A fiber beam element with axial, bending and shear interaction for seismic analysis of RC structures

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ABSTRACT :

Inelastic failure of reinforced concrete (RC) structures under seismic loadings can be due either to loss of flexural, shear or bond capacity. Specifically, the effect of combined loadings can lead to a complex failure mechanism that plays a vital role in concrete mechanics. This paper describes the formulation of an inelastic nonlinear beam element with axial, bending, and shear force interaction. The element considers shear deformation and is based on the section discretization into fibers with hysteretic material models for the constituent materials. The steel material constitutive law follows the Menegotto-Pinto model. The concrete material model uses an orthotropic constitutive relation in which the directions of orthotropy are the principal directions of total strain. These directions will change during the loading history, in accordance with the well-known rotating crack model. The concrete model accounts for the biaxial state of stress in the directions of orthotropy, in addition to degradation under reversed cyclic loading. Shear deformations are coupled with bending effects. Transverse strains are internal variables determined by imposing equilibrium at each fiber between the concrete and the vertical steel stirrups. Element forces are obtained by performing equilibrium based numerical integration on section axial, flexural, and shear behavior along the length of the element. In order to establish the validity of the proposed model correlation studies were conducted between analytical results and experimental tests of columns tested under cyclic loading. A structural analysis of a shear sensitive bridge pier subjected to ground input motion is also presented.

KEYWORDS: Fiber element, combined loadings, shear, nonlinear beams, stirrups

1. INTRODUCTION

The behavior of RC structures is very complicated due to the nonlinear material nature and the mutual interaction of axial, bending and shear loadings. The establishment of nonlinear constitutive models for RC elements and the development of corresponding nonlinear finite element models are essential to predicting the correct behavior of RC structures. Many researchers have developed different types of analytical models for RC structures such as truss models, orthotropic models, nonlinear elastic models etc. The orthotropic models stand out both in accuracy and in efficiency as compared to other models. Vecchio and Collins (1982), and Vecchio(1990,1992) have developed a concrete model which addresses compressive strength increase due to biaxial compression and compressive strength degradation as a function of tensile strain after cracking.

The overall objective of this present paper is to develop a nonlinear finite element program which is able to accurately predict the behavior of RC structures subjected to static, reversed cyclic and dynamic loadings. During an earthquake, RC structures undergo large cyclic deformations with the concrete undergoing crack opening and closing. The problem is further complicated by considering additional effects such as: softening of concrete in compression, confinement, tension stiffening, and dilatancy, which accompanies large shear strains induced in the stirrups. This study tackles the extension of the fiber method formulation to account for axial, bending, and shear interaction. Timoshenko beam theory with quadratic shape functions for deformation and linear shape functions for rotation has been used in the model. After a brief description of the model, numerical analysis was performed using the developed model to investigate the effect of the structural variables. This model was developed by modifying the general purpose finite element program FEAPpv (Taylor 2005).

2. BASIC ASSUMPTIONS

At first glance, Concrete is considered as an orthotropic material. The 3D problem is converted to a 2D problem with plane stress orthotropic approximation. All the stresses and strains are assumed to be smeared across the thickness.

3. GENERAL PROCEDURE

3.1 Smeared Crack Approach

Tensile cracking of the concrete is one of the main sources of nonlinearity in RC, and it can be modeled with a discrete cracking approach or else using a smeared cracking approach. In a discrete crack model, a crack is modeled discretely as a geometric discontinuity which leads the finite element topology as the crack propagates during the loading. The discrete model is based on the stress-strain relationship of plain concrete and steel, and their interaction through bond slip along with shear sliding along the cracked surface. This discrete crack model is very expensive and is best suited to model the materials in which fracture is dominated by very few cracks. This approach is not suitable to model the primary portions of the members such as shear wall since it is very difficult to model bond slip and shear sliding. From a scientific point of view, the primary region is one where stress and strains are distributed regularly so that they can be expressed with equilibrium and compatibility conditions. Discrete models can be more useful to model local regions in which stress and strains are so disturbed and irregular that they are not amenable to use compatibility conditions.

The modified compression field theory (MCFT), and softened truss model (STM) are fully smeared crack models that treat the cracked reinforce concrete as a continuum material. In this research a fiber element was developed based on these smeared crack models since they determine the overall stiffness and strength properties of cracked RC elements without dealing with tedious crack width simulation, crack spacing and bond slip.

3.2 Rotating Crack Approach

Rotating crack models, such as MCFT and STM, were developed based on the assumption that the direction of the cracks is inclined at an angle that follows the directions of principal compressive strains in concrete. This angle is a rotating angle because the coordinates of principal strain in concrete will rotate with increasing proportional loading and the crack direction is assumed to follow these principal strain directions of concrete.

3.3 Non-linear Finite Element Analysis

The present study uses the fiber models or semi-local models in which the cross section is subdivided into layers or fibers. The finite element analysis of RC structures is a highly nonlinear problem because of the nonlinear behavior of concrete and steel. For a nonlinear finite element problem, an incremental procedure is used. The structural tangent stiffness matrix relates the increments of load to corresponding increments of displacement. For solving the nonlinear equations, the well-known Newton-Raphson solution algorithm is adopted.

4. CONCRETE STRAIN STATE

The values of the concrete uniaxial strains in principal directions 1-2 have three conditions, and the strength in one direction is affected by the strain state in the other direction. Three concrete strain conditions exist as follow:

4.1 1-TENSION, 2-COMPRESSION

In this case, the uniaxial strain of concrete $\in_1$ in principal direction 1 is in tension, and the uniaxial strain $\in_2$ of concrete in principal direction 2 is in compression. Due to this condition, the uniaxial concrete stress σ_1 in direction 1 is calculated from $\in_1$, and is not a function of the perpendicular concrete strain $\in_2$. The compressive strength in direction 2, σ_2 will soften due to tension in the orthogonal direction. Vecchio (1990) derived a softening equation in the tension-compression region, which is implemented in the current model (Figure. 1). Based on panel testing as proposed by Hsu et al. (1995), Belarbi and Hsu (1995) also developed a similar expression based on the softened concrete constitutive laws. The lateral tensile stress reduces the compressive strength in the orthogonal direction. The equation for compressive strength reduction factor proposed by Vecchio (1990) is:

$$\beta = \frac{1}{0.8 + \dfrac{0.34|\in_1|}{|\in_0|}} \leq 1 \tag{4.1}$$

The ultimate stresses in the orthogonal directions is

$$\sigma_{2p} = \beta f_c^{'} \tag{4.2}$$

Where β is softening coefficient, $\in_1$ is the lateral tensile strain, $\in_0$ is the concrete strain at peak compressive strength and σ_{2p} is the softened concrete compressive strength.

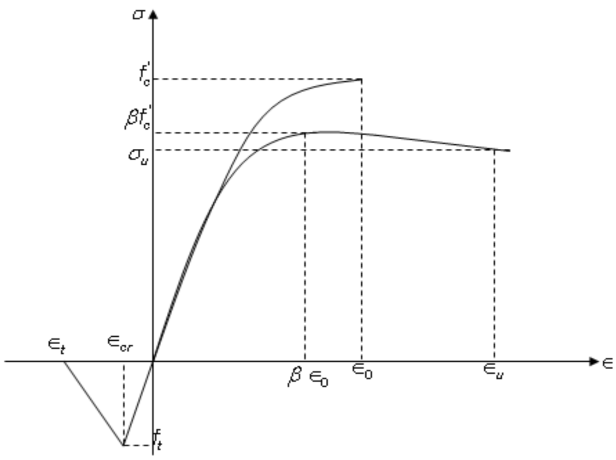


Figure 1 Stress-Strain curve with softening (1990)

4.2 1-TENSION, 2-TENSION

The uniaxial strain of concrete $\in_1$ in direction 1 is in tension, and the uniaxial strain $\in_2$ of concrete in direction 2 is also in tension. Due to this condition, the uniaxial concrete stress σ_1 in direction 1 is calculated from $\in_1$, and σ_2 in direction 2 is calculated from $\in_2$. Both σ_1 and σ_2 are functions of the orthogonal concrete strains $\in_2$ and

$\in_1$ respectively.

4.3 1- COMPRESSION, 2- COMPRESSION

The uniaxial strains of concrete in principal directions 1 and 2 are both in compression. The current research uses the Vecchio's (1992) simplified version of Kupfer et al. (1969) biaxial compression strength equation. The concrete compressive strength increase in one direction depends on the confining stress in the orthogonal direction. The strength enhancement and increase in ductility are dependent on the biaxial compressive stresses. Concrete in compression exhibits lateral expansion and increase in the value of Poison ratio. An upper limit of 0.5 has been considered for Poison ratio.

5. VERTICAL STIRRUP CONTRIBUTION

A beam can be divided into a web, top stringer and bottom stringer, in which the web can be treated as a shear element since the shear force V is typically resisted by the web of the beam, while the moment is resisted by the top and bottom stringers. The shear flow in the web is distributed uniformly over the depth of the web in the transverse direction. Since the shear flow is constant over the depth, the shear force V can be calculated by integrating the shear flow along the depth of the web. The shear flow in the main body is distributed uniformly along the length of the web in the longitudinal direction. Hence, the transverse steel stress $\sigma^i_{y,s}$ and the stresses in the diagonal concrete struts $f_c^{'}$ vary uniformly along their lengths. Transverse strains are internal variables determined by imposing the equilibrium at each fiber between concrete and vertical steel stirrups. Transverse strains are not known in advance and because of the non-linear behavior of the concrete and steel, an iterative procedure is needed to satisfy the equilibrium in the transverse direction expressed as follow:

$$\sigma^i_{y,c} A^i_{y,c} + \sigma^i_{y,s} A^i_{y,s} = 0 \tag{5.1}$$

Where as,

$\sigma^i_{y,c}$ = Concrete stress in transverse direction at fiber i

$\sigma^i_{y,s}$ = Stress in stirrup at fiber i

$A^i_{y,c}, A^i_{y,s}$ = Area of concrete and steel in transverse direction at fiber i

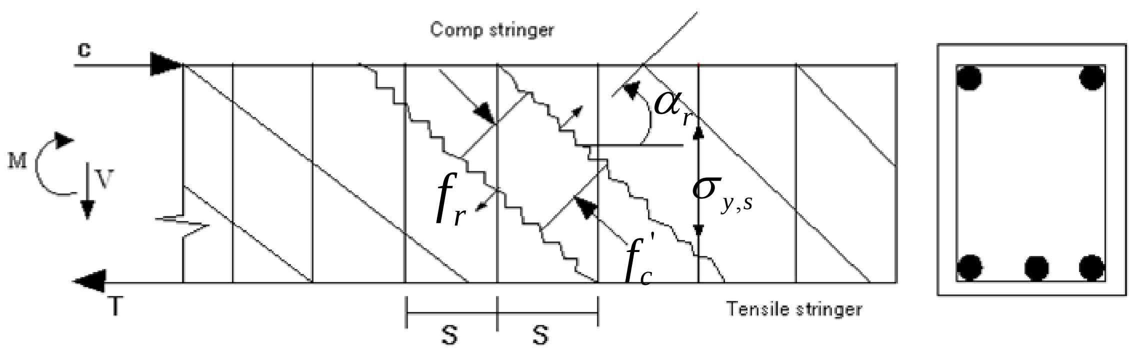


Figure 2 Beam with vertical stirrup and concrete stress.

6. MATERIAL CONSTITUTIVE LAW

The relationship between stress and strain of concrete can be related through a plane stress assumption. The stresses in the orthogonal directions are:

$$\sigma = D_{LO} \in \tag{6.1}$$

Where σ is the local stress vector, ∈ is the local strain vector and D_{lo} is a uniaxial constitutive matrix of concrete (Chen 1976).

By using the transformation matrix R, the stresses and strains can be transformed between different coordinates. RC structures are considered orthotropic materials with axes oriented at an angel θ between the principal direction and the x-axes. The transformation matrix R is introduced as:

$$R = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -2\cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{6.2}$$

The orthotropic stiffness matrix in global direction D_{gl} is:

$$D_{gl} = R^T D_{lo} R \tag{6.3}$$

The sectional stiffness $K_{Section}$ can be formulated by coupling the axial, shear and bending terms in D_{gl}.

7. STIFFNESS AND FORCE RESULTANTS

The total stiffness of the section is the sum of concrete and steel stiffness, and is evaluated as follows.

$$K_{Section} = \sum_{1}^{nC} (K_{Section})_C + \sum_{1}^{nS} (K_{Section})_S \tag{7.1}$$

Where the subscripts C, S denotes the parameters related to concrete and steel respectively, nC and nS are the number of concrete and steel fibers in the section and $K_{Section}$ is the section stiffness.

The total force of the section is the sum of concrete and steel forces in the respective direction, and is evaluated as follows.

$$F_{Section} = \sum_{1}^{nC} (F_{Section})_C + \sum_{1}^{nS} (F_{Section})_S \tag{7.2}$$

Where $F_{Section}$ is the section force

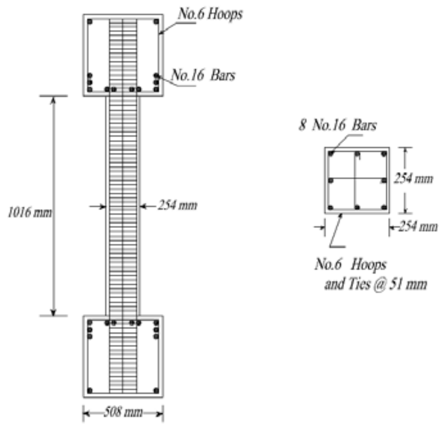
8. NUMERICAL VERIFICATIONS

8.1 Xia and Martirossyan Specimen

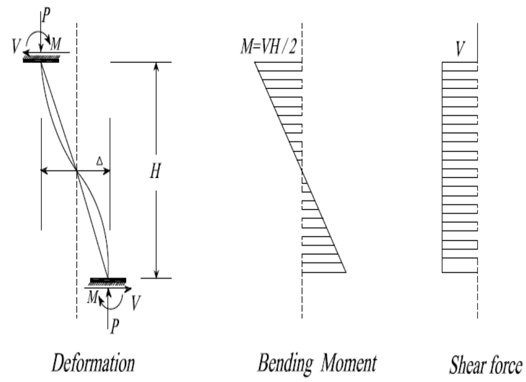
The proposed element was validated by modeling a high strength reinforced concrete squat column (Column HC4-8L 16-T6-0.1P) tested by Xiao and Martirossyan (1998) at the University of Southern California, Los Angels (Figure. 3a). An axial load of 534 kN was applied constantly to the column. Rotations were fixed at the column bottom and top so that the column deforms anti-symmetrically with respect to the mid height under combined axial and lateral loading (Figure. 3b). This column failed with a shear cracks following the degradation of the transverse shear reinforcement. In the numerical analysis, the column was divided into several finite elements until converged was achieved.

The width of the column section is 254 mm and its depth is also 254 mm. The longitudinal reinforcement consists of 8 No.16 (15.9 mm diameter) bars, uniformly spaced along the perimeter with a clear cover of 13mm. No. 6 (6.4 mm diameter) steel stirrups are provided with a spacing of 51mm.

An average concrete compressive strength $f_c^{'}$=86 MPa is used to analyze the reinforced concrete column. A yield stress f_y=510 MPa and 449 MPa are being used for the longitudinal and transverse reinforcements respectively. Young's modulus of concrete and steel are E_c = 45500 MPa and E_s = 200,000 MPa respectively.

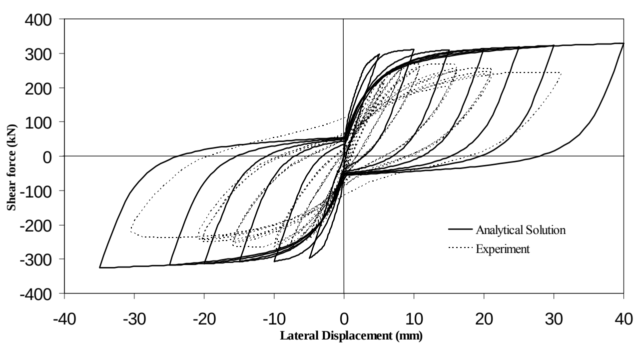


(a)

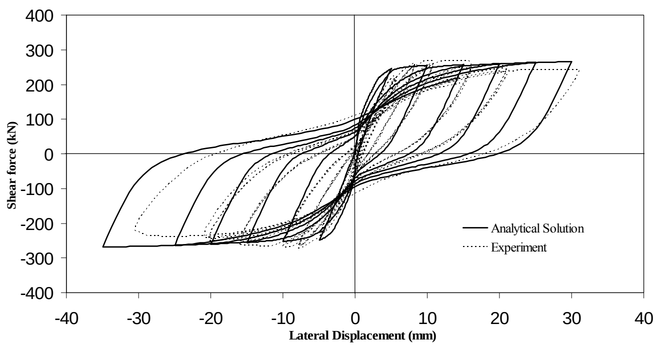


(b)

Figure 3 (a) Column details and (b) Loading pattern.



(a)



(b)

Figure 4 Cyclic shear force displacement hysteresis comparison with experiment (a) flexural element (b) shear element

Column HC4-8L 16-T6-0.1P has low shear reinforcement ($\rho_s = 1.63\%$) and is tested under cyclic uni-axial bending. The experimental and analytical cyclic force-displacement results are shown in Figure 4. Figure 4(a) refers to the Bernoulli type flexural element with infinite shear strength which tends to overestimate the strength and ductility, while

Figure 4(b) refers to the shear element which shows a very good correlation with the experimental results. The shear model showed a flexural shear failure mode, where shear failure was observed at the column ends with the increase of transverse strains, followed by core degradation and stirrup yielding. The new element predicted accurately the amount of energy dissipation and displaced shape. Because of the formation of a plastic hinge after yielding of the reinforcement, the resisting load is stable with a capacity of 260 kN until a ductility level of 6. Figure 5 shows the analytical stress-strain behavior of the transverse reinforcement using the shear element. Both, the analytical and experimental behavior predict yielding of the transverse bars, with the analytical model estimating well the maximum transverse strain.

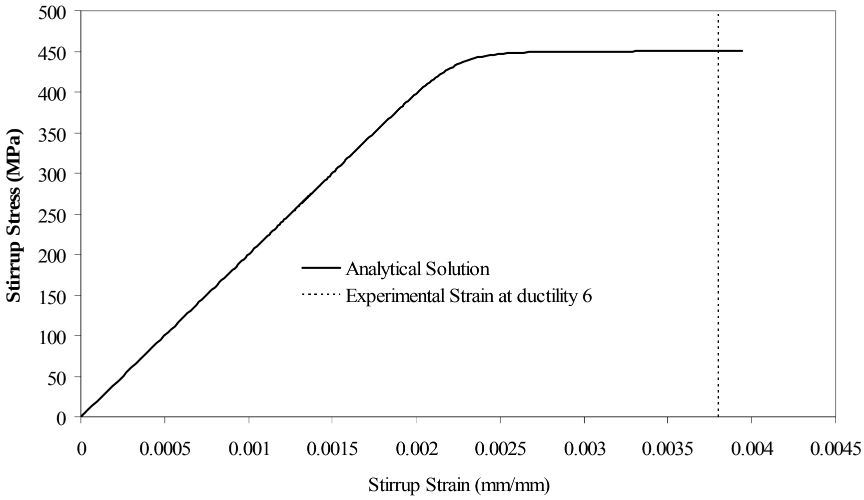
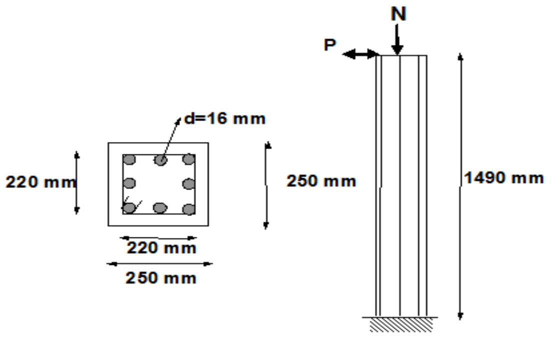


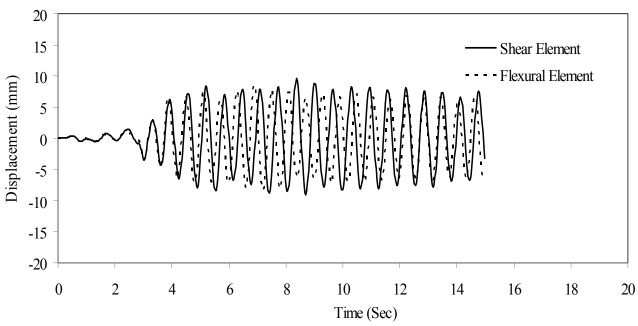
Figure 5 Transverse reinforcement stress-strain behavior

8.2 Bousias Specimen

An earthquake analysis was conducted with the RC column tested by Bousias et al. (1995) using the Erzincan (1992) record recorded in Turkey. The column is 1490 mm long with a rectangular cross section of 250x250 mm. A cover of 15 mm in all directions is provided. The longitudinal reinforcement consists of 3 bars 16mm in diameter placed at the top and bottom, and 2 bars 16 mm in diameter placed at the center. The concrete compressive strength f_c' is equal 30.75 MPa, the yield strength of the steel rebars f_y is equal 460 MPa, and the steel elastic modulus E_s is equal 210,000 MPa. The column is subject to a constant axial force N= 300 kN



(a)



(b)

Figure 6 (a) Column details (b) Earthquake response of Bousias column with Erzincan earthquake record

Figure 6(b) shows the column displacement time history. The shear effect is small in the elastic region, but becomes predominant after yielding of the reinforcement.

SUMMARY

This work represents a new approach for the numerical static and dynamic analysis of structures made up of non-linear materials. A Fiber beam element was developed to analyze reinforced concrete structures with the incorporation of mechanisms of shear deformation and strength. The emphasis of the paper is on the evaluation of the effect of orthotropic model parameters to account for shear sensitive structures and the action of transverse reinforcement. The section deformations are determined from the strain state using the classical plane section hypothesis. The proposed model is able to capture the interaction between axial, flexural, and shear responses. The results of a numerical correlation with the experimental data of a bridge pier section show that the model is able to capture the shear failure and provides accurate global and local results.

Future studies will concentrate on the effect of combined loadings on shear sensitive sections and that of bond slip at the interface between the reinforcing bars and concrete following the model proposed by Ayoub (2006) and Ayoub and Filippou (2003).

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