GDEE-BASED SEISMIC RESPONSE ANALYSIS AND RELIABILITY EVALUATION OF STRUCTURES

J.B. Chen[1], F. Kong[2] and J. Li[3]

[1] *Associate Professor,* [2] *Ph.D. Student,* [3] *Professor*
State Key Laboratory for Disaster Reduction in Civil Engineering, School of Civil Engineering, Tongji University, Shanghai, China
Email: chenjb@mail.tongji.edu.cn, kfcoffee@sina.com, lijie@mail.tongji.edu.cn

ABSTRACT :

GDEE-based seismic response analysis and reliability evaluation of structures are outlined. The stochastic ground motion can be represented by a random Fourier function where basic random variables are involved via the physical stochastic model. The principle of preservation of probability can then be applied to the augmented system composed of any arbitrary physical response quantities related to the system and the basic random parameter set. A family of generalized density evolution equation can thus be derived and solved using numerical methods. Further, introducing appropriate virtual stochastic process, the extreme value distribution of response of the system can be obtained and thus the dynamic reliability and the global reliability can be evaluated by a simple one-dimensional integral. The approach is applied to seismic response analysis and reliability evaluation of a practical structure located in a city where the seismic fortification intensity is 8 degree. The reliabilities of the structure with and without vibration mitigation system are compared.

KEYWORDS: Stochastic ground motion, probability density evolution, extreme value distribution, reliability

1. INTRODUCTION

Safety of structures under strong earthquake is of paramount importance to reduce the life and property loss. One of the most important issues for the earthquake is the large degree of randomness involved in the time, location and intensity (magnitude) (Li and Li, 1992). The recent Wenchuan earthquake strengthened the necessity of capturing the performance of engineering structures once again, particularly in the sense of reliability. In the past over one century, many researches have been done in seismology and earthquake engineering, resulting in a variety of results and approaches that improve the seismic design of engineering structures, both in the seismic risk analysis (Dowrick, 2003) and structural dynamics (Clough and Penzien, 1993). However, precise seismic reliability evaluation for practical engineering structures is still unavailable in that: (1) the models for stochastic ground motion are mainly based on statistics and are not mature enough (Kanai, 1957; Jennings et al, 1968); (2) the approaches for stochastic response analysis of large complex structures are not available (Schuëller, 1997; Lutes and Sarkani, 2004); and (3) the traditional dynamic reliability theory can not obtain the precise dynamic reliability, let alone the global reliability (Madsen et al, 1986).

In the past years, the probability density evolution theory for stochastic dynamic response analysis and reliability evaluation has been developed (Li and Chen, 2004, 2008; Chen and Li, 2007, Chen et al, 2007). The theory starts with the principle of preservation of probability, and yields a family of generalized density evolution equation, via which not only the probability density function and its evolution of any arbitrary response quantities, but also the extreme value distribution, can be obtained. Combined with the physical stochastic ground motion, the seismic response analysis and reliability evaluation can be carried out. The present paper outlines such an approach and applies it to a practical engineering structure.

2. GENERALIZED DENSITY EVOLUTION EQUATION FOR DYNAMIC RESPONSE ANALYSIS

The equation of motion of an engineering structure can usually be written as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{f}(\mathbf{X}) = -\mathbf{M}\mathbf{I}\ddot{x}_{g}(t) \tag{2.1}$$

where **M** and **C** are the mass and damping matrix; $\ddot{\mathbf{X}}, \dot{\mathbf{X}}, \mathbf{X}$ are the relative acceleration, relative velocity and relative displacement, respectively; **f** is the restoring forces, in the case $\mathbf{f} = \mathbf{K}\mathbf{X}$, the structure is linear, otherwise nonlinear; **I** is the column vector with all components being 1; $\ddot{x}_{g}(t)$ is the earthquake ground acceleration, which should be regarded as a stochastic process.

2.1 Physical Stochastic Model for Ground Motion

The frequency ω_0 and the damping ratio ζ of the site soil are usually random variables, which are two of the major sources of randomness involved in the ground motion on the surface of the site. The other major source of the randomness is the amplitude of the motion on the bedrock of the site. Based on this understanding, the random ground motion in the surface of the site can be regarded as the process on the bedrock filtered through the site soil (Li and Ai, 2006). Thus, the Fourier transform of the absolute acceleration on the surface of the site can be given by

$$\ddot{X}(\omega) = \frac{\omega_0^2 + i2\zeta\omega_0\omega}{\omega_0^2 - \omega^2 + i2\zeta\omega_0\omega}\ddot{U}_{g}(\omega) \tag{2.2}$$

Here $\ddot{U}_{g}(\omega)$ is the Fourier transform of the acceleration of the input seismic waves. Introducing the concept of random Fourier function, the equation above can be transformed as

$$F_X(\omega) = \left[\frac{1 + 4\zeta^2(\frac{\omega}{\omega_0})^2}{(1 - \frac{\omega^2}{\omega_0^2})^2 + 4\zeta^2(\frac{\omega}{\omega_0})^2}\right]^{\frac{1}{2}} F_{g}(\eta, \omega) \tag{2.3}$$

where η is a random variable related to the amplitude of the input seismic waves, $F_{g}(\eta,\omega)$ is the Fourier spectrum of the accelerations on the bedrock.

Notice that the inverse Fourier transform of $F_X(\omega)$ will yield the time history of ground acceleration

$$\ddot{X}_{g}(\mathbf{\Theta}, t) = \mathscr{F}^{-1}[F_X(\mathbf{\Theta}, \omega)] \tag{2.4}$$

where $\mathbf{\Theta} = (\omega_0, \zeta, \eta)$ is the involved random vector.

Substituting Eqn. 2.4 in Eqn. 2.1 yields

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{f}(\mathbf{X}) = -\mathbf{M}\mathbf{I}\ddot{X}_{g}(\mathbf{\Theta}, t) \tag{2.5}$$

2.2 Generalized Density Evolution Equation

The equation of motion Eqn. 2.5 is usually well-posed if the initial condition

$$\mathbf{X}(t_0) = \mathbf{x}_0,\ \dot{\mathbf{X}}(t_0) = \dot{\mathbf{x}}_0 \tag{2.6}$$

is given. Clearly, the physical solution will depend on $\boldsymbol{\Theta}$ and can be written in the form

$$\mathbf{X}(t) = \mathbf{H}_{\mathbf{X}}(\mathbf{x}_0, \dot{\mathbf{x}}_0, \boldsymbol{\Theta}, t) \tag{2.7}$$

Likewise, the velocity can be written in the form

$$\dot{\mathbf{X}}(t) = \mathbf{h}_{\mathbf{X}}(\mathbf{x}_0, \dot{\mathbf{x}}_0, \boldsymbol{\Theta}, t) \tag{2.8}$$

In essence, Eqns. 2.7 and 2.8 is the Lagrangian description of the original dynamical systems.

In the analysis, one might be interested in some physical quantities other than the displacement and velocity, for instance, the strain or stress at a point of the structure, or the internal force or deformation in a section of a member of the structure. In this case, we denote the interested physical quantities as $\mathbf{Z} = (Z_1, Z_2, \cdots, Z_m)^{\mathrm{T}}$. Clearly, these physical quantities are determined by the displacement and velocity vectors, i.e.

$$\dot{\mathbf{Z}}(t) = \mathscr{Z}[\mathbf{X}(t), \dot{\mathbf{X}}(t)],\ \mathbf{Z}(t_0) = \mathbf{z}_0 \tag{2.9}$$

Substituting Eqns.2.7 and 2.8 in 2.9 yields

$$\begin{aligned}\dot{\mathbf{Z}}(t) &= \mathscr{Z}[\mathbf{H}_{\mathbf{X}}(\mathbf{x}_0, \dot{\mathbf{x}}_0, \boldsymbol{\Theta}, t), \mathbf{h}_{\mathbf{X}}(\mathbf{x}_0, \dot{\mathbf{x}}_0, \boldsymbol{\Theta}, t)] \\ &= \mathbf{h}(\boldsymbol{\Theta}, t)\end{aligned} \tag{2.10}$$

It is seen that Eqn. 2.10 is also a dynamical system, in which the randomness involved comes completely from $\boldsymbol{\Theta}$. Because of this, the augmented system $(\mathbf{Z}(t), \boldsymbol{\Theta})$ is a probability preserved system. From the random event description of the principle of preservation of probability, we have (Li and Chen, 2008)

$$\frac{\mathrm{D}}{\mathrm{D}t}\int_{\Omega_t \times \Omega_\theta} p_{\mathbf{Z}\boldsymbol{\Theta}}(\mathbf{z}, \boldsymbol{\theta}, t) d\mathbf{z} d\boldsymbol{\theta} = 0 \tag{2.11}$$

where $\mathrm{D}/\mathrm{D}t$ is the total derivative, $p_{\mathbf{Z}\boldsymbol{\Theta}}(\mathbf{z}, \boldsymbol{\theta}, t)$ is the joint PDF of $(\mathbf{Z}(t), \boldsymbol{\Theta})$, Ω_t is the domain at time t corresponding to Ω_0, Ω_0 is any arbitrary domain in the state space at time t_0, Ω_θ is any arbitrary domain in the distribution domain of $\boldsymbol{\theta}$.

After a series of mathematical manipulations, a family of generalized density evolution equation can be derived as follows

$$\frac{\partial p_{\mathbf{Z}\boldsymbol{\Theta}}(\mathbf{z}, \boldsymbol{\theta}, t)}{\partial t} + \sum_{j=1}^{m} Z_j(\boldsymbol{\theta}, t)\frac{\partial p_{\mathbf{Z}\boldsymbol{\Theta}}(\mathbf{z}, \boldsymbol{\theta}, t)}{\partial z_j} = 0 \tag{2.12}$$

The initial condition is

$$p_{\mathbf{Z}\boldsymbol{\Theta}}(\mathbf{z}, \boldsymbol{\theta}, t_0) = \delta(\mathbf{z} - \mathbf{z}_0) p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \tag{2.13}$$

It is worth noting that the dimension of Eqn. 2.12 is independent to the dimension of the original system Eqn. 2.1. In many cases, we have $m = 1$ and thus Eqn. 2.12 reduces to

$$\frac{\partial p_{Z\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t)}{\partial t}+Z(\boldsymbol{\theta},t)\frac{\partial p_{Z\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t)}{\partial z}=0 \tag{2.14}$$

The initial condition is

$$p_{Z\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t_0)=\delta(z-z_0)p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \tag{2.15}$$

Once $p_{Z\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t)$ is obtained by solving Eqns. 2.14 and 2.15, the PDF of Z can be given by

$$p_Z(z,t)=\int_{\Omega_{\boldsymbol{\Theta}}} p_{Z\boldsymbol{\Theta}}(z,\boldsymbol{\theta},t)d\boldsymbol{\theta} \tag{2.16}$$

3. SEISMIC RELIABILITY EVALUATION

As a dynamic reliability problem, the seismic reliability can be evaluated according to the first-passage criterion or the low-cycle fatigue criterion. In the present paper, we consider the first-passage reliability defined by

$$R=\Pr\{X(\tau)\in\Omega_{\mathrm{s}},\tau\in[0,T]\} \tag{3.1}$$

where T is the time duration, X is the interested physical quantity, Ω_{s} is the safe domain. For instance, in the seismic reliability evaluation, X might be any inter-story drift.

The traditional theory for dynamic reliability based on the level-crossing process needs the computation of expected crossing rate and the assumption on the nature of crossing events. This makes the error not be guaranteed. On the other hand, if the dynamic reliability is viewed from the angle of extreme event, the above problems do not exist. In fact, Eqn. 3.1 is equivalent to

$$R=\Pr\{X_{\mathrm{ext}}\in\Omega_{\mathrm{s}}\} \tag{3.2}$$

where X_{ext} is the extreme value corresponding to the failure criterion in Eqn. 3.1. For instance, if Eqn. 3.1 is

$$R=\Pr\{|X(\tau)|\le x_{\mathrm{B}},\tau\in[0,T]\} \tag{3.3}$$

then

$$X_{\mathrm{ext}}=\max_{\tau\in[0,T]}|X(\tau)| \tag{3.4}$$

If a virtual stochastic process is introduced, the PDF of X_{ext}, dented by $p_{X_{\mathrm{ext}}}(x)$, can be obtained (Chen and Li, 2007). Thus, the reliability can be evaluated by

$$R=\Pr\{X_{\mathrm{ext}}\in\Omega_{\mathrm{s}}\}=\int_{\Omega_{\mathrm{s}}} p_{X_{\mathrm{ext}}}(x)dx \tag{3.3}$$

Thus, the dynamic reliability can be transformed to the problem of a one-dimensional integral. Likewise, when the equivalent extreme value event is introduced, then the global reliability (system reliability) can be evaluated without essential difficulties (Li et al, 2007).

4 SEISMIC RESPONSE AND RELIABILITY EVALUATION OF A PRACTICAL STRUCTURE

Seismic response and reliability evaluation of a practical engineering structure are carried out. It is a high-rise building with the height of 94.95m, located in a city of Eastern China where the earthquake fortification intensity is 8 degree. With the routine seismic design, it is very hard to meet the seismic requirement. Thus, earthquake mitigation system composed of damping walls is designed and installed.

The 3-dimensional finite element model of the building is shown in Figure 1, where the 3 stories in the bottom is the basement.

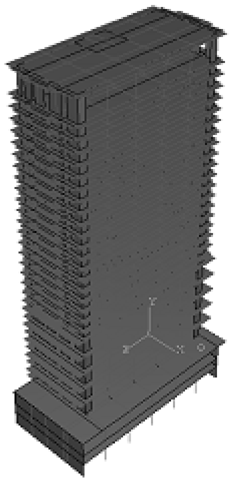

Figure 1 Three-dimensional finite model

Using the physical stochastic model for ground motion as described in Section 2.1, when we consider the situation of frequently occurred earthquake of intensity of 8 degree, i.e. the peak acceleration value with exceedance probability of 63.5%, the peak acceleration on the bedrock should be 0.11g according to Chinese Code for Seismic Design. But for the present site, investigations show that the peak acceleration with exceedance probability of 63.5% is 0.084g. Thus, in this case, only the randomness involved in the frequency and damping ratio of the site soil are considered. Using the strategy of selecting representative points via tangent spheres 221 points can be determined. Simultaneously the corresponding assigned probabilities can also be specified. 221 representative acceleration time histories can be generated. Once this is done, the response of the structure subjected to the 221 acceleration time histories can be computed and then the generalized density evolution equation can be solved to obtain the probability density function of the responses. In the present investigations, we evaluate the PDF of the inter-story drifts.

Figure 1 shows the mean and standard deviation of the inter-story drift of 22nd story. It is seen that the inter-story drift can be regarded as a mean-zero but non-stationary process because the standard deviation varies against time. Shown in Figure 2 is the PDF of the inter-story drift at three typical time instants. Clearly, the PDF at different time is quite different not only in the distribution range, but also in the shape, some times seems regular but sometimes quite irregular. Figure 3 pictures the PDF over the time interval [18, 20] s while Figure 4 pictures the contour of the PDF surface over the time interval [15, 20] s.

Employing the approach of evaluating the extreme value distribution based on the generalized density evolution equation (Chen and Li, 2007), the PDF of each maximum inter-story drift can be obtained. Simultaneously the cumulative distribution functions are available. Notice that the threshold of the inter-story drift angle is 1/800 for the present structure, the threshold of the inter-story drift is then $d_0 = h/800$, where h is the story height. Thus the value of the CDF as the coordinate is d_0 is actually the reliability. Figures 5 and 6 show the PDF and CDF of the inter-story drift of 22nd story. The reliabilities of different stories are listed in Table 4.1.

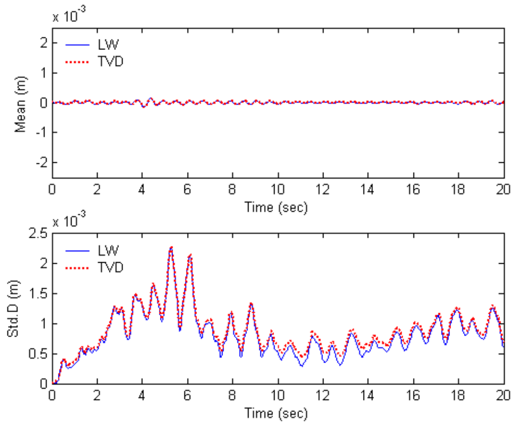

Figure 1 Mean and Standard deviation

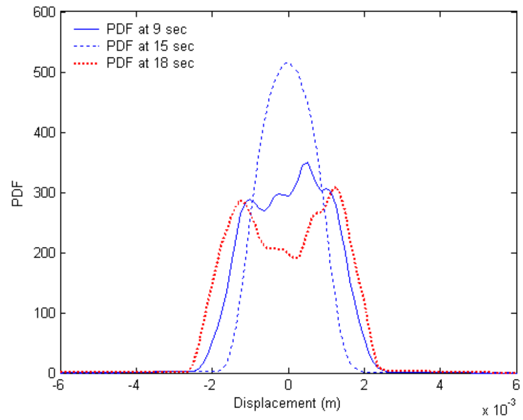

Figure 2 PDF at different time instants

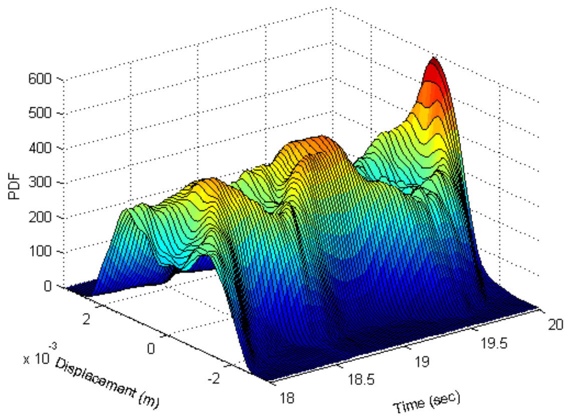

Figure 3 PDF evolution surface

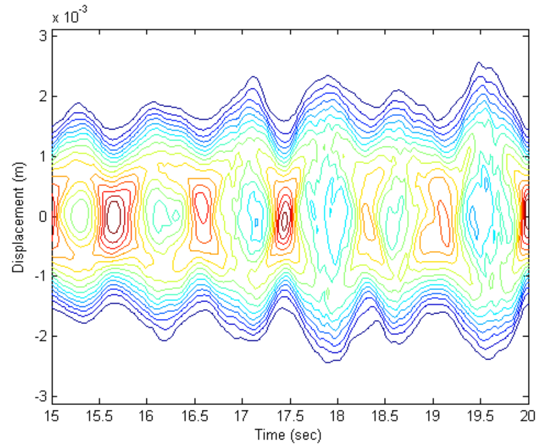

Figure 4 Contour of the PDF surface

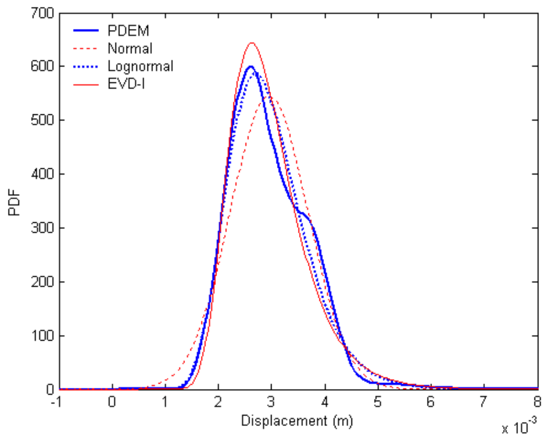

Figure 5 PDF of the maximum inter-story drift

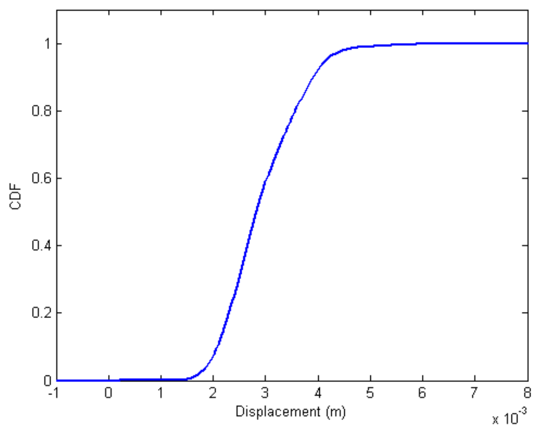

Figure 6 CDF of the maximum inter-story drift

Likewise, the global reliability can be evaluated when the principle of equivalent extreme value event is employed (Li et al, 2007). It is 0.9416 in the present case. Clearly, it is seen that it is less than the smallest one in Table 4.1.

The global reliability is less than 95%. This makes it necessary to reduce the seismic response by installing additional devices. In this regard, totally 61 W2000×H2000 damping walls are installed in the structure, 35 in X direction and 26 in Y direction. Investigations show that the equivalent damping ratio of the structure exceeds 7.5%. Thus, we assume the equivalent damping ratio is 7.5% and carry out the seismic reliability evaluation in a similar way. Table 4.2 lists the reliability of all the stories. Simultaneously, the global reliability of the structure is now promoted to 0.9856. Comparing Table 4.2 and Table 4.1 shows that the reliabilities of all the stories are increased by installing the damping wall system.

Table 4.1 Reliability of different stories (without damping walls)

story	reliability	story	reliability
1	1.0000	15	0.9864
2	1.0000	16	0.9858
3	1.0000	17	0.9844
4	0.9998	18	0.9811
5	0.9891	19	0.9808
6	0.9875	20	0.9749
7	0.9899	21	0.9568
8	0.9891	22	0.9459
9	0.9891	23	0.9520
10	0.9882	24	0.9681
11	0.9872	25	0.9808
12	0.9865	26	0.9865
13	0.9867	27	0.9876
14	0.9866	28	0.9896

Table 4.2 Reliability of different stories (with damping walls)

story	reliability	story	reliability
1	1.0000	15	0.9928
2	1.0000	16	0.9927
3	1.0000	17	0.9928
4	1.0000	18	0.9930
5	0.9963	19	0.9922
6	0.9935	20	0.9898
7	0.9929	21	0.9878
8	0.9923	22	0.9875
9	0.9928	23	0.9877
10	0.9926	24	0.9885
11	0.9925	25	0.9907

12	0.9923	26	0.9939
13	0.9926	27	0.9956
14	0.9928	28	0.9975

5. CONCLUDING REMARKS

The approach based on the probability density evolution theory for seismic response and reliability evaluation of engineering structures is outlined. The implementation procedures are discussed. A practical structure located in a city where the seismic fortification intensity is 8 degree is investigated. The probability density functions and their evolution of the inter-story drifts are evaluated. The dynamic reliability of all the stories and the global reliability are evaluated. Moreover, the reliabilities of the structure with and without damping wall systems are compared, showing the improvement of installment of the vibration mitigation system in a quantitative way in the sense of reliability.

ACKNOWLEGEMENTS

Financial supports from the National Natural Science Foundation of China for Innovative Research Groups (Grant No.50601062) and the New Century Excellent Scholars Plan by the Ministry of Education of China are acknowledged. Profs. W.Q. Liu, S.G. Wang and D.S. Du from Nanjing University of Technology are highly appreciated for their cooperation and discussions in the project.

REFERENCES

Chen, J.B. and Li, J. (2007). The extreme value distribution and dynamic reliability analysis of nonlinear structures with uncertain parameters. *Structural Safety* **29**: 77-93.
Chen, J.B., Liu, W.Q., Peng Y.B. and Li, J. (2007). Stochastic seismic response and reliability analysis of base-isolated structures. *Journal of Earthquake Engineering* **11**:**6**, 903-924.
Clough, R.W. and Penzien, J. (1993). Dynamics of Structures (2nd Ed.), McGraw-Hill College.
Dowrick, D.J. (2003). Earthquake Risk Reduction, John Wiley & Sons, Ltd.
Jennings, P.C., Housner, G.W. and Tsai, N.C. (1968). Simulated earthquake motions. Report of Earthquake Engineering Research Laboratory, California Institute of Technology.
Kanai, K. (1957). Semi-empirical formula for the seismic characteristics of the ground. Bull. Earthquake Research Institute, University of Tokyo, **35**: 309-325.
Li, J., Ai, X.Q. (2006). Study on random model of earthquake ground motion based on physical process. *Earthquake Engineering and Engineering Vibration* **26**:**5**, 21-26 (in Chinese).
Li, J. and Chen, J.B. (2004). Probability density evolution method for dynamic response analysis of structures with uncertain parameters. *Computational Mechanics* **34**, 400-409.
Li, J. and Chen, J.B. (2008). The principle of preservation of probability and the generalized density evolution equation. *Structural Safety* **30**: 65-77.
Li, J., Li, G.Q. (1992). Introducing to Earthquake Engineering, Beijing, Earthquake Press (in Chinese).
Li, J., Chen, J.B. and Fan, W.L. (2007). The equivalent extreme-value event and evaluation of the structural system reliability. *Structural Safety* **29**: 112-131.
Lutes, L.D. and Sarkani, S. (2004). Random Vibrations: Analysis of Structural and Mechanical Systems, Elsevier, Amsterdam.
Madsen, H.O., Krenk, S. and Lind, N.C. (1986). Methods of Structural Safety, Prentice-Hall.
Schuëller, G.I. (Ed.). (1997). A state-of-the-art report on computational stochastic mechanics. *Probabilistic Engineering Mechanics* **12**:**4**, 197-321.