# TO WHAT TYPE OF BEAM CAN BE ASSOCIATED A BUILDING?

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**ABSTRACT:**

This article is devoted to the study of the dynamics of usual buildings made up of identical stories. The aim is to build analytical beam models enabling to describe the first horizontal modes of vibrations. The homogenization method of periodic discrete media is applied to a class of unbraced framed structures. Depending on the order of magnitude of the shear force and two bending moments, seven families of beam are proved to be possible. The macroscopic parameters of the homogenized model are expressed in function of the static mechanical and geometrical characteristics of the frame elements. Simple criteria are established to identify the relevant model for real structures. Lastly, the models are validated by comparison with numerical calculations and experimental data.

**KEYWORDS:** Discrete structure, periodic homogenization, continuous model, beam, Timoshenko

## 1. INTRODUCTION

A great part of existing buildings was built before the definition of seismic codes. So the question of their seismic assessment is of first importance. The large number of buildings and the general lack of information make inappropriate the use of sophisticated computing methods. However, the experimental modal shapes suggest using continuous beam models to describe the first horizontal modes of vibration of usual regular buildings as the one presented on the figure 1. The interest of such modelling lies in the efficiency of the analytical formulation for understanding the dynamics of a real structure. Applications concern as well prediagnosis as strategies of reinforcement. The shear beams and bending beams are the two simple models usually considered in this approach. But can buildings behave differently? And how can we determine the more appropriate model for a given structure?

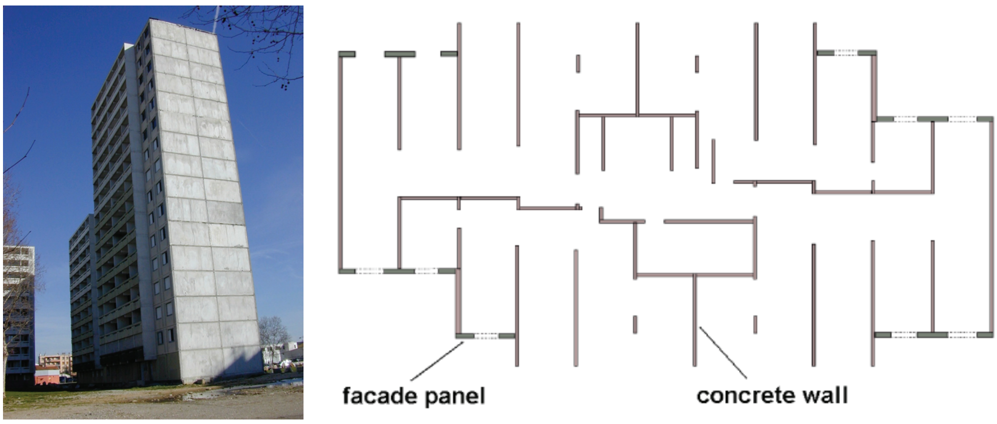


Figure 1 Studied building and its typical floor view

The problem is treated using the homogenization method of periodic discrete media (HPDM). This choice is motivated by the fact that, very frequently, the structure of ordinary concrete buildings is periodic in height. The advantage of the HPDM is to derive rigorously the continuous description in direct relation with the characteristics of the elements without any assumption on the behaviour of the structure. Furthermore, the passage to the continuous modelling takes place without loss of information at the local level, so that it is possible to go back to the deformations and efforts in the elements of the structure. Like any other homogenization method, the HPDM is based on the condition of scale separation, meaning that the height of a story is small in comparison with the modal wavelength. This condition limits the study to a number of modes of the order of the third of the number of stories. But it is sufficient because only the first modes are shaken during earthquakes. In this study, the HPDM is applied to a class of framed structures without bracing corresponding to "idealized buildings". The principles of the method are exposed in the first part of the paper. In the second part, the different beam models and the identification criteria are presented. Finally, this approach is validated in the third part by comparison with numerical calculations and experimental data.

## 2. PRESENTATION OF THE STRUCTURES AND THE HPDM

The study focuses on the harmonic transverse vibrations of a family of structures made up of an infinite periodic pile of unbraced frames called cells. The frames are formed by three plates: two identical walls bearing a floor (Figure 2). The walls and the floor are made of the same material but can have different dimensions. The cell elements are linked by rigid massless connections and behave as Euler-Bernoulli beams – i.e. pure bending beams. The study is conducted within the framework of linear elasticity and small deformations. The various physics of the basic frame are introduced through the walls and the floors thicknesses. This method induces a large range of contrast of stiffness between elements, in compression and bending.

The HPDM was developed by Caillerie and is presented in (Verna 1991), (Tollenaere 1994) et (Tollenaere, Caillerie 1998). It was modified (Boutin, Hans 2003) to take into account the influence of the microstructure on the global behaviour. The modal analysis of a periodic lattice of interconnected beams is performed in two steps: first, the discretization of the momentum balance, second, the actual homogenization, leading to a continuous model elaborated from the discrete description. A brief outline of this method is given hereafter. Detailed presentations can be found in (Tollenaere, 1994), (Hans, 2002) or (Boutin, Hans 2003).

As the connections between the elements are perfectly stiff, the displacements and rotation of each extremity linked to the same node are identical and are in fact the discrete kinematic variables of the system. The discretization step consists in integrating the dynamic balance – in harmonic regime – of the beams, taking the unknown displacements and rotations at their extremities as boundary conditions. Efforts applied by an element on its extremities are then expressed explicitly as functions of the nodal kinematic variables. The dynamic balance of each element being henceforth satisfied, it only remains to express the balance of efforts applied by the elements connected to the same node. This process reduces the balance of the whole structure to the balance of the set of nodes without any assumption. Consequently, there is a full equivalence between the discrete description – that uses the nodal variables only – and the complete description.

The second phase makes an explicit use of the homogenization principles. The key assumption of the HPDM lies in the scale separation. This means that the cell size $l$ – in the direction of periodicity – should be small compared to the unknown characteristic size $L$ of the deformation of the structure under vibrations. In other words, the scale ratio $\varepsilon = l/L$ is small compared to 1, that is $\varepsilon \ll 1$. The condition of scale separation implies that the method is limited to the first modes of vibrations, whose wavelengths are much larger than the cell size. In this case, the values of efforts and displacements vary slowly from a node to another and these discrete variables can be considered as the discrete values of continuous functions to determine. For this purpose, a macroscopic space variable $x$ along the periodicity axis is introduced and the researched motions are expressed as continuous functions of $x$ coinciding with the discrete variables at any node. These functions, assumed to converge when $\varepsilon$ tends to zero, are expanded in powers of $\varepsilon$. It is worth noting that any unknown among which the modal frequency is expanded in powers of $\varepsilon$. Furthermore, $l = \varepsilon L$ being a small increment with respect to $x$, the variations of the physical variables between neighbouring nodes are expressed using Taylor's series, that introduces in turn the macroscopic derivatives. Finally, the scale separation requires that, at the modal frequency of the global system, the wavelengths generated in the local elements are much larger than their own length. Consequently, the nodal efforts can be developed in Taylor's series with respect to the scale ratio $\varepsilon$.
The method of asymptotic expansions is based on the identification of terms of the same power of $\varepsilon$ in the

balance equations. So it is necessary to integrate correctly the properties of the cell through a normalization process. It consists in scaling the geometrical and mechanical characteristics of the elements according to the powers of ε. As for the modal frequency, the scaling is imposed by the balance of the inertia forces and elastic forces at the macro level. Such normalization insures that each mechanical effect appears at the same order whatever the ε value is. Therefore, the same physics is kept at the limit $\varepsilon \rightarrow 0$, i.e. for the homogenized model.

The whole expansions in ε powers are introduced in the discrete nodal balance equations. The obtained relations being valid for any enough small value ε, the separation of different orders can be operated. This leads for each order to a set of balance equations whose resolution, done by increasing order, defines naturally the relative orders of magnitude of the macroscopic variables.

## 3. TRANSVERSE DYNAMICS OF MULTI-FRAMED STRUCTURES

### *3.1. Description of the beam models*

The application of the HPDM to several framed structures with different contrasts of stiffness between the walls and the floors shows various behaviours, among which the shear and bending beams but also unusual models. They are all generated by the combination of three basic mechanisms presented on the figure 2. The shear of the cell is generated at the microscale by the local bending of the walls and of the floors. The other mechanisms are two different types of bending. For the global bending, the whole structure behaves as a classical bending beam. This phenomenon comes from the simultaneous traction-compression of the walls. The local bending is unusual. Each wall behaves as a bending beam and they are synchronized by the floors.

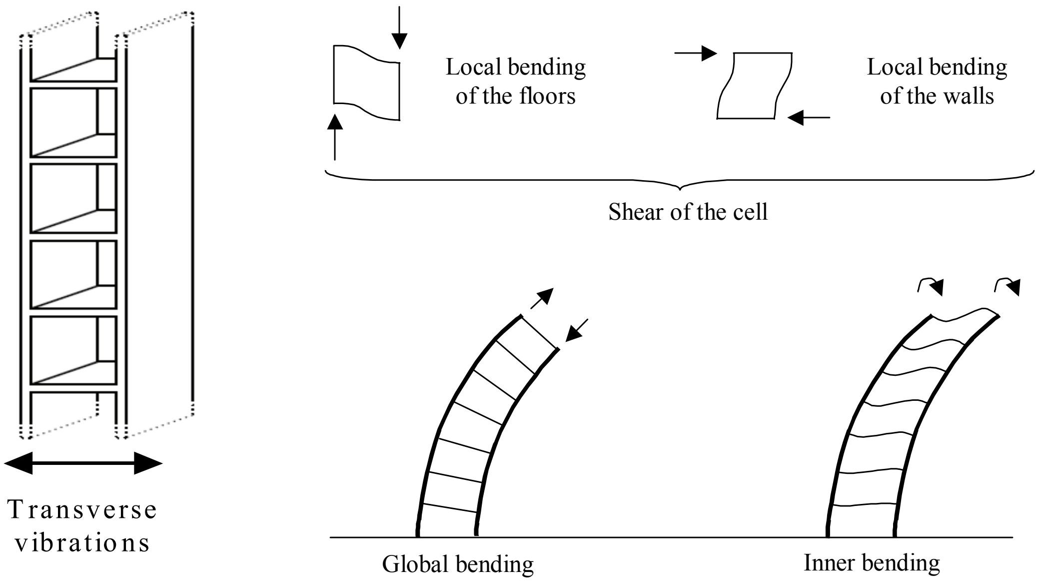


Figure 2 Left: class of studied structure, right: mechanisms governing the transverse vibrations

We can notice here the differences between massive beams and hollow beams. The phenomenon of simultaneous bending of the walls does not exist in the classical beam theory and represents an additional degree of freedom. Moreover, for massive beams, the shear correction taken into account in the Timoshenko beam theory only arises in case of small slenderness. Here the shear effect is present even for framed structures of large slenderness. This is due to the lack of bracing which makes the shear deformability of the frame much larger. Finally, the rotation inertia is much smaller than in a massive beam and this effect does not appear at the leading order of the expansions of the balance equations.

Each of the three mechanisms can be associated with macroscopic characteristics: generalized effort and deformation, stiffness and the corresponding constitutive law. For the shear of the cell, equations suggest defining a macroscopic shear force T – linked to the shear force in the walls – dual to the section translation U, which is the average horizontal displacement of the two nodes of a cell. These quantities are related by a global shear stiffness K corresponding to the association in series of the static stiffness of the two walls and the static stiffness of the floor, which work as bi-embedded beams. In the case of global bending a macroscopic moment is defined by the difference of the axial forces in the walls. It produces the rotation of the section α measured by the difference of the vertical displacements of the nodes of a cell. The corresponding inertia $I_{macro}$ is exactly that of a beam made of the two walls distant of the length of the floor. Finally, the inner bending is associated to an inner bending moment, which is the sum of the bending moments in the walls. The corresponding variable is the rotation of the nodes θ and the inertia is the sum of the inertias of the walls $\Sigma I_m$. The balance equations given by the HPDM are not written in this short presentation but the macroscopic stiffnesses appear naturally through the expansions. The reader may refer to (Boutin, Hans 2003).

By combining these three mechanisms, a generic model governed by a differential equation of the sixth degree is built. If one or two mechanisms are negligible, the generic model degenerates into simpler models presented on the figure 3. In the equations, $E_m$ is the elastic modulus of the material of the walls, Λ the linear mass of the structure and ω the angular frequency. The left part of the chart contains the classical beams: the bending beam, the shear beam and their association, the slender-Timoshenko beam. The right part of the chart describes the unusual models, which include the inner bending. It is worth noting that the generic model has three degrees of freedom and needs six macroscopic boundary conditions. For the simpler models, the number of degrees of freedom decreases and the "hidden" variables are completely defined by the driving variables. When the thicknesses of the elements vary, the behaviour of the structure evolves gradually from one model to another.

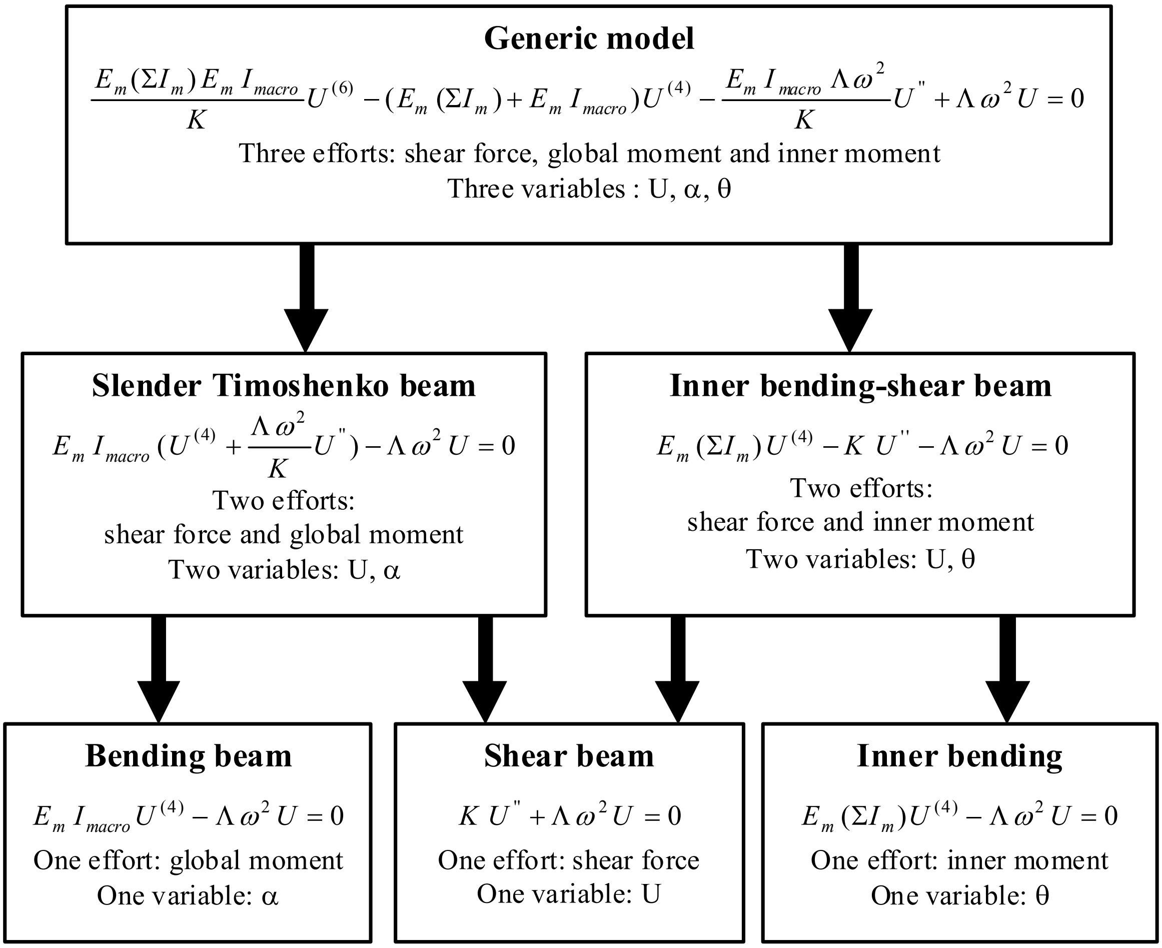


Figure 3 Beam models with the driving efforts and variables

Now, we have an exhaustive list of the possible behaviours for a particular class of structures. The second part of the problem is to determine the domains of validity of these models and to find how the results can be generalized to real buildings.

### *3.2. Behaviour of real structures*

We begin with the case of a "realistic" structure whose cells have the same geometry as in the previous part. The difference lies in the fact that, here, the structure is made up of a finite number of elements themselves of finite size. The ideal condition of homogenization stipulating that the scale ratio should tend to zero is only imperfectly satisfied. The first consequence is that the homogenized model does not provide an exact description of the reality but simply reasonable approximations of the behaviour. This point will be discussed in the next part through the numerical validation. The second consequence concerns the choice of the relevant model for a given structure. During the homogenization process this choice is determined by the scaling according to the powers of $\varepsilon$ of the physical parameters and especially the thickness of the elements. Therefore, the physical scale ratio $\varepsilon_p = l/L$ has to be evaluated for a real structure. $l$ is the length of the cell in the direction of periodicity and is perfectly known. The problem is the estimation of the characteristic size of the vibrations $L$. (Boutin, Auriault *1990*) shows that $L = O(U)/O(\partial_x U)$. The application of this result to a structure embedded at base and free at top (to model a building) gives $L \approx 2H/\pi$ where $H$ is the height of the structure. This evaluation is valid whatever the model. So it is possible to estimate a priori the scale ratio $\varepsilon_p$ and the scaling for a real structure and then to apply the homogenization. Smaller $\varepsilon_p$ is, better the approximation is.

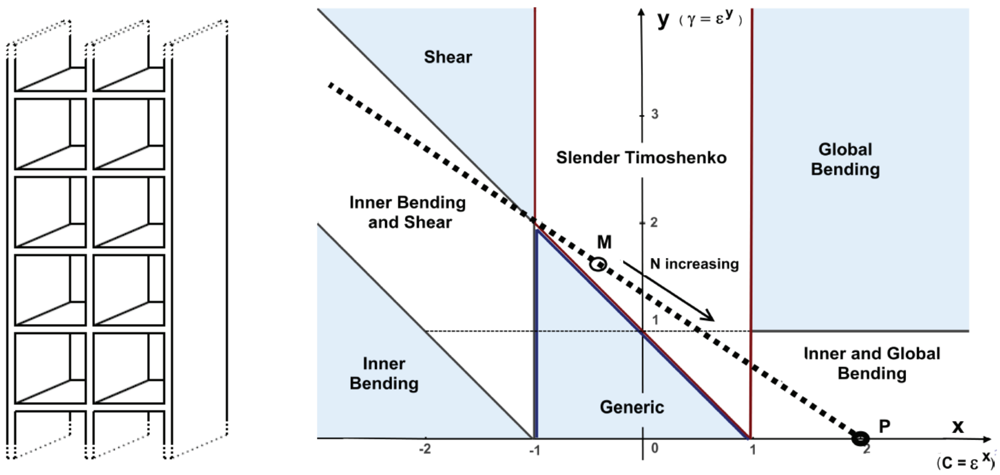


Figure 4 Left: double frame structure, right: domains of validity of the beam models

The structures studied in this work are very simple compared with buildings. To see the influence of the complexity of the cell, the same method was applied to structures made up of double frames formed by three walls bearing two floors (Figure 4). The same mechanisms and beam models are obtained. The only difference is the expression of the macroscopic stiffnesses, especially the shear stiffness. Thus, the previous results can be generalized to other structures with different cell geometry. In this aim, the domains of validity of the models are determined by working on the macroscopic elastic coefficients rather than the element thicknesses. Thanks to an adimensional analysis realized on the generic beam – which includes all the possible effects – two dimensionless parameters are defined:

$$C = \frac{E_m I_{macro}}{K L^2} \qquad\qquad \gamma = \frac{E_m(\Sigma I_m)}{E_m I_{macro}} = \frac{(\Sigma I_m)}{I_{macro}}$$

The parameter $C$ evaluates the global bending effect compared to the shear effect and $\gamma$ the inner bending effect compared to the global bending effect. $\gamma$ is an intrinsic characteristic of the cell whereas $C$ includes also the slenderness of the beam, i.e. the number of cells. The orders of magnitude with respect to $\varepsilon_p$ of these two dimensionless parameters supply efficient identification criteria of behaviour. The domains of validity of each model are mapped on the figure 4. Beams made up of a given cell are located on a straight line issued from the point P (x = 2, y = 0 with $C = \varepsilon_p{}^x$ and $\gamma = \varepsilon_p{}^y$). On this line the representative point moves closer to P when increasing the number of cells and away from P when increasing the order of the mode. For higher modes, the characteristic size of the deformation *L* is smaller, which is equivalent to reducing the slenderness of the beam. Consequently, the modes of a given structure are not necessarily described by a unique macroscopic model.

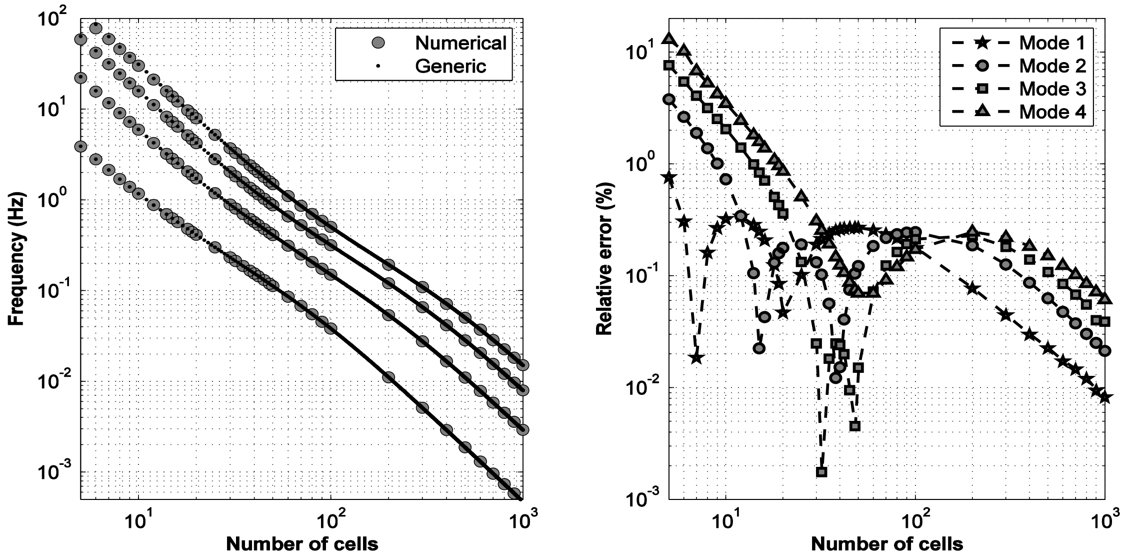


Figure 5 Comparison between numerical calculations and generic model
according to the number of cells for the four first modes
Left: eigenfrequencies values, right: relative errors

## 4. VALIDATION OF THE APPROACH

### *4.1. Numerical validation*

In order to investigate the relevancy of the homogenized descriptions for structures of finite number of cells, comparisons with numerical simulations have been performed. To illustrate the great variety of effective behaviours, a large number of fictitious structures made up of the same basic frame were modeled. The elements of the frame have the same length (3m) and the same elastic modulus (E = $2.10^{11}$ GPa) but the walls are thicker (1m) than the floors (0.15m). For each of these structures, embedded at base and free at top, the four first eigenfrequencies have been calculated using three independent methods:

- a direct numerical treatment performed with the finite element code RDM6,
- the use of the generic model, in which the dimensionless parameters determined from the basic frame are introduced. The boundary conditions are expressed with the macroscopic variables. The resolution follows the usual modal method,
- the identification of the relevant model for each structure thanks to the dimensionless parameters, which reduces significantly the calculations. The studied structures are described by an inner-bending beam for a small number of cells (less than 20), then they go gradually from an inner bending-shear beam to a slender Timoshenko beam and finally tend to a global bending beam for more than 300 cells.

The comparison of the first two methods (Figure 5) shows an excellent agreement for the wide range of tested cases. Even with a poor scale separation (5 cells), the generic beam provides good results: the error on the first mode is always less than 1% and about 20% for the fourth mode in the most unfavourable case. The results concerning the degenerated models are not presented here but they are very satisfactory in the validity range established theoretically. Thus, even if the models were built for infinite structure, they can be applied for the study of real structures.

### *4.2. Experimental validation*

This part illustrates the relevance of the beam models to describe the dynamic behaviour of a 16-storied building, dating from the seventies (Figure 1). It presents floors, longitudinal and transversal shear walls in reinforced concrete. Precast panels with very poor steel reinforcement constitute the facades. A similar building with 11 stories is located close to this building, across a 5 cm gap. The first horizontal modes were determined from accelerometric records of the response of the structure to ambient vibrations, whose contribution of the soil-structure interaction was removed (Hans 2005). The experimental frequencies for the both directions are summarized in the table 1.

As the study of the plan of a story suggests no simple mechanism of inner bending, the Timoshenko beam is enough to model the behaviour of the building. Usual values for the characteristics of the reinforced concrete are assumed, in particular for the Young modulus (E = 20 GPa). Thanks to a static calculation on one story of the building, the parameter $C$ is evaluated: $C$ = 1.79 for the longitudinal direction and $C$ = 0.08 for the transversal direction. The both values correspond to a Timoshenko beam and not to ones of the degenerated models. The eigenfrequencies obtained by this method are presented in the table 1 and are relatively close to the experimental ones.

Table 1 Frequencies (Hz) of the first horizontal modes of the studied building

| | Longitudinal direction | Transversal direction |
|---|---|---|
| Experimental data (ratio $f_i/f_1$) | 2.15 – 7.24 – 13.97 – 20.5 (1 – 3.36 – 6.49 – 9.53) | 1.56 – 6.64 – 14.0 (1 – 4.25 – 8.97) |
| Estimation from the plan (ratio $f_i/f_1$) | 2.58 – 7.91 – 14.1 (1 – 3.06 – 5.46) | 2.24 – 10.54 – 23.07 (1 – 4.7 – 10.3) |
| Fit to the experimental data | **2.15 – 7.24** – 13.97 – 20.5 | **1.56 – 6.64** – 14.0 |

Another method for the determination of the most relevant model consists in working directly on the experimental data. In fact, the distribution of the modal frequencies of a beam depends strongly on its nature and can be used as an identification criterion. For instance, in the case of a beam embedded at base and free at top, the ratios fi/f1 follow the series of the odd numbers (1, 3, 5, … , 2i-1) for a pure shear beam and the series of the squares of the odd numbers (1, $3^2/1.2^2$, $5^2/1.2^2$,…, $(2i-1)^2/1.2^2$) for of a pure bending beam. For more complex models, as a Timoshenko beam, the distribution is intermediate between these two extremes (Figure 6). Thus, it is possible to calculate $C$ from the ratio f2/f1. Then the Young modulus is estimated so as to find again the first modal frequency. The same value of 21 GPa is obtained in the both directions, which is very realistic. Furthermore, the higher frequencies (Table 1) and the modal shapes (Figure 7) show an excellent agreement with the experimental data.

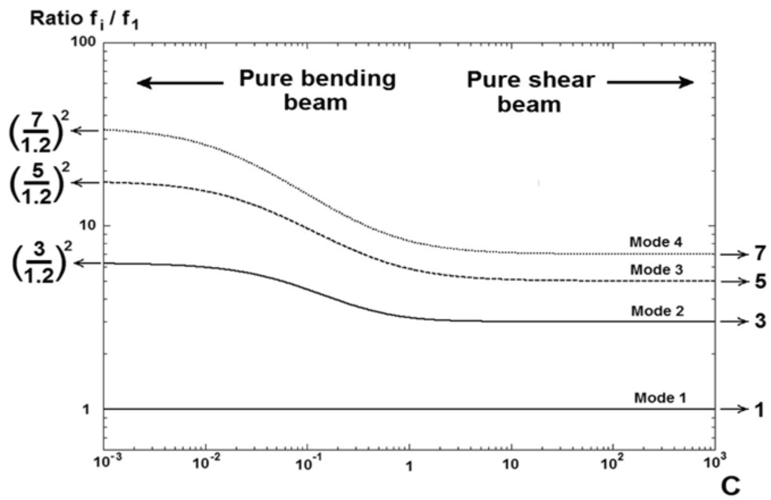


Figure 6 Distribution of the eigenfrequencies of a Timoshenko beam according to the dimensionless parameter $C$

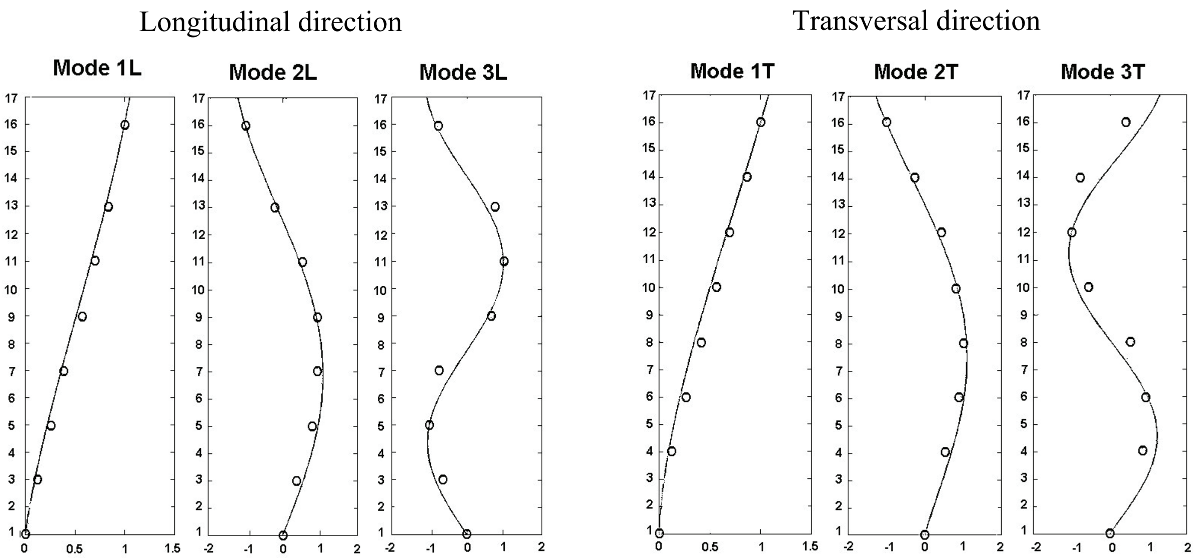


Figure 7 Comparison between the theoretical (line) and experimental (points) modal shapes

## CONCLUSION

The application of the HPDM to idealized buildings provides an exhaustive list of the possible continuous beam models for the description of their dynamic behaviour. This list includes the classical bending beam and shear beam but also unusual models with inner bending. A generic beam of the sixth degree takes all the possible mechanisms into account. More than the actual frame cell geometry; the global behaviour is governed by the three macroscopic elastic coefficients of the cell, which can be used as identification criteria. Therefore the results can be generalized to other symmetric and periodic structures. In particular the study of the modes of real buildings is considerably simplified. Instead of performing dynamic calculations on the whole structure, a static analysis of only one story enables to determine the most appropriate model and its parameters. Then it just remains to solve a 1D problem. This approach is also an efficient tool for the interpretation of in situ measurements and can be used for the assessment of the seismic vulnerability.

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