# PARAMETRIC SYSTEM IDENTIFICATION OF MULTISTOREYED BUILDINGS WITH NON-UNIFORM MASS AND STIFFNESS DISTRIBUTION

**K. Suresh[1], Sajal K. Deb[2] and Anjan Dutta[2]**

[1] *Graduate Student,* [2] *Professor, Dept. of Civil Engineering, Indian Institute of Technology Guwahati, India*
*Email :skdeb@iitg.ernet.in , skd.iitg@gmail.com*

**ABSTRACT :**

In this paper, structural health monitoring of multi-storeyed shear buildings based on system identification by parametric state space modeling has been presented. The earthquake ground acceleration and the acceleration response histories of various floors of buildings are required as input. The method evaluates the modal parameters like the eigen frequencies and mode shapes using a sub-space iteration technique (N4SID) after appropriate re-sampling and filtering of the recorded response. Using the Least Square Technique, the stiffness of the structure is evaluated and localized damage can be identified, if present. The method of system identification proposed in this study is capable of identification of system parameters of buildings with non-uniform mass and stiffness distribution. The method is applied for identification of a nine storeyed reinforced concrete building of BSNL Guwahati instrumented with limited numbers of sensors. Structural response data recorded during earthquakes on 11th February and 12th August, 2008 have been used for identification of system parameters of the building.

**KEYWORDS:** Structural health monitoring, parametric identification, multistoreyed building

## 1. INTRODUCTION

Based on the procedure of transforming input output data into a mathematical relation, the field of identification can be divided broadly into two classes: parametric identification and nonparametric identification. In parametric identification, the structure of the mathematical relation is fixed a priori and parameters of the structure are fitted to the data. In nonparametric identification, no (or very little) assumption is made with respect to the model structure. So the above mentioned non-parametric identification approaches do not require knowledge of the excitation and hence it is applicable to the general problem where the excitation is from ambient sources, which is generally immeasurable. On the other hand, there are different types of parametric models in system identification like Auto Regressive eXogenous model (ARX), Auto Regressive Moving Average eXogenous model (ARMAX), Box-Jenkins model, State Space Model etc. Most of the researches carried out using parametric models were confined to the evaluation of damping and frequency of damaged / undamaged structures as an indicative of damaged state of the structure. However, in this study the stiffness of the structure is calculated to detect the location and extent of the damage in the structure. Parametric modeling can be used as a faster way to detect the state of a structure soon after it experiences an earthquake. Researches on parametric modeling in recent years are more towards the use of these models in damage detection by using active sensing. Lynch (2004) introduced the concept of linear classifications of poles obtained from the parametric identification of a structure. The poles of ARX time-series models describing modal frequencies and damping ratios were plotted upon the discrete-time complex plane and perception linear classifiers were employed to determine if poles of the structural element in an unknown state (damaged or undamaged) could be distinguished from those of the undamaged structure. But, it is observed the damage cannot be localized by this method.

In the State Space form, the relationship between the input, noise and output signals is written as a system of first order differential or difference equations using an auxiliary state vector $x(t)$ (Ljung, 1987). Unlike other parametric models, the State Space model has the advantage wherein the state vectors provide more insight to the physical state of the system. The state vector $x(t)$ has the physical significance like positions, velocities etc. and the outputs can be expressed as the known combinations of the states.

Among various parametric state space modeling algorithms (e.g. algorithm based on block Hankel matrix with Markov parameters), numerical algorithms for subspace state space system identification (N4SID), introduced by Overschee and Moor (1993) has been used in this study. The major advantage with N4SID is that it is non-iterative, with no non-linear optimization part involved and also unlike classical identification, when estimating a state space system from data measured, it does not need the initial condition to be zero. These are the reasons for adopting State Space Modeling based on N4SID algorithm as the parameterized method in damage detection of the structure in the present work. Further, an iterative procedure has been developed to determine the eigenvectors and the stiffness coefficients of multistoreyed shear buildings with non-uniform mass and stiffness distribution in vertical direction.

## 2. PARAMETRIC IDENTIFICATION

The system transfer function $\mathbf{H}(z)$ relating input $\mathbf{u}(z)$ and output $\mathbf{y}(z)$ is expressed as:

$$\mathbf{y}(z) = \mathbf{H}(z)\,\mathbf{u}(z) \tag{2.1}$$

The values of $z$ for which transfer function $\mathbf{H}(z)$ is infinity are called poles. For a stable system, all poles must have a magnitude less than one and should be located within the unit circle.

The $j^{th}$ pole of the system is given by

$$z_j = e^{(-\xi_j \omega_j \pm i\omega_j \sqrt{1-\xi_j^2}\Delta t} \tag{2.2}$$

where, $\xi_j$ and $\omega_j$ are damping ratio and frequency of the $j^{th}$ mode of vibration. The frequency and damping ratio can be determined as follows:

$$\omega_j = \frac{1}{\Delta t}\sqrt{\ln^2 r_j + \theta_j^2} \tag{2.3}$$

$$\xi_j = -\frac{\ln r_j}{\sqrt{\ln^2 r_j + \theta_j^2}} \tag{2.4}$$

where, $r_j = |z_j|$ the magnitude; and $\theta_j = \tan^{-1}\left[\operatorname{Im} g(z_j)/\operatorname{Re}(z_j)\right]$, the phase angle of the $j^{th}$ pole.

## 3. SYSTEM IDENTIFICATION WITH LIMITED NUMBERS OF SENSORS

An iterative procedure suggested by Caicedo *et. al.* (2004) has been adopted to determine the eigenvectors and the stiffness terms of the building systems for limited sensors availability. The steps in this procedure are as follows:

1. As an initial value, set the mode shape matrix equal to the full mode shape of undamaged system. Alternatively, one could use modal characteristics of an analytical or numerical model of the structure to set initial value.
2. Insert known values from the identified eigenvectors into the eigenvector matrix.
3. Use these mode shapes to determine the stiffnesses of the floors of the structure.
4. From the stiffness matrix, $\mathbf{K}_i$ **,** using the stiffnesses identified in step 3, and compute the corresponding eigen vectormatrix, $\Phi_i$ .
5. Set $i = i+1$ , and return to step 2 using the eigen vectors computed in step 4.

## 4. DETAILS OF SAMPLE BUILDING AND INSTRUMENTATION

A sample nine storey building of BSNL Guwahati has been considered for this study is shown in Fig.1. First storey height of the building is 2.75 m while height of remaining stories is 3.0 m. The building does not have any infill wall in the first storey, but all the upper stories have similar infill wall. Thus the distribution of stiffness and mass is not uniform along the height. The building has been instrumented to record the acceleration response histories of different floors during earthquakes. A tri-axial accelerometer has been mounted at the ground level to collect the ground acceleration. Uni-axial accelerometers are installed only at some limited floors in both X (longer) and Y (shorter) directions of the building. Four uni-axial accelerometers are installed at first, third, seventh and top floor of the building along the Y direction while three uni-axial accelerometers are installed at first, fifth and top floor of the building along the X direction of the building. The data recorded from the mounted sensors in the building during earthquakes on 11th February and 12th August, 2006 have been utilized for the current study.

Figure 1 *BSNL building in Guwahati*

## 5. RESULTS AND DISCUSSION

First the various floor responses obtained from the accelerometers are filtered using the sixth order Butterworth filter of MATLAB tool box (version 7.0) and then the responses obtained are re-sampled at frequency of 60 Hz. By re-sampling to that frequency any external noises present beyond this frequency have been filtered without affecting the modal properties. The filtered re-sampled floor accelerations of various floors and the ground accelerations are given as input for N4SID model and the frequencies and the corresponding mode shapes are evaluated. The poles as extracted corresponding to acceleration data along the X and Y directions for the earthquake on 11th February, 2006 are plotted in Fig.2 and Fig.3.

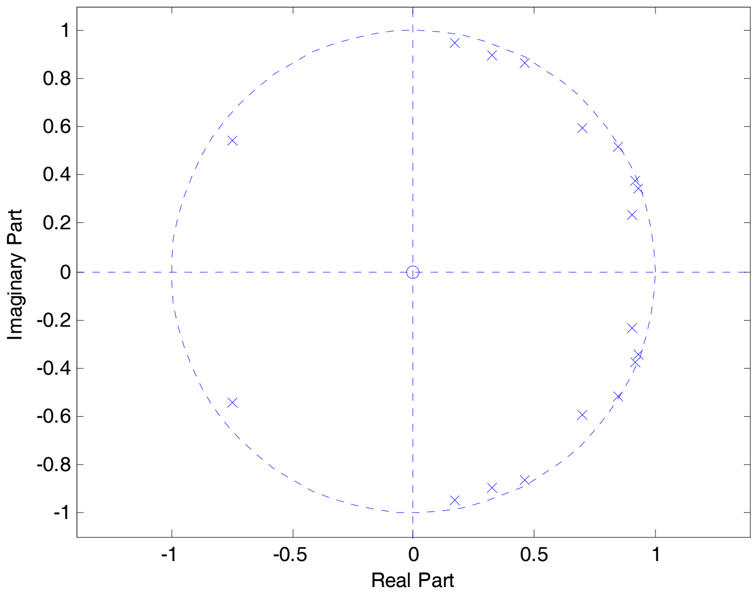


Figure 2 Poles obtained from earthquake data recorded in X direction on 11th February, 2006

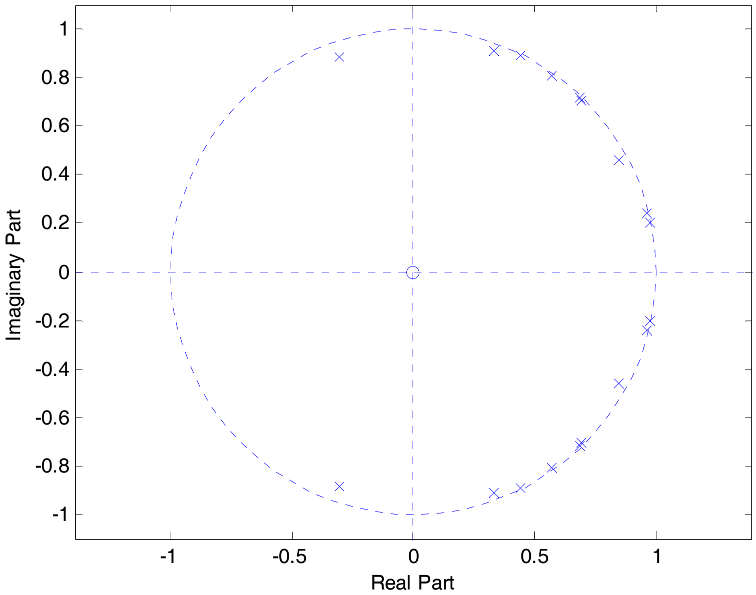


Figure 3 Poles obtained from earthquake data recorded in Y direction on 11th February, 2006

It is observed that the system is a stable system as the poles are within the unit circle. The obtained frequency and damping ratios corresponding to the acceleration along X and Y directions have been shown in Table 1.

Table 1 Identified frequencies and damping with data recorded on *11th February, 2006*

| Longer (X) direction | | Shorter (Y) direction | |
|---|---|---|---|
| Frequencies (rad/sec) | Damping (%) | Frequencies (rad/sec) | Damping (%) |
| 13.08 | 5.05 | 14.35 | 1.58 |
| 17.78 | 1.43 | 17.25 | 2.92 |
| 19.37 | 1.97 | 34.88 | 7.53 |
| 27.51 | 1.41 | 55.44 | 1.50 |
| 35.61 | 2.62 | 56.57 | 0.85 |
| 54.19 | 1.68 | 66.74 | 1.33 |
| 61.24 | 3.70 | 77.89 | 0.91 |
| 69.68 | 2.52 | 85.75 | 2.48 |
| 125.86 | 3.11 | 133.21 | 3.65 |

Similarly, modal parameters have also been obtained from the earthquake data on 12th August, 2006. The poles have been plotted in Fig.4 and Fig. 5, while the frequencies and damping have been shown in Table 2.

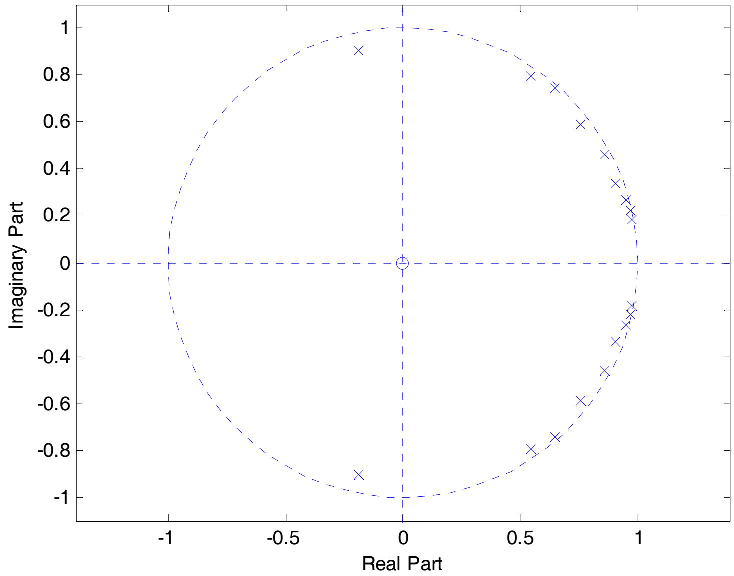


Figure 4 Poles obtained from earthquake data recorded in X direction on 12th August, 2006

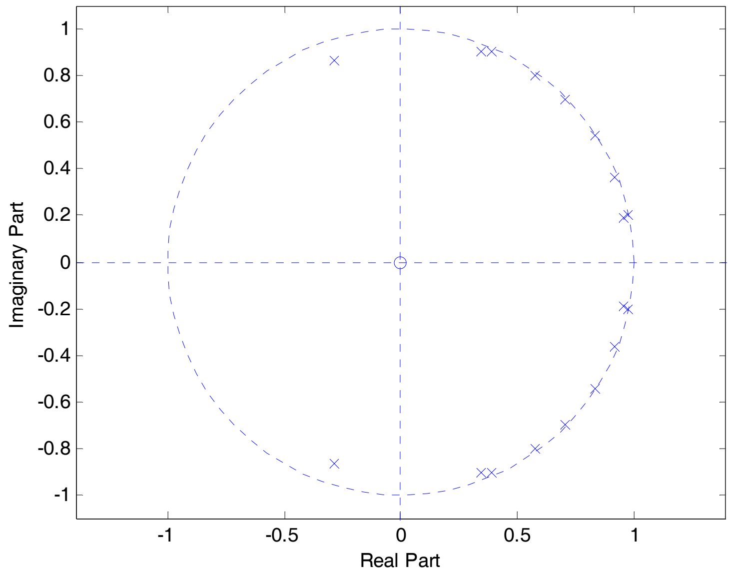


Figure 5 Poles obtained from earthquake data recorded in Y direction on 12th August, 2006

From Table 1 and Table 2, it is observed that there is not much of change in the modal parameters of the building as intensities at the site for both earthquakes have been very small and structure has not been undergone any damage.

Table 2 Identified frequencies and damping with data recorded on *12th August, 2006*

| Longer Direction | | Shorter Direction | |
|---|---|---|---|
| Frequencies (rad/sec) | Damping (%) | Frequencies (rad/sec) | Damping (%) |
| 13.07 | 3.64 | 14.06 | 3.63 |
| 15.90 | 3.18 | 14.29 | 1.13 |
| 19.31 | 4.10 | 26.25 | 3.04 |
| 24.96 | 9.29 | 40.36 | 0.63 |
| 34.51 | 4.55 | 54.72 | 1.28 |
| 46.47 | 6.79 | 66.10 | 1.35 |
| 59.86 | 2.17 | 81.42 | 1.13 |
| 67.98 | 3.83 | 84.58 | 2.62 |
| 124.52 | 4.36 | 132.45 | 5.05 |

The next step is to evaluate the stiffness of the building. Characteristics equations have been solved to evaluate the stiffness from the identified frequencies and mode shapes following the iterative procedure presented in the section 3. The identified stiffness at various floor levels of the nine storeyed sample building in both X and Y directions obtained using data recorded during 11th February 2006 earthquake are presented in the Table 3 and Table 4 respectively.

Table 3 Stiffness in X (longer) direction (in $10^8$ N/m)

| K1 | K2 | K3 | K4 | K5 | K6 | K7 | K8 | K9 |
|---|---|---|---|---|---|---|---|---|
| 15.04 | 11.58 | 11.56 | 11.63 | 11.66 | 11.64 | 11.64 | 11.53 | 11.61 |

Table 4 Stiffness in Y (shorter) direction (in $10^8$ N/m)

| K1 | K2 | K3 | K4 | K5 | K6 | K7 | K8 | K9 |
|---|---|---|---|---|---|---|---|---|
| 19.78 | 15.22 | 15.18 | 15.24 | 15.28 | 15.27 | 15.23 | 15.18 | 15.23 |

Similarly, the identified stiffness at various floor levels of the sample nine storeyed building in both X and Y directions obtained using data recorded during 12th August 2006 earthquake are presented in the Table 5 and Table 6 respectively.

Table 5 Stiffness in X (longer) direction (in $10^8$ N/m)

| K1 | K2 | K3 | K4 | K5 | K6 | K7 | K8 | K9 |
|---|---|---|---|---|---|---|---|---|
| 14.47 | 11.15 | 11.13 | 11.20 | 11.22 | 11.21 | 11.19 | 11.11 | 11.18 |

Table 6 Stiffness in Y (shorter) direction (in $10^8$ N/m)

| K1 | K2 | K3 | K4 | K5 | K6 | K7 | K8 | K9 |
|---|---|---|---|---|---|---|---|---|
| 19.14 | 14.72 | 14.68 | 14.76 | 14.78 | 14.80 | 14.75 | 14.68 | 14.74 |

It is observed that the stiffness coefficients of the sample building at various floor levels in both X and Y directions as identified from the data recorded during the earthquakes on 12th February 2006 and 14th August 2006 are very close in terms of magnitude and distribution.

## 6. CONCLUSIONS

The system identification using sub-space iteration (N4SID) and least square technique is an effective method of structural health monitoring for a multi-storeyed shear building. The data must be properly re-sampled and filtered to remove noise from recorded signals. The iterative procedure adopted in this study is found to be effective for the evaluation of structural stiffness for buildings with non-uniform mass and stiffness distribution. The method can be very effectively used for structural health monitoring even when limited numbers of sensors are available for structural response recording.

## REFERENCES

1. Caicedo, J.M., Dyke, S.J. and Johnson, E.A.(2004), "Natural Excitation Technique and Eigensystem Realization Algorithm for Phase I of the IASC-ASCE Benchmark Problem: Simulated Data", *J. Engng. Mech., ASCE*, **130: 1**, 49-60.

2. Ljung L. (1987), System Identification: Theory for the User, Prentice-Hall Inc.

3. Lynch J.P. (2004) "Linear Classification of System Poles for Structural Damage Detection using Piezoelectric Active Sensors", *SPIE 11th Annual International Symposium on Smart Structures and Materials*, San Diego, CA, USA, March

4. Overschee, P. V., Moor, B. D. (1993) "N4SID: Numerical Algorithms for State Space System Identification", *Proceedings of the World Congress of the International Federation of Automatic control, IFAC*, Sydney, Australia.