Numerical Study of Localized Deformation in Granular Materials by Discrete Element Method

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ABSTRACT :

Granular materials exhibit very complex behavior under external load. The macro- and micro-mechanical characteristics of granular assembly are strongly dependent on the properties of the constituent individual particles. The objective of this paper is to explore the deformation in microscale using discrete element method (DEM). Quasi-static biaxial compression test is carried out to investigate the structured deformation pattern. The simulation results show that the micro-scale deformation is nonuniform but not random or unrelated. The localized deformations represented by slip deformation in local void cells appear as microbands at small strain and shear bands at large strain. During the biaxial compression, the negative slip occurs in the neighboring regions of elastic unloading and plastic flow can occur along a plane of discontinuity in the velocity gradient field.

KEYWORDS: granular materials, discrete element method, slip deformation, micro-band, shear band

1. INTRODUCTION

Granular materials are encountered in nature and engineering such as soil, mining and construction industries. It is well known that deformation within granular materials is nonuniform, particularly at the micro-scale of particle groups. When granular materials are subjected to large deformation localized deformation structures appear as microbands at small strain and shear bands at large strain (Kuhn, 1999). So far, most of the published research on localized deformation of granular materials has focused on the evolution of the motion of individual particle (Oda et al., 1997, Zhou and Chi, 2003). It should be emphasized, however, the slip deformation is an important deformation feature in granular materials. Thus, the evolution of slip deformation in void cell should be investigated further. In this paper, discrete element method based on void cell is used to investigate the localized phenomenon represented by slip deformation.

2. DISCRETE ELEMENT METHOD

Discrete element method (DEM) idealizes granular material as an assembly of infinite particle elements. The interactions between the neighboring particles are modeled as a dynamic process, and an explicit central difference time integration scheme is adopted for the time evolution of the particles. A simplified linear elastic contact mechanism is employed between particles in contact and the tangential force is limited via Mohr-Coulomb criterion. The rolling resistance at the contacts is not considered. The relationships between the contact forces and the contact displacements can be expressed as follows

$$\begin{aligned} F_n &= k_n \delta_n \\ F_t &= k_t \delta_t \le F^s_{\max} = \mu |F_n| \end{aligned} \tag{2.1}$$

where, k_n and k_s are the normal and the tangential stiffness respectively, and δ_n and δ_t represent the normal and tangential components of the overlap between particles in contact respectively.

In a two-dimensional granular array, the topology can be described by the particle graph which is associated with the Voronoi-Dirichlet tessellation of a two-dimensional region (Kuhn, 1997). The faces of the planar graph are polygonal void cells, which are enclosed by the circuits of contacting particles. The vertices are the centers of the contacting particles and the edges connect the vertices (Bagi, 1996), as shown in Fig. 1.

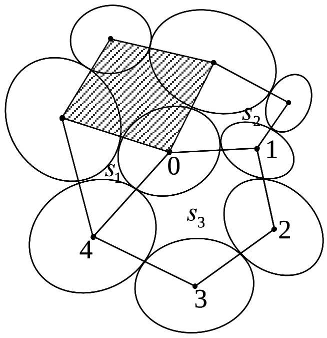


Figure 1 Particle graph of a two-dimensional particulate array

In the DEM adopted, the micro-scale deformations are measured based on void cells and the deformations occur in them (Kuhn, 1997). Deformations within individual void are computed from the relative motions of the surrounding particles. The average stress in a granular assembly and can be computed as a sum of dyadic products associated with its contacts (Bardet and Vardoulakis, 2001), i.e.

$$\sigma_{ij} = \frac{1}{V}\sum_{c \in V} l_j^c f_i^c \tag{2.2}$$

where, V is the assembly volume. In this notation, $f_i^c = f_i^{nm} = -f_i^{mn}$ is the contact force on particle n exerted by particle m, and $l_j^c = x_j^m - x_j^n$ is the branch vector connecting the reference point x_j^n on particle n and x_j^m on particle m.

3. DEM BIAXIAL COMPRESSION TEST

Dense particulate array of 4000 sand particles is chosen for the DEM biaxial compression test. The specimen is surrounded by two pairs of periodic boundaries which can eliminate the effect of the boundaries, as shown in Fig. 2. Particle size ranges from 0.5 to 1.5 of d_{50}, where d_{50} is the median particle diameter and the uniformity coefficient $C_u = d_{60}/d_{10} = 1.69$. The initial void ratio is 0.15 and the initial overlap is $1.01\times10^{-2}D_{50}$. The density of sand is 2.0×10^3kg/m^3 and the frictional coefficient is 0.5 (corresponding to the frictional angle $\phi = 26.57^\circ$). The normal stiffness is 2.5×10^8N/m and the ratio of the tangential contact stiffness to the normal contact stiffness is 0.25. The time step is 1×10^{-6} second. During the compression simulations, the height of the array is slowly reduced at a constant rate, while the width is continually adjusted to maintain a constant horizontal stress σ_{11}.

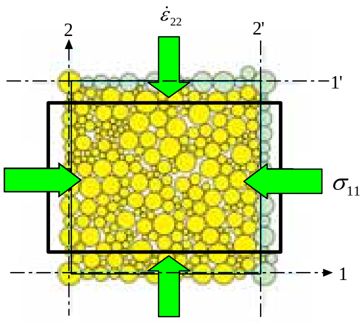


Figure 2 Biaxial compression test of granular materials with periodic boundaries

The stress ratio (σ_{22}/σ_{11}) versus axial strain (ε_{22}) curve is presented in Fig. 3. It can be seen that the compression test experienced the following stages, e.g. the initial loading stage, the peak state, the softening stage and the critical state. During the initial loading stage the mechanical response of granular materials is elastic. With the development of loading the stress ratio increases gradually and decrease after peak state. The softening phenomenon is due to the failure of the interlocking force between the neighboring particles in dense particular array. With the compression proceeds the ratio remains constant and it retains steady state which is also called critical state.

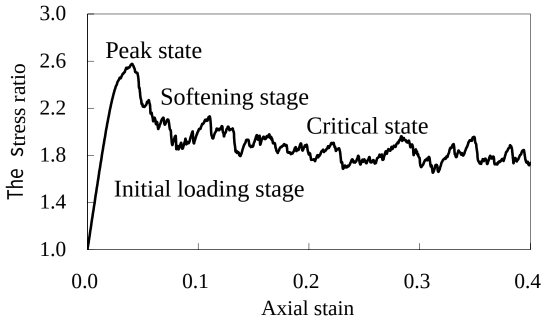


Figure 3 The stress ratio versus axial strain

Figure 4 shows the curve of the void ratio versus the axial strain (ε_{22}). It can be seen that the void ratio decreases at the initial compression. With the increase of the axial strain the dense particulate assembly dilates until the critical state is reached and then the variation of the void ratio is very small.

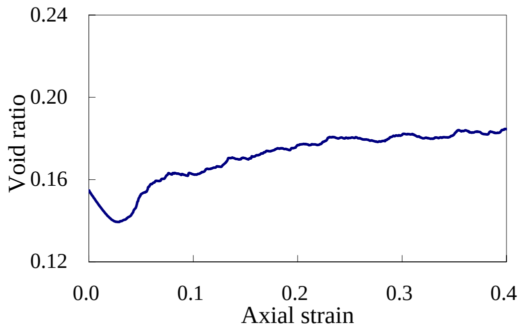


Figure 4 The void ratio versus axial strain

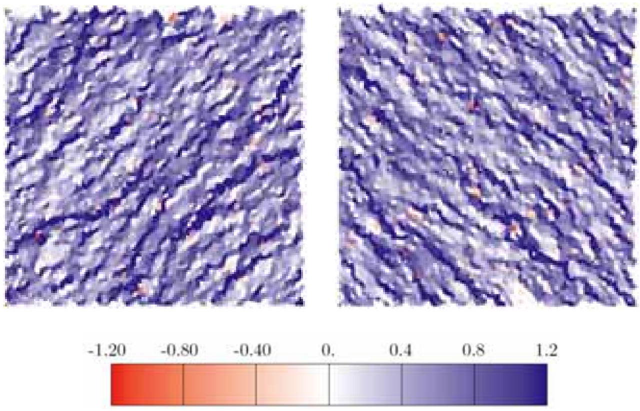
4. LOCALIZED DEFORMATION IN GRANULAR MATERIALS

In this section, the deformation of granular materials at the microscale (within the local void cell) will be discussed in detail. According to the method adopted by Kuhn (1999), the deformation are represented by computing the inner product of the local velocity gradient $\overline{L}^i$ with a selected deformation mode $\boldsymbol{\Phi}$. The dimensionless scalar measure can be obtained as the following equation,

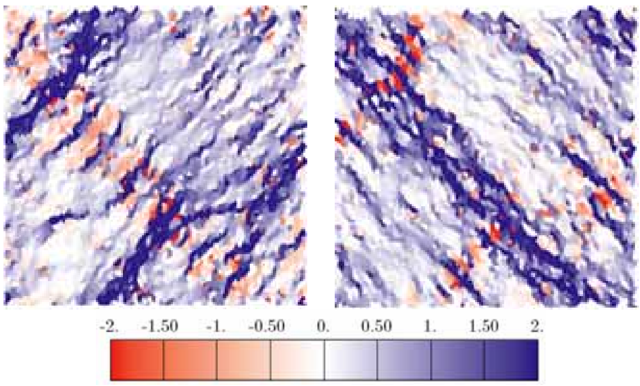
$$\Phi^{\beta\pm,i} = \overline{\boldsymbol{L}}^i \cdot \boldsymbol{\Phi}^{\beta\pm} / \left|\overline{\boldsymbol{D}}\right| \left|\boldsymbol{\Phi}^{\beta\pm}\right| \tag{4.1}$$

where, $\left|\overline{\boldsymbol{D}}\right|$ is the mean deformation of the whole granular assembly, $\Phi^{\beta\pm} = \begin{bmatrix} \cos\beta\sin\beta & \mp\cos^2\beta \\ \pm\sin^2\beta & -\cos\beta\sin\beta \end{bmatrix}$ and $\beta^{\pm} = 45^{\circ}$.

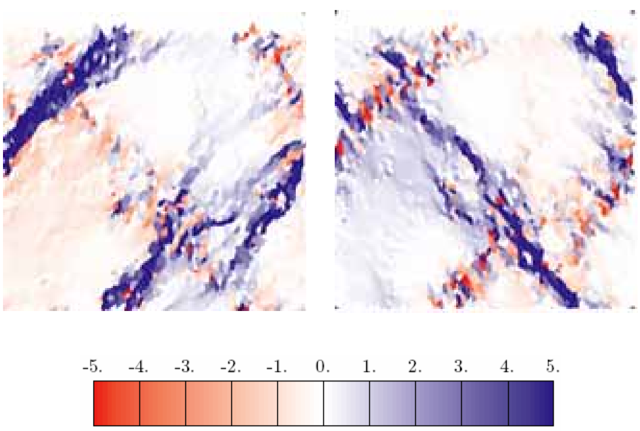
The slip deformation in local void cells of granular materials is presented in Figs. 5 (a-c). Figs. 5 (a-c) correspond to the initial loading stage (a), the peak state (b) and the soft stage (c) in stress ratio versus axial strain curve (see Fig. 3), respectively. The intense of the color represents the magnitude of the slip deformation characterized by normalized velocity gradient $\Phi^{\beta\pm,i}$. Blue zones ($\Phi^{\beta\pm,i}$ is positive) represent that the local velocity gradient $\overline{L}^i$ in the void cell is in the same direction with the average velocity gradient $\overline{L}$, while red zones ($\Phi^{\beta\pm,i}$ is negative) denote that the direction of $\overline{L}^i$ is opposite to that of $\overline{L}$.



(a) ε_{22}=0.01



(b) ε_{22}=0.04



(c) ε_{22}=0.06

Left slip deformation Right slip deformation

Figure 5 Left and right slip deformations in particulate assembly during biaxial compression

It can be observed that the nonuniform deformation is manifested obviously by the intense of the colour filled in the local void cells. In the initial stage of biaxial compression, there exist obvious microbands in granular materials. With the development of the biaxial compression, the void cells with large slip deformation concentrate and become thicker. In this stage discontinuous bands emerge as shown in Fig. 5 (b). As deformation proceeds, the localized zones come in to being continuous shear bands (Fig. 5 (c)). It should be pointed out that the localized deformation represented by slip deformation in void cells is seldom reported in published papers. It can be also found that there exist red zones in all the loading stages which indicate that the negative slip occurs even in the small deformation stage.

5. CONCLUSIONS

This paper has considered the evolution of internal micro-scale deformation in densely distributed assemblies at different loading stages. The phenomenon of localized deformation has been investigated using the discrete element method based on void cells. The numerical results reveal that the micro-scale deformations within local void cells are organized into oblique bands. During the biaxial compression, the negative slip occurs in the neighboring regions of elastic unloading and plastic flow along a plane of discontinuity in the velocity gradient field.

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ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation (50679013, 10772041 and 50609001), the National Key Basic Research Special Foundation (2005CB321704), and the Program for Changjiang Scholars by the Ministry of Education of China.