ESTIMATING PARK-ANG DAMAGE INDEX USING EQUIVALENT SYSTEMS

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ABSTRACT:

The performance-based seismic design (PBSD) philosophy aims at achieving a reliable performance from a structure for a given seismic hazard. There are two primary concerns of PBSD – a proper quantification of the uncertainties associated in the whole design process, and a better characterization of the potential structural damage. A probabilistic seismic hazard analysis (PSHA) and the use of structural parameters better correlated to seismic damage are the preferred tools for seismic demand estimation as per PBSD. For the present work, the Park-Ang damage index is selected as the seismic damage measure since it is considered to be one of the most realistic measures of structural damage. Hazard response spectra (probabilistic or deterministic) are the most common mode of characterizing the seismic hazard at a site, and these spectra represent the demand on single degree oscillators. In order to use these spectra for estimating the demand on a multi-degree of freedom system (MDOF), an equivalent single degree of freedom (ESDOF) system-based method is proposed here. The proposed method is tested on three two-dimensional steel moment resisting frames under several ground motion scenarios. For each frame, demands, in terms of Park-Ang damage index, on the corresponding ESDOF system are obtained under different ground motion scenarios and are compared with the actual demands on the MDOF structure. The accuracy of the estimation is judged through bias factor statistics for these three buildings, and the variation of effectiveness of the procedure is studied. Finally, the prospective use of these bias factors in reliability based design of structures considering Park-Ang damage index is discussed.

KEYWORDS: Park-Ang damage index, static pushover analysis, equivalent system methodology, bias factor, reliability-based design

1. INTRODUCTION

Over the last decade, performance-based seismic design (PBSD) has emerged as the accepted concept in earthquake resistant design of structures [1]. It is a general design philosophy in which the design criteria are expressed in terms of achieving probabilistically defined performance objectives when the structure is subjected to stated levels of seismic hazard. A better quantification of seismic damage in a structure is emphasized in PBSD. For better assessment of seismic damage and cost effectiveness, inelastic damage parameters are preferred to the elastic ones. For example, displacement ductility demand and hysteretic energy demand are two important parameters that characterize inelastic seismic demand better than previously used elastic parameters, such as design base shear.

For structural analysis and design, damage can also be quantified in terms of a numerical "damage index". Damage indices may be based on the results of a nonlinear dynamic analysis, on the measured response of a structure during an earthquake etc. In most of the cases damage indices are dimensionless parameters intended to range between 0 for the undamaged (elastic) state and 1 for a collapsed state of a structure, with intermediate values giving some measure of the degree of damage. The most valuable outcome would be the accurate estimation of consequences related to a numerical value of damage index predicted for a postulated earthquake [2].

2. PARK-ANG DAMAGE INDEX (D_{PA})

As a structure is weakened or damaged by a combination of stress reversals and high stress excursion, a damage criterion should include not only the maximum response but also the effect of repeated cyclic loading [3]. Consistent with the dynamic behavior, Park and Ang expressed seismic structural damage as a linear combination of the damage caused by excessive deformation and that contributed by repeated cyclic loading effect. In terms of damage index this is represented as:

$$D_{PA} = \frac{\delta_M}{\delta_u} + \frac{\beta}{Q_y \delta_u} \int dE \tag{2.1}$$

where, δ_M = maximum deformation under earthquake; δ_u = ultimate deformation capacity under monotonic loading; Q_y = calculated yield strength (if the maximum strength, Q_u, is smaller than Q_y, Q_y is replaced by Q_u); dE = incremental absorbed hysteretic energy; β = non-negative parameter representing the effect of cyclic loading on structural damage - this is determined experimentally; $\delta_u = \mu d_y$ where μ is displacement ductility and d_y yield displacement.

Under elastic response, the value of D_{PA} should theoretically be zero. $D_{PA} \geq 1.0$ signifies complete collapse or total damage. Therefore structural damage is a function of the responses δ_M and dE that are dependent on the loading history. The parameters, δ_u and Q_y are independent of the loading history. The cyclic loading effect at different deformation levels is assumed to be uniform [3].

D_{PA} can be defined for an element, for a story or for the overall building. For example the D_{PA} at a plastic hinge location can be defined as Eqn. 2.2. For real life MDOF structures, for the element and section damage, the following modifications to the original model were introduced [4]:

$$D_{PA} = \frac{\theta_m - \theta_r}{\theta_u - \theta_r} + \frac{\beta}{M_y \theta_u} E_h \tag{2.2}$$

where θ_m is the maximum rotation attained during the loading history; θ_u is the ultimate rotation capacity of the section; θ_r is the recoverable rotation when unloading; M_y is the yield moment and E_h is the dissipated energy in the section. The element damage is then selected as the biggest damage index of the end sections. D_{PA} for a story or the overall building can be obtained from the D_{PA} values at each plastic hinge locations as given in previous literature [5, 6]. However the relation between sectional D_{PA} to the story or building D_{PA} is difficult to establish [4].

For the present work, the damage index is considered for the structure as a whole.

$$D_{PA} = \frac{\delta_M - \delta_y}{\delta_u - \delta_y} + \frac{\beta}{V_y \delta_u} \sum_{i=1}^{n} \int_0^t dE_i \tag{2.3}$$

Where δ_M = peak roof displacement obtained from non-linear response history analysis for a specific record; δ_u= monotonic roof displacement capacity based on ductility capacity assumed; δ_y = yield displacement obtained from non-linear static pushover analysis; V_y = yield base shear based on pushover analysis; E_i = hysteretic energy at i-th plastic hinge; n = number of plastic hinges.

For the MDOF system, pushover analysis is performed for a pre-defined large value (large enough to exceed displacement capacity under feasible ductility capacity value) of roof displacement. The lateral load distribution {f} recommended in the International Building Code [7] is adopted for pushover analyses. The curvilinear pushover plot (base shear versus global drift) is approximated (Figure 1) by a bilinear (elastic perfectly plastic) curve by equating the area under the original and the approximating curves. From the bilinear curve, yield displacement of the MDOF system is determined and then target roof displacement capacity (δ_u) is obtained for already specified ductility values. From initial slope of bilinearised pushover plot, global stiffness of the MDOF structure is obtained and V_y is determined. Then dynamic analysis is performed for the structure and the maximum displacement (under a ground motion) in the horizontal direction is considered as δ_m.

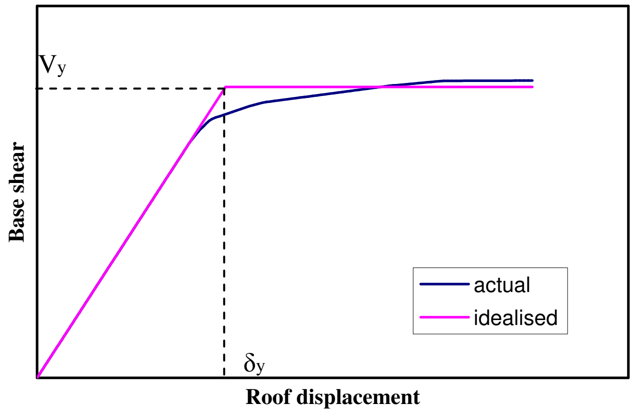

Figure 1 Idealization of pushover plot to obtain δ_y and V_y

3. PROPOSED EQUIVALENT SYSTEM METHODOLOGY

In order to use the information provided by response spectra, which are for SDOF systems, for the design of real-multistory buildings, some relationship correlating the behavior of a SDOF oscillator and that of a MDOF system is needed. Equivalent system methodology is the solution for this problem. An equivalent single degree of freedom (ESDOF) system is a trimmed down idealized representation of an actual MDOF structure. The idealization is based on properties of the real structure, such that the ESDOF system is capable of representing certain responses of the MDOF structure. For example, the maximum roof displacement of a building under a given earthquake can be obtained, in a statistical sense, from the displacement time history of its ESDOF system under same earthquake.

Several methodologies have been proposed for constructing an ESDOF, which are based on MDOF characteristics, such as the fundamental or any other mode shape of the structure or its response to a static pushover analysis. This study considers an approximate methodology, which is based on nonlinear static pushover analysis of the MDOF system, and is a variant of the method proposed by Qi and Moehle [8] and used by Ghosh and Collins [9]. Equivalent system based on nonlinear static pushover analysis is considered for obtaining the drift and strength demand on a structure.

The formulation of the equivalent system starts from the dynamic response of a two-dimensional MDOF cantilever-type structure subjected to horizontal base motion:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+\{R\}=-[M]\{1\}\ddot{u}_g \tag{3.1}$$

where [M] = mass matrix (assumed to be diagonal); {u} = {u(t)} = lateral displacement vector (one displacement for each floor); [C] = damping matrix; {R} = {R(t)} = restoring force vector; {1} = unit vector and $u_g = u_g(t)$ = ground displacement.

In the generalized system the displacement vector, {u(t)}, is replaced with a single displacement, for example, the roof displacement D(t), and a time-invariant displacement profile or shape vector, $\{\Phi_1\}$. Thus, the lateral displacement vector {u(t)} is assumed to be replaced by $\{\Phi_1\}D(t)$. The shape vector $\{\Phi_1\}$ is chosen on the basis of nonlinear static pushover analysis of the MDOF structure. A pushover analysis is carried out by incrementally scaling a prescribed lateral force distribution, {f}, which has been normalized in a way such that it corresponds to a base shear of unity (i.e., base shear = $\{1\}^T\{f\} = 1$). At any stage of the pushover analysis this unit base shear is scaled by a factor V to obtain the actual base shear (V), and the actual lateral force vector applied to the structure becomes V{f} or {Vf}. It is assumed that this same set of forces can be used to represent the restoring force vector, {R}, from Eqn. 3.1, since{R} can be interpreted as the static nodal forces associated with the nodal displacements {u}. Substituting {R} and {u} by V{f} and $\{\Phi_1\}$D, respectively, Eqn. 3.1 becomes:

$$[M]\{\Phi_1\}\ddot{D}+[C]\{\Phi_1\}\dot{D}+V\{f\}=-[M]\{1\}\ddot{u}_g \quad (3.2)$$

The relationship between V and D can be represented mathematically as, V = KG(D), where K is the initial slope of the pushover curve and G is the scalar mathematical function of D describing the shape of the curve. So Eqn. 3.2 can be written as

$$[M]\{\Phi_1\}\ddot{D}+[C]\{\Phi_1\}\dot{D}+KG(D)\{f\}=-[M]\{1\}\ddot{u}_g \quad (3.3)$$

Pre-multiplying both sides by a second vector $\{\Phi_2\}^T$, this vector equation is reduced to a single equation:

$$\{\Phi_2\}^T[M]\{\Phi_1\}\ddot{D}+\{\Phi_2\}^T[C]\{\Phi_1\}\dot{D}+KG(D)\{\Phi_2\}^T\{f\}=-\{\Phi_2\}^T[M]\{1\}\ddot{u}_g \quad (3.4)$$

Eqn 3.4 is simplified by defining the following terms: $M^* =\{\Phi_2\}^T[M]\{\Phi_1\}$; $C^* =\{\Phi_2\}^T[C]\{\Phi_1\}$; $K^* =K\{\Phi_2\}^T\{f\}$; $L^* =\{\Phi_2\}^T[M]\{1\}$; $P^* = L^*/M^*$; $(\omega^*)^2 = K^*/M^*$ and $C^*/M^* = 2\xi\omega^*$; and the equation becomes:

$$M^*\ddot{D}+C^*\dot{D}+K^*G(D)=-L^*\ddot{u}_g \quad (3.5)$$

After dividing by M*, Eqn. 3.5 becomes,

$$\ddot{D}+2\xi(\omega^*)\dot{D}+(\omega^*)^2G(D)=-P^*\ddot{u}_g \quad (3.6)$$

Eqn. 3.6 can be interpreted as the equation of motion of a SDOF oscillator with linear elastic frequency ω* and damping ratio ξ.

The vector $\{\Phi_2\}$ is chosen as $\{\Phi_2\} = \{\Phi_1\}$. So Eqn. 3.5 is consistent with the equivalent SDOF equation derived using the principle of virtual work. This formulation of ESDOF system is referred as the "virtual work" formulation in this study. However, the term K*G(D) no longer represents the base shear V, since K* is not equal to K.

In the present work equivalent system parameters are calibrated from the results of nonlinear pushover analysis. The initial or the elastic stiffness for the approximate curve is adopted to be the same as in the original plot. Yield drift and yield strength for the equivalent system is obtained from this approximating curve. The equivalent system scheme (ESS) is adopted for the virtual work formulation based on the bilinear approximation procedure and the choice of shape vector $\{\Phi_1\}$. For the ESS, pushover analysis is carried upto a global drift of 2.5% and it is approximated by an elastic-perfectly plastic bilinear curve. $\{\Phi_1\}$ is the displacement profile at 1% global drift.

4. VALIDATION OF EQUIVALENT SYSTEM METHODOLOGY

The proposed ESDOF methodology is validated for three steel moment resisting frame buildings under several strong motion earthquake scenarios. The buildings considered for this study are the 3-, 9- and 20-story "Pre-Northridge" design steel moment frames. These frames were considered for various recent research works and details of these building frames are available in published literature [10]. The North-South frames of each building are considered for this study.

Park-Ang damage indices under different ground motion scenarios are considered for both MDOF models of the SAC building frames and their ESDOF counterparts. The validity of proposed ESDOF models are tested by comparing these two sets of damage index outputs. DRAIN-2DX is used to perform nonlinear static pushover analyses and also for the nonlinear dynamic analyses for several selected ground motions. Gravity load effects, P-delta effects, flexibility of joint panel zones of these buildings and stiffness contribution from gravity-frame members are not considered for analyses. Members of the moment frame are considered to be elastic-perfectly plastic (that is, no strain hardening). Rayleigh damping is assumed for dynamic analysis. The mass proportional and the stiffness proportional damping coefficients (α_m and β_k, respectively) are obtained by adopting a modal damping factor (ξ) of 0.05 in first two modes.

Three different ductility capacity (μ) values (4, 6 and 7.5) are considered for defining δ_u of D_{PA} (Eqn. 2.3). A total of 28 real ground motions are considered for the comparison of D_{PA} values of MDOF and ESDOF systems. The D_{PA} values are obtained for all the MDOF systems for each of the 28 ground motion scenarios. However, only D_{PA} values greater than 0.05 are used for obtaining bias statistics, since D_{PA} less than that indicates very little or almost no inelasticity. The D_{PA} values are obtained for the ESDOF scheme for the ground motion scenarios and are compared with damage indices of the actual MDOF model. Information on the comparison between the MDOF response and the response of ESDOF system are illustrated using scatterplots (Figures 2, 3 and 4). In each plot, a data point represents a comparison of the MDOF and the ESDOF responses for a single earthquake and a specific ductility capacity. The diagonal line across scatterplots represents the ideal ESDOF response, which is exactly the same as the MDOF response in terms of Park-Ang damage index. A data point above that line signifies that the ESDOF model overpredicts the MDOF response, and vice versa. The trend of overestimation or underestimation by the ESDOF model is analyzed through bias factor (N) statistics, where the bias factor is defined as the ratio of the MDOF system response to the ESDOF system response for a particular earthquake. The bias statistics for all the frames are discussed below.

Table 4.1 presents the bias statistics for the three frames. As stated earlier, three different ductility capacity values are considered for each frame and the statistics for each frame include the results for different ductility capacities. Only $D_{PA} > 0.05$ are considered for this statistics, since lower values will amount to negligible inelasticity. This reduces the number of records for the 9- and 20-story frames to 19 and 8, respectively.

Table 4.1: Statistics for the bias factor (N), for D_{PA}, for nonlinear inelastic response

Frames	Mean	Standard Deviation	Coefficient of Variation	Maximum	Minimum
3-story	0.898	0.258	0.287	1.69	0.499
9-story	0.801	0.552	0.690	2.80	0.228
20-story	0.727	0.423	0.583	1.16	0.472

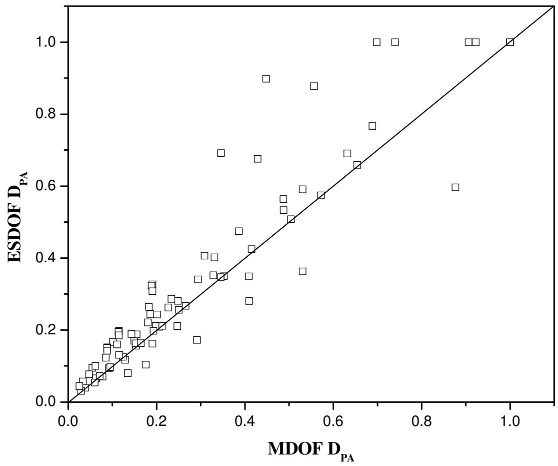

Figure 2 Scatterplot comparing D_{PA} values of MDOF and ESDOF system for the 3-story frame

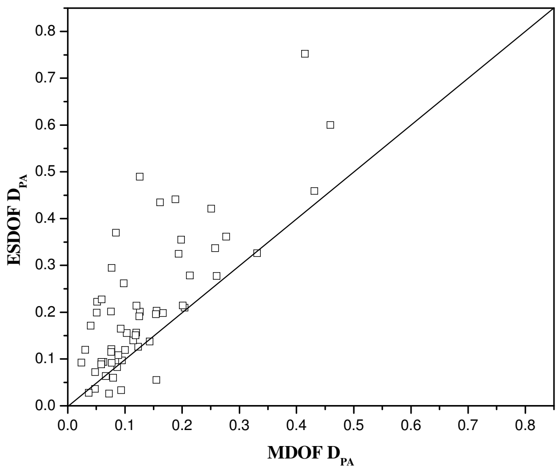

Figure 3 Scatterplot comparing D_{PA} values of MDOF and ESDOF system for the 9-story frame

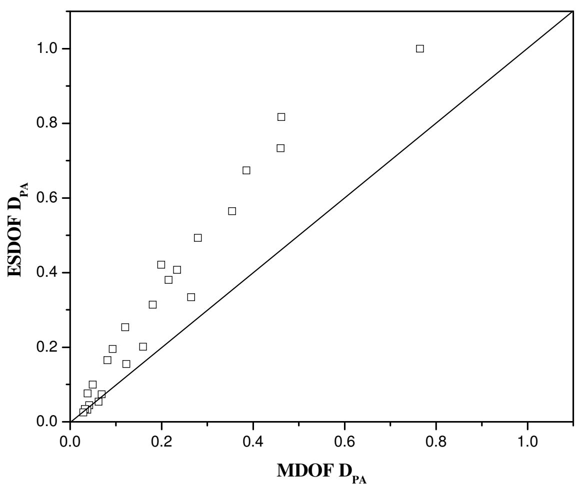

Figure 4 Scatterplot comparing D_{PA} values of MDOF and ESDOF system for the 20-story frame

5. OBSERVATION AND DISCUSSION

The Concept of equivalent system methodology for design of real MDOF systems is presented here. In the present study, only one equivalent system scheme is described. The mean bias is not far from the ideal value 1. On average, the ESDOF system overestimates the D_{PA} of the MDOF system. The overestimation is more as we go from lower to higher story. The coefficient of variation (COV) presents the spread in error. The COVs show that for the 3-story building the estimates are more reliable compared to the 9- and 20-story estimates.

The mean bias factor can be used to obtain MDOF D_{PA} demand from D_{PA} response spectrum. The SDOF D_{PA} based on a response spectrum can be multiplied with the mean bias in order to obtain the D_{PA} demand for the MDOF structure with certain level of confidence. The confidence level in this estimation can be obtained from the corresponding standard deviation.

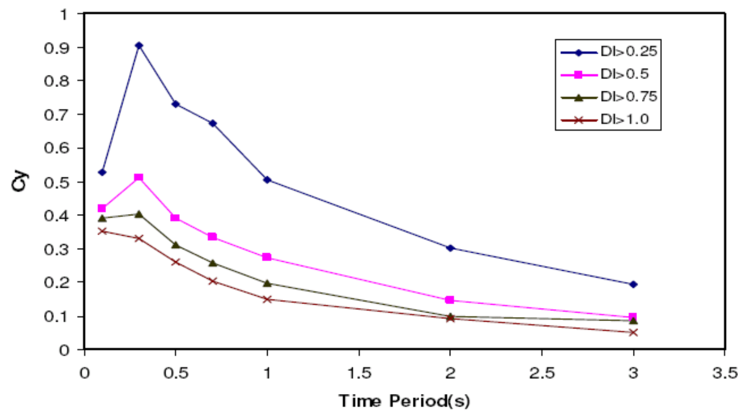

Figure 5 UHS for 10% exceedance probability in 50 years for μ = 4 [11]

This ESDOF scheme can be utilized in developing reliability-based methodology considering D_{PA} demand. For this a probabilistic response spectrum such as uniform hazard spectrum for D_{PA} can be used [11] (Figure 5). Several alternative equivalent system schemes can also be developed for this estimation similar to those in literature [12]. These schemes can be similarly tested for their effectiveness in estimating D_{PA} demand in MDOF systems.

REFERENCES

[1] Structural Engineers Association of California (SEAOC). VISION 2000 Committee (1995), Performance Based Seismic Engineering of Buildings, Volume. I, Sacramento, CA, USA.
[2] Williams, M. and Sexsmith, R. (1995). Seismic damage indices for concrete structures: a state of the art review. *Earthquake Spectra*, **11:2**, 319-349.
[3] Park, Y.J. and Ang, A.H.-S. (1985). Mechanistic seismic damage model for reinforced concrete, *Journal of Structural Engineering*, ASCE, **111:4**, 722-739.
[4] Kunnath, S.K., Reinhorn, A.M., and Lobo, R.F. (1992). IDARC Version 3.0: A Program for the Inelastic Damage Analysis of Reinforced Concrete Structures, Report No. NCEER-92-0022, National Center for Earthquake Engineering and Research, State University of New York at Buffalo, NY, USA.
[5] Park, Y.J., Ang, A. H-S. and Wen, Y.K.(1987), Damage-limiting aseismic design of buildings, *Earthquake Spectra*, **3:1**,1-26.
[6] Fajfar, P. and Gašperšič, P., (1996), The N2 method for the seismic damage analysis of RC buildings, *Earthquake Engineering and Structural Dynamics*, **25:1**, 31-46.
[7] International Code Council (ICC) (2006), *International Building Code*.
[8] Qi, X. and Moehle, J.P. (1991), Displacement Design Approach for Reinforced Concrete Structures Subjected to Earthquakes, Report No. UCB/EERC-91/02, University of California at Barkeley, CA, USA.

[9] Ghosh, S. and Collins, K.R. (2006), Merging Energy-Based Design Criteria and Reliability-Based Methods: Exploring a New Concept, *Earthquake Engineering and Structural Dynamics*, **35:13**, 1677-1698.
[10] Gupta, A and Krawinkler, H. (1999), Seismic demands for Performance Evaluation of Steel Moment Resisting Frame Structures (SAC Task 5.4.3), Report No. 132, John A Blume Earthquake Engineering Center, Department of Civil and Environmental Engineering, Stanford University, Stanford, CA, USA.
[11] Datta, D. and Ghosh, S. (2006), Inelastic Uniform Hazard Spectra based on Park-Ang damage index, 13th Symposium on Earthquake Engineering (13SEE), Roorkee, India, 1211-1219.
[12] Ghosh, S. (2003), Two alternatives for implementing Performance Based Seismic Design of buildings- life cycle cost and seismic energy demand, Ph.D. Thesis, Department of Civil Engineering, University of Michigan, Ann Arbor.