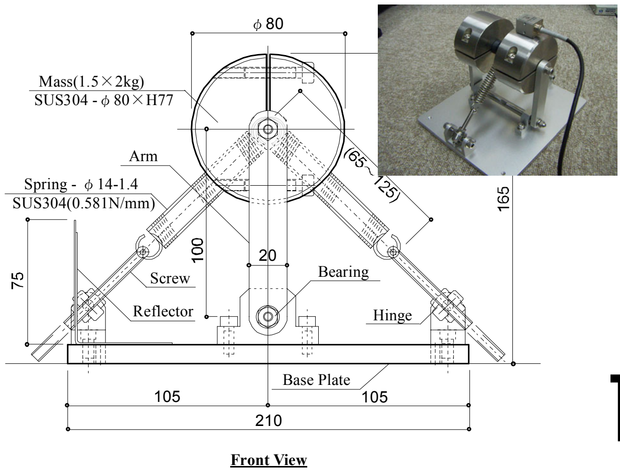

Figure 6 Physical Vibration-Sliding Model

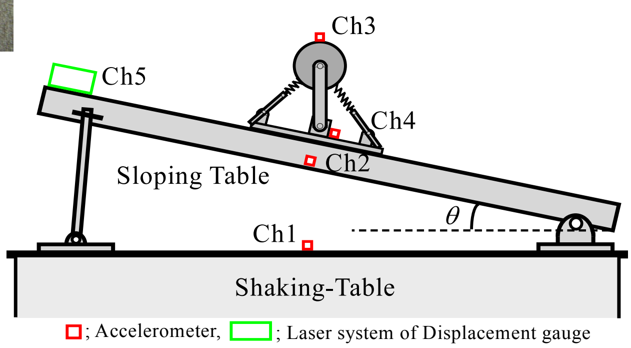

Figure 7 Set up of Physical model with Sloping Table

3. PHYSICAL VIBRATION-SLIDING MODEL

3.1 Outline of Physical Vibration-Sliding model

A physical vibration-sliding model which was developed according to the mechanical properties of the mathematical vibration-sliding model is shown in Figure 6. The physical model was 210 mm in length, 165 mm in height, and 180 mm in width, and was made of stainless steel and aluminum alloys. The physical model fundamentally forms the inversed pendulum system with a mass supported with arms and springs on a base plate as shown the figure. The photograph shows the physical model equipped with two stainless steel weights of 1.5 kg, which corresponds to the mass of pendulum in the inversed pendulum. The base plate was 1.52 kg in weight. The natural frequency of the physical model could be varied by changing the number of the weights or the stiffness and/or angle of the springs. In order to evaluate the vibration properties, the natural frequency ω_o and damping constant h were measured by analyzing the time history of acceleration of the weight under free vibration condition with the base plate fixed.

3.1.1 Sloping table

The setup of the physical vibration-sliding model and a sloping table on a shaking table is shown in Figure 7. The physical model is put on the sloping table and subjected to the base shaking. Therefore, when the sloping table is subjected to the base shaking, the physical model is vibrated and slid under the application of a constant horizontal thrust. The sloping table made of steel was 1500 mm in length and 250 mm in width, and is coated by lacquer, and was connected rigidly with bolts to the shaking table. A resin plate was laid on the slide table surface to ensure a constant coefficient of friction in the bottom of the model. As the frictional properties depended on a subtle condition of dust and humidity on the sloping table surface, the surface was wiped lightly by soft and dry cloth just before each of the shaking tests conducted. The coefficient of friction was measured in the tests preliminary conducted under static condition with variable slope angle, in order to find the fundamental sliding properties between the slide table and the physical model.

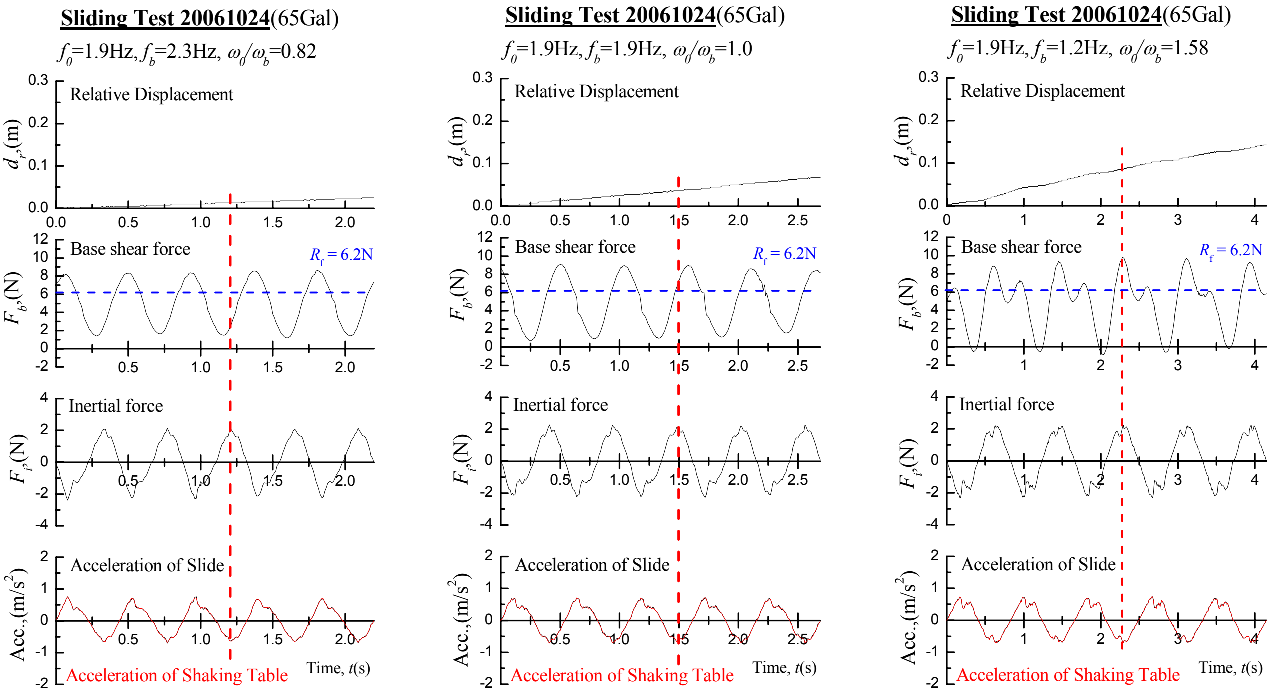

Figure 8 Time histories of observed results with base shear force and inertial force
(a) flexible model f_0/f_b=0.82, (b) resonant model f_0/f_b=1.00, (c) hard model f_0/f_b=1.58

3.2 Experimental Condition

Single weight of 1.48kg was fixed to the top of the physical model. Two sets of springs used have a free length of 65 mm, maximum length of 61.3mm and spring constant of 0.581N/mm. A free vibration behavior of the physical model under this condition was observed; as a result, it was found that the natural frequency f_o was 1.9Hz through Fourier analysis, and the damping constant h was 0.03 by using logarithmic decay method. The total mass of the physical model m was 3.16kg, and the mass ratio was γ=0.52. The coefficient of static friction between the sloping table surface and the bottom of the physical model was about 0.2 in the static tests conducted preliminary. The angle of the slide table was set to be θ=9 degrees, so that the thrust to the model was equivalent to T= 0.16 mg.
Three test series were conducted with different sinusoidal base shakings, where the amplitude was 50, 65 or 80Gal. In each of the test series, the frequency of the sinusoidal base shaking f_b was parametrically varied from 0.6 to 2.5Hz.
In the observation of the vibration-sliding behavior of the physical model, four accelerometers with the frequency range of measurement of 0-46Hz, and a laser system of displacement gauge with the resolution of ±0.5mm were employed. The installation positions of the sensors are shown in Figure 7; accelerometers were placed on the centers of shaking table and slide table, on the surface of base plate and the top of the weight. The laser system of displacement gauge is fixed rigidly with bolts to the upper sloping table for the measurement of relative displacement of the physical model to the sloping table. The time interval of the measurement was 0.01 second; data sampling frequency was 100Hz.

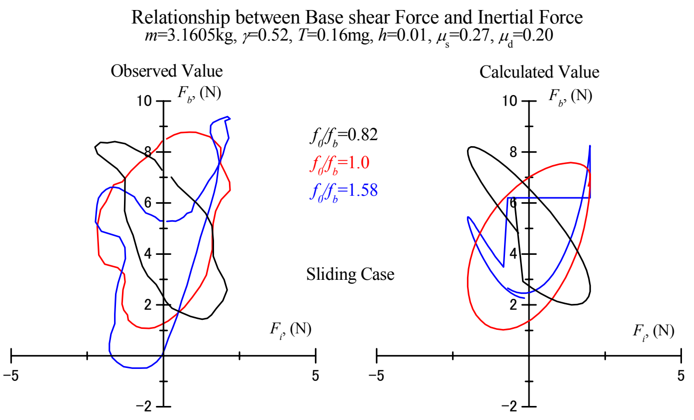

Figure 9 Relationship between Base shear force and Inertial force in sliding case
(a) Observed, (b) Calculated

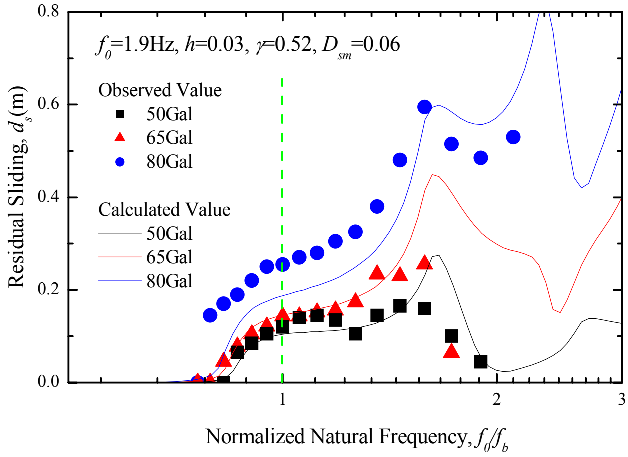

Figure 10 Sliding Response Spectrum

4. EXPERIMENTAL RESULTS

The typical time histories observed for 5 cycles of the sinusoidal base shaking application are shown in Figure 8. In Figuer 8 (a), (b) and (c), acceleration amplitude were common as 65Gal; however the base shaking frequency were different. Figure 8 (a) was for a flexible model where the frequency ratio f_o/f_b was less than unity, Figure 8 (b) was for a model under resonant condition where f_o/f_b was equal to unity, and Figure8(c) was for a hard model where f_o/f_b was greater than unity. The acceleration of the slide table and the shaking table, the inertial force, the base shear force, and the sliding displacement are plotted against time from bottom to top in the figures. The inertial force was calculated from acceleration of shaking table and the weight of the physical model. The base shear force was calculated from the inertial force, acceleration of the weight and the base plate where the slope angle of the sloping table was taken into consideration. When the base shear force and inertial force indicate positive values in the figure, the physical model is subjected to forces with downward direction in Figure 7.
From the comparison of the figures, the accumulated sliding displacement was largest for the relatively hard model of the three models. The sliding displacement of the hard model was over eight times as large as that of flexible model. A vertical dotted red line was drawn for the peak of the inertial force in Figure 8. In the hard model with the largest sliding displacement, the inertial force and the base shear force reach a peak at almost the same time; the difference between phase angles is practically zero between the inertial force and the base shear force, on the other hand the phase angle difference is perceptible for the resonant model and the flexible model. It should be noted that in the flexible model the phase angle difference is about 180 degrees and the sliding and inertial force are opposite in direction as shown in Figure 8(a). As explained above, the sliding displacement is remarkably dependent on the flexibility of the model, and the phase angle difference between inertial force and base shear force plays an important role in the sliding behavior.
The relationships between the base shear force and the inertial force in the physical model and the mathematical model during sliding vibration are shown in Figure 9. The behaviors of the flexible model, the resonant model and the hard model conditions are presented by black, red and blue lines, respectively. The counterclockwise ellipsoidal loops can be seen due to the energy loss as a result of frictional sliding of the model. The blue loop with the largest length for the hard condition has a positive slope and the loop tapered toward upper right. This means that the base shear force and the inertial force are same in the direction, and the inertial force contributes more in the sliding behavior. The red loop for the resonant model indicates a positive gradient; however, the enclosed area of the red loop is greater than that of the blue loop, due to the greater phase angle difference as shown in Figure 8. The black loop for the flexible model has negative slope. This means that the phase angle difference between the base shear force and the inertial force was about 180 degrees, and during the sliding the inertial force is opposite to the sliding win direction. Therefore, the sliding displacement of the flexible model is rather small compared with other two models.

The loops calculated with the mathematical model are shown in Figure 9 (b). In the figure, the loops at cyclic steady condition are drawn. The loops were calculated with a coefficient of static friction μ_s=0.27, and a coefficient of dynamic friction μ_d=0.20. A good agreement of the loops between the calculated and measured behaviors can be seen.

The observed residual sliding displacement of the physical vibration-sliding model is plotted against the ratio of the natural frequency to the base shaking frequency f_0/f_b, for the application of sinusoidal base shaking of 10 cycles in Figure 10. In the figure, the three cases with different base shaking acceleration amplitudes are plotted. The residual sliding displacements calculated with the mathematical vibration-sliding model are also shown in the figure with lines. In the calculations the parameters were determined from the test conditions of the three series of shaking table tests on the physical model. The resonant condition for f_0/f_b = 1.0 is expressed by a vertically broken line in the figure.

It cane bee seen that the amount of the residual sliding displacement depends significantly on the frequency ratio f_0/f_b. In the figure, the left-hand side and right-hand side of the resonant condition correspond to the relatively flexible condition (flexible model) and relatively hard condition (hard model), respectively. As explained above, it can be said that the hard model is generally subjected to larger residual sliding displacement compared with the flexible model in dependent of the intensity of the base shaking. The good agreement can be recognized between the physical model and the mathematical model.

5. APPLICATION TO PRACTICAL PROBLEMS

As shown in the previous sections, the flexibility or rigidity of the structure has significant effect on the vibration-sliding behavior of the structure, and the residual sliding displacement of the structure through base shaking vibration is much dependent on the relationship of its natural frequency with the frequency properties of the base shaking. And the mathematical vibration-sliding model is able to predict the vibration-sliding behavior of structure and to calculate the amount of residual sliding displacement of the structure. Then the application of the mathematical vibration-sliding model to practical earthquake engineering problems may be useful to estimate the structural damage associated with sliding during shaking, which is needed for rational performance based design. In this section two examples of the application of the mathematical vibration-sliding model are presented; a land slide and a quay wall in port area are selected.

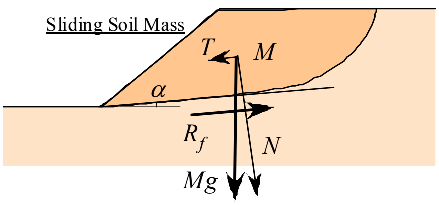

Figure 11 Sliding Soil Mass

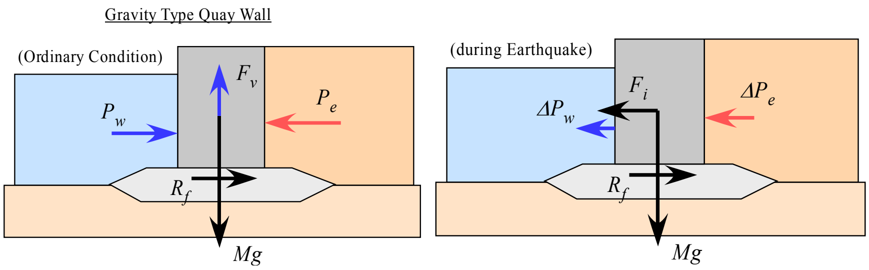

Figure 12 Gravity Type Quay Wall

5.1 Application to soil slope

Sliding of soil mass is shown in Figure 11 with gravity force and shear resistance. The sliding soil mass is M, the gravity force is W=Mg and the slope angle of the sliding surface is α. In this problem the soil mass is subjected to the thrust T as a function of the slope angle α. The natural frequency and damping ratio of the soil mass can be evaluated from through the measurement of microtremor and/or elastic wave exploration. The shear resistance can be evaluated from shear strength of the soil in a conventional manner.

5.2 Application to Gravity Type Quay Wall

The section of a typical gravity type quay wall with caisson is shown in Figure 12; the caisson is subjected to the gravity forces and some external forces, and figures present the ordinary and seismic conditions. The mass of caisson is M, the angle of the frictional resistance on the bottom of the caisson is φ_μ, and the buoyancy to the caisson is F_v. For the mathematical model, the frictional resistance force and the thrust are shown as $R_f=(Mg-F_v)\tan\varphi_\mu$ and $T=(P_e-P_w)$, respectively, therefore the application to the mathematical vibration-sliding model becomes possible. The dynamic water pressure and the dynamic earth pressure during an earthquake can be calculated as the additional mass corresponding to the inertial force. For the evaluation of the natural frequency of caisson, the microtremor measurement will be utilized. Kohama, et al. (2002) conducted a series of microtremor measurement on the gravity type quay walls in the port and harbor areas in Japan, and found a close relationship between the natural frequency of the caisson and its size and shape.

6. CONCLUDING REMARKS

Mathematical and physical vibration-sliding models both were devised with mass, spring, dashpot and slider to take account of the effect of vibration properties of structures on the sliding amount during earthquake vibration. The vibration-sliding behavior calculated with the mathematical model was closely examined. And the vibration-sliding behavior observed in shaking table tests on the physical model was comparatively examined with the calculated vibration-sliding behavior. The findings in this study are summarized as follows:

-- The significance of vibration properties in the sliding response of structure during shaking was demonstrated with the mathematical vibration-sliding model subjected to sinusoidal base shaking motion. In the case of the model with single degree of freedom, the resonant vibration behavior led to dominant sliding displacement. When the natural frequency of structure was about 1.5 times the frequency of base shaking (f_o/f_b=1.5), the amount of sliding displacement attained maximal.
-- The comparison of the vibration-sliding model with the rigid-sliding model proposed by Newmark where calculated sliding displacement was rather small, the vibration-sliding mathematical model is effective for the simplified evaluation of the sliding displacement with taking account of the vibration properties of structure.
-- In the vibration-sliding behavior observed in some series of shaking table tests on the physical model, the significant effect of the vibration properties on the residual sliding displacement was recognized. The phase angle difference between inertial force and base shear force depends strongly on the frequency ratio f_o/f_b; in the case of hard model with $f_o/f_b>1$ inertial force becomes larger during sliding, and contributes the residual sliding displacement. The mechanism sliding during shaking can be explained by means of the mathematical model.
-- The applicability of the mathematical vibration-sliding model was verified with the observed behavior. Good agreement between the observed behavior and calculated behavior was recognized both in the relationship of base shear force with inertial force and in the variation of residual sliding displacement as a function of amplitude and frequency of sinusoidal base shaking.

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