RESPONSE OF NONSTRUCTURAL COMPONENTS IN DUCTILE LOAD-BEARING STRUCTURES SUBJECTED TO ORDINARY GROUND MOTIONS

C. Adam[1] and T. Furtmüller[2]

[1] *Professor, University of Innsbruck, Faculty of Civil Engineering, Innsbruck, Austria*
[2] *PhD student, University of Innsbruck, Faculty of Civil Engineering, Innsbruck, Austria*
Email: christoph.adam@uibk.ac.at, thomas.furtmueller@uibk.ac.at

ABSTRACT:

The dynamic seismic behavior of nonstructural components attached to elastic-plastic load-bearing structures is assessed. The assessment is based on floor spectra of two-degree-of-freedom (2DOF) systems with assigned ductility of the supporting structure. Frame structures are transformed into equivalent single-degree-of-freedom (ESDOF) systems, and thus floor spectra of 2DOF systems become applicable for the prediction of the seismic peak response of vibratory nonstructural components. The utilized floor spectra are the outcome of a parametric study involving a set of 40 ordinary ground motions with strong motion characteristics. Example problems show the capability of this method to predict the seismic response of nonstructural elements with sufficient accuracy.

KEYWORDS: Coupled seismic response, Floor spectra, Inelastic deformations, Secondary structures

1. INTRODUCTION

Strong motion earthquakes of the last decades have left the serviceability of many buildings substantially impaired, because nonstructural components were damaged. These components such as large antennas, supply lines, electrical equipment, etc., are exposed to the amplified seismic response of the load-bearing structure, and thus they are vulnerable to seismic failure, Villaverde (2004). The development of methods for the rational quantification of seismic effects on vibration-prone nonstructural elements is a scope of ongoing research, see e.g. Medina et al. (2006). Most of the publications deal with the prediction of the seismic behavior of nonstructural components on unlimited elastic load-bearing structures. However, modern seismic design standards allow targeted inelastic deformations in load-bearing structures when subjected to severe earthquake excitations. The aim of this study is to develop a simple methodology for the prediction of the seismic peak response of nonstructural components taking into account elastic-plastic deformations of the load-bearing structure. Floor spectra for simple oscillators attached to single-degree-of-freedom (SDOF) primary structures with assigned ductilities are derived. These spectra are utilized to estimate the peak response of nonstructural components mounted on multi-degree-of freedom (MDOF) frame structures. Subsequently, nonstructural components are denoted alternatively as secondary structures, and primary structure is a synonym for the load-bearing structure.

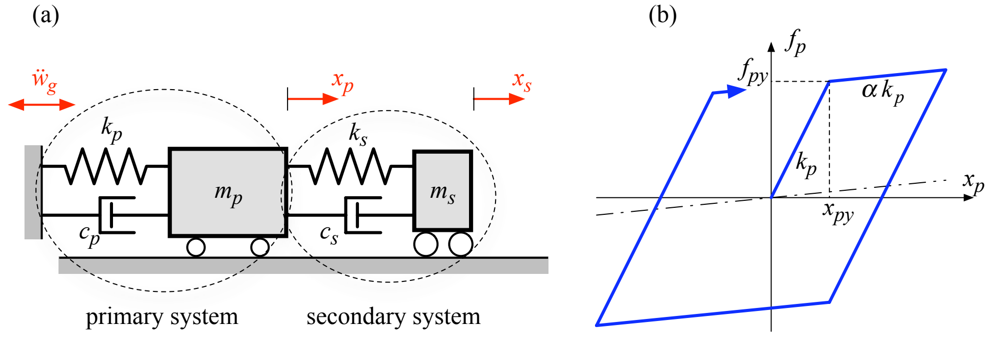

Figure 1 (a) SDOF inelastic primary structure equipped with elastic SDOF secondary structure; (b) Bilinear cyclic behavior of the spring of the primary structure

2. INELASTIC SINGLE-DEGREE-OF-FREEDOM PRIMARY STRUCTURES

2.1. Structural model and equations of motion

In the first part of this study the primary structure is modeled as an elastic-plastic SDOF oscillator with mass m_p, spring with initial stiffness k_p, and viscous damper with parameter c_p, see Figure 1(a). The spring exhibits bilinear cyclic behavior as shown in Figure 1(b), and its post-yielding stiffness is characterized by the hardening ratio α. An additional elastic SDOF oscillator with mass m_s, stiffness k_s, and viscous damping parameter c_s serves as vibration prone secondary structure. Both oscillators are connected in series, and they represent a single dynamic unit with two-degrees-of-freedom (2DOF), see Figure 1(a). The base of the primary structure is subjected to the ground acceleration $\ddot{w}_g$, which induces time varying displacements (relative to the base) x_p and x_s of the primary and secondary mass, respectively. The equations of motions of the non-classically damped inelastic 2DOF system are derived as

$$\begin{bmatrix} 1 & 0 \\ 0 & \bar{m} \end{bmatrix} \begin{Bmatrix} \ddot{x}_p \\ \ddot{x}_s \end{Bmatrix} + \begin{bmatrix} 2\zeta_p\omega_p + 2\zeta_s\omega_s\bar{m} & -2\zeta_s\omega_s\bar{m} \\ -2\zeta_s\omega_s\bar{m} & 2\zeta_s\omega_s\bar{m} \end{bmatrix} \begin{Bmatrix} \dot{x}_p \\ \dot{x}_s \end{Bmatrix} + \begin{bmatrix} \omega_p^2 + \omega_s^2\bar{m} & -\omega_s^2\bar{m} \\ -\omega_s^2\bar{m} & \omega_s^2\bar{m} \end{bmatrix} \begin{Bmatrix} x_p \\ x_s \end{Bmatrix} - \begin{Bmatrix} \omega_p^2 \\ 0 \end{Bmatrix} x_p^{pl} = -\begin{Bmatrix} 1 \\ \bar{m} \end{Bmatrix} \ddot{w}_g \quad (1)$$

where

$$\bar{m} = m_s / m_p \quad (2)$$

denotes the secondary to primary mass ratio, which is in general significantly smaller than one: $\bar{m} \ll 1$. The decoupled natural circular frequencies ω_p, ω_s and damping coefficients ζ_p, ζ_s of both substructures are:

$$\omega_s = \sqrt{k_s / m_s} \quad , \quad \omega_p = \sqrt{k_p / m_p} \quad , \quad \zeta_p = c_p / \left(2\omega_p m_p\right) \quad , \quad \zeta_s = c_s / \left(2\omega_s m_s\right) \quad (3)$$

x_p^{pl} denotes the plastic part of deformation of x_p. The characteristic response parameters of a system according to Figure 1(a) are the mass ratio $\bar{m}$, damping coefficients ζ_p, ζ_s, periods of the decoupled substructures $T_s = 2\pi / \omega_s$, $T_p = 2\pi / \omega_p$, strain hardening coefficient α, and ductility μ of the primary structure. Ductility μ is defined as the ratio of the absolute maximum relative displacement $|x_{p\max}|$ of mass m_p during a single time history analysis related to the corresponding displacement x_{py} at onset of yielding,

$$\mu = |x_{p\max}| / x_{py} \quad (4)$$

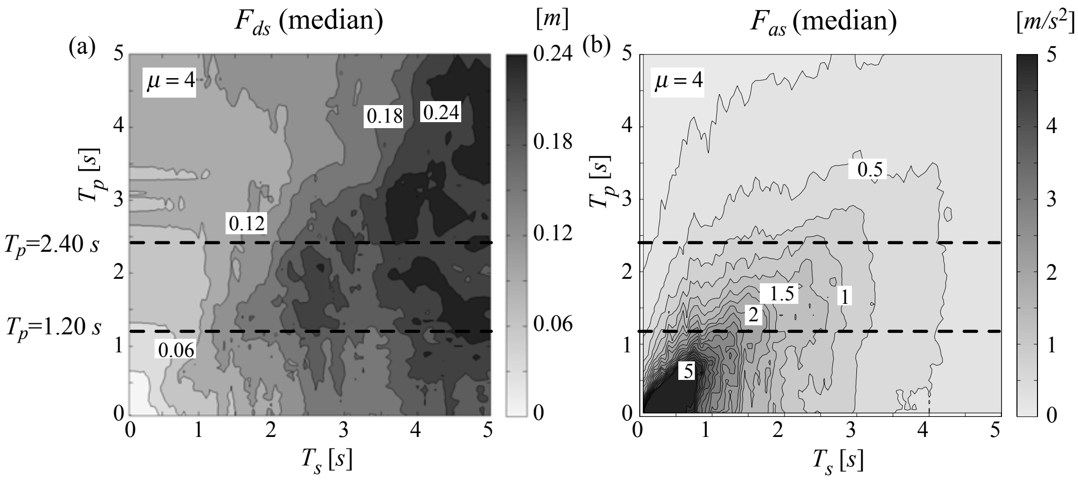

Figure 2 (a) Median displacement floor spectrum, and (b) median acceleration floor spectrum for inelastic 2DOF systems with the following parameters: $\bar{m} = 0.05$, $\zeta_p = 0.05$, $\zeta_s = 0.005$, $\mu = 4$, $\alpha = 0.03$

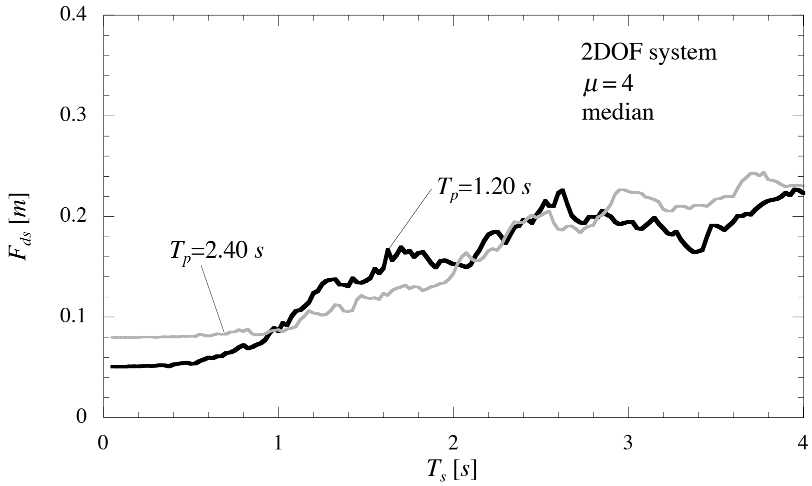

Figure 3 Median displacement floor spectra for inelastic 2DOF systems with periods of the primary structure $T_p = 1.20s$ and $T_p = 2.40s$, and parameters $\bar{m} = 0.05$, $\zeta_p = 0.05$, $\zeta_s = 0.005$, $\mu = 4$, $\alpha = 0.03$

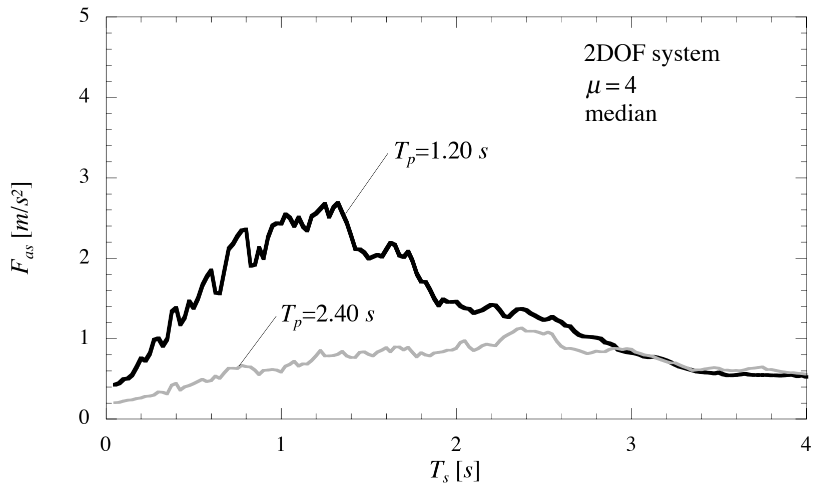

Figure 4 Median acceleration floor spectra for inelastic 2DOF systems with periods of the primary structure $T_p = 1.20s$ and $T_p = 2.40s$, and parameters $\bar{m} = 0.05$, $\zeta_p = 0.05$, $\zeta_s = 0.005$, $\mu = 4$, $\alpha = 0.03$

2.2. Floor spectra for oscillators on inelastic single-degree-of-freedom primary structures

Floor spectra are derived for simple oscillators mounted on primary structures with preassigned ductilities. These spectra are denoted as 2DOF floor spectra. The seismic input is based on a set of 40 ordinary ground motions, which were recorded in California on NEHRP site class D during earthquakes of moment magnitude between 6.5 and 7 and closest distance to the fault rupture between 13 *km* and 40 *km*. This set of records has strong motion duration characteristics insensitive to magnitude and distance. Details are given by Medina and Krawinkler (2003). For each 2DOF system with fixed parameters $\bar{m}$, T_p, T_s, ζ_p, ζ_s, μ, α and each earthquake record time history analyses are performed. Thereby, the yield strength of the primary spring has to be determined iteratively to provide for the preassigned ductility. The peak displacement gives one value of the displacement floor spectrum, the peak acceleration renders one value of the acceleration floor spectrum,

$$F_{ds,i} = \max |x_s|_i \ , \quad F_{as,i} = \max |\ddot{x}_g + \ddot{x}_s|_i \ , \quad i = 1,...,40 \tag{5}$$

The statistical evaluation of the peak responses from the 40 ground motions yield median displacement floor spectra $F_{ds}(med)$, and median acceleration floor spectra $F_{as}(med)$. Floor spectra are presented in three-

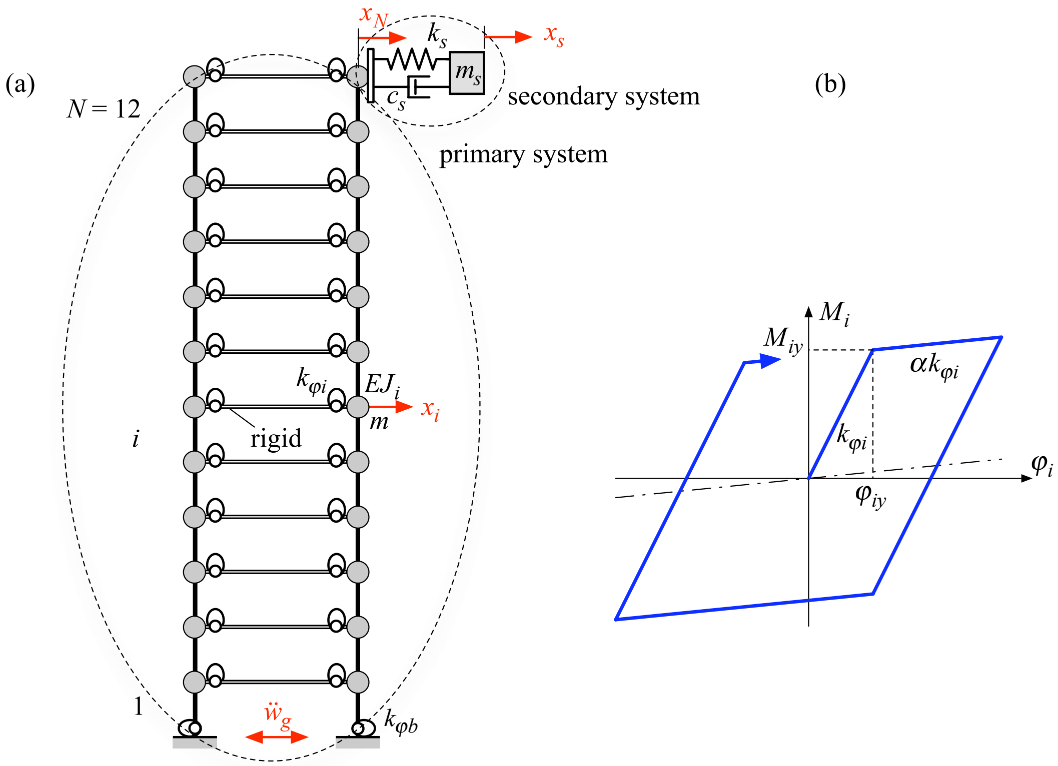

Figure 5 (a) Generic inelastic multi-story frame structure equipped with elastic SDOF secondary structure; (b) Bilinear cyclic behavior of the rotational springs

dimensional form as function of the decoupled periods T_p and T_s of the primary and the secondary system, respectively. Examples are shown in Figure 2 for inelastic 2DOF structures with the parameters $\bar{m} = 0.05$, $\zeta_p = 0.05$, $\zeta_s = 0.005$, $\alpha = 0.03$, and a primary structure ductility μ of 4. For periods of the primary structure $T_p = 1.20s$ and $T_p = 2.40s$ median displacement and median acceleration floor spectra are shown two-dimensionally in Figures 3 and 4. They represent a vertical section of Figures 2 at these periods, depicted by dashed lines. In Adam and Furtmüller (2008) the outcomes of an intensive study on 2DOF floor spectra including inelastic deformations of the primary structure are presented.

3. INELASTIC MULTI-DEGREE-OF-FREEDOM PRIMARY STRUCTURES

3.1. Structural model and equations of motion

In the second part of this study 2DOF floor spectra are utilized to estimate the response of nonstructural components attached to elastic-plastic regular plane multi-story moment resisting frame structures. Starting point of the developed methodology are the coupled equations of motion of a frame structure with N stories and lumped masses at its story corners as shown in Figure 5(a), whose kth floor is equipped with a SDOF vibratory secondary structure of mass m_s, stiffness k_s, and viscous damping parameter c_s (Adam and Fotiu, 2000)

$$\begin{gathered}\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} - \mathbf{G}\boldsymbol{\varphi}^p - k_s\left(x_s - x_k\right)\mathbf{g}_k - c_s\left(\dot{x}_s - \dot{x}_k\right)\mathbf{g}_k = -\mathbf{M}\mathbf{e}\ddot{x}_g \\ m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s - k_s x_k - c_s\dot{x}_k = -m_s\ddot{x}_g\end{gathered} \tag{6}$$

In the model of Figure 5(a) inelastic deformations are confined to the rotational springs located at the base and at both ends of the beams. The horizontal story displacements x_i, $i = 1,\ldots,N$, relative to the base are the N dynamic degrees of freedom of the primary structure, which are assembled in vector $\mathbf{x}$. Matrix $\mathbf{G}$ is the influence matrix of the plastic spring rotations expressed by the vector $\boldsymbol{\varphi}^p$. $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the mass, damping, and initial stiffness matrix, respectively, of the frame structure. Influence vector $\mathbf{e}$ represents the displacements of the primary masses resulting from a static unit ground displacement in direction of the seismic excitation. The *k*th component of influence vector $\mathbf{g}_k$ is one with all other components being zero. This vector identifies the location of the nonstructural element in the *k*th story of the primary structure. The horizontal displacement x_s of the mass of the secondary system with respect to the base represents the $(N+1)$th degree-of-freedom of the coupled system.

3.2. Approximation of floor spectra for oscillators on inelastic multi-story frame structures

Basic assumption of the following considerations is that the dynamic properties of the frame structure can be described by an equivalent SDOF (ESDOF) system. In the ESDOF system approximation the deflection shape of the frame structure follows a time-independent shape vector $\boldsymbol{\phi}$ regardless of the magnitude of deformation, Fajfar (2002). Thus, deformation vector $\mathbf{x}$ of the primary structure is a function of the roof displacement x_N,

$$\mathbf{x} = \boldsymbol{\phi} x_N \quad , \quad \phi_N = 1 \tag{7}$$

After pre-multiplication of the upper equation (6) by the transposed shape vector $\boldsymbol{\phi}^T$ equation (7) transforms equations (6) into the following coupled equations of motion,

$$\begin{bmatrix} 1 & 0 \\ 0 & \bar{m}^* \end{bmatrix} \begin{Bmatrix} \ddot{x}_N \\ \ddot{x}_s \end{Bmatrix} + \begin{bmatrix} 2\zeta_p \Omega^* + 2\zeta_s \omega_s \phi_k^2 \bar{m}^* & -2\zeta_s \omega_s \phi_k \bar{m}^* \\ -2\zeta_s \omega_s \phi_k \bar{m}^* & 2\zeta_s \omega_s \bar{m}^* \end{bmatrix} \begin{Bmatrix} \dot{x}_N \\ \dot{x}_s \end{Bmatrix} + \begin{bmatrix} \Omega^{*2} + \omega_s^2 \phi_k^2 \bar{m}^* & -\omega_s^2 \phi_k \bar{m}^* \\ -\omega_s^2 \phi_k \bar{m}^* & \omega_s^2 \bar{m}^* \end{bmatrix} \begin{Bmatrix} x_N \\ x_s \end{Bmatrix}$$
$$- \begin{Bmatrix} \Omega^{*2} \\ 0 \end{Bmatrix} x_N^{pl} = - \begin{Bmatrix} \Gamma^* \\ \bar{m}^* \end{Bmatrix} \ddot{w}_g \tag{8}$$

In (8) $\bar{m}^*$ is the equivalent mass ratio, Ω^* denotes the equivalent circular frequency of the ESDOF primary structure, and Γ^* represents the equivalent effective participation factor,

$$\bar{m}^* = m_s / m^* \ , \ \Omega^* = \sqrt{k^* / m^*} \ , \ \Gamma^* = \boldsymbol{\phi}^T \mathbf{M}\,\mathbf{e} / m^* \ , \ m^* = \boldsymbol{\phi}^T \mathbf{M} \boldsymbol{\phi} \ , \ k^* = \boldsymbol{\phi}^T \mathbf{K} \boldsymbol{\phi} \tag{9}$$

m^* is the equivalent mass and k^* the equivalent stiffness of the ESDOF system. A further approximation of the ESDOF system concerns the inelastic deformations. It is assumed that the distributed plastic deformations can be expressed in full analogy to a SDOF system, i.e.

$$\boldsymbol{\phi}^T \left(\mathbf{K}\,\mathbf{x} - \mathbf{G} \boldsymbol{\varphi}^p \right) = k^* \left(x_N - x_N^{pl} \right) \tag{10}$$

and furthermore, that hysteretic cyclic behavior of the ESDOF system complies with global hysteretic cyclic behavior of the frame structure. Since the equations of motions of the ESDOF primary system coupled with a SDOF nonstructural component, equations (8), and their counterparts for the system of Figure 1(a), equations (1), have both two degrees-of-freedom it may be concluded that 2DOF floor spectra derived in the first part of this study can also be applied for the prediction of floor spectra of SDOF secondary systems mounted on regular frame structures. Instead of the mass ratio $\bar{m}$ according to equation (2) the effective equivalent mass ratio

$$\bar{\bar{m}}^* = m_s \phi_k^2 / m^* \left(= \bar{m}^* \phi_k^2 \right) \tag{11}$$

must be employed. Moreover, different weighting of the excitation intensity of the primary structure must be considered. While the ground acceleration $\ddot{w}_g$ of the upper equation (8) is multiplied by the effective participation factor Γ^*, the excitation of the corresponding equation of the “real” 2DOF system is just $\ddot{w}_g$, compare equations (1) and (8). Hence, 2DOF floor spectra must be modified before they may be applied for estimating the floor response of secondary structures mounted on MDOF structures. For a rigid secondary system ($T_s = 0$) the response of this substructure is identical to the response of the primary structure at the attachment point, and hence, the spectral values of 2DOF floor response spectra are to be multiplied by Γ^*. At tuned periods $T^* \approx T_s$, $T^* = 2\pi / \Omega^*$, interaction between x_N and x_s is strong. In particular, the secondary response x_s is affected by x_N, because the corresponding primary mass $\bar{m}^*$ is in general much larger than the secondary mass m_s. Thus, it is reasonable to weight 2DOF floor spectra by Γ^* for $T^* \approx T_s$. However, for systems with $T_s \gg T^*$ the primary structure behaves much more rigid compared to the secondary element, and thus, the response depends mainly on the parameters of this element, which is hardly affected by Γ^*. It is proposed to multiply the 2DOF floor spectra by a period depending effective participation factor $\bar{\Gamma}^*(T_s)$, which is found from empirical considerations, see Figure 6. Up to the period $3/2T_s$ $\bar{\Gamma}^*(T_s)$ complies with the absolute value of Γ^*. Afterwards follows a linear ramp, which ends at $3T^*$. $\bar{\Gamma}^*(T_s) = 1$ for $T_s \geq 3T^*$.

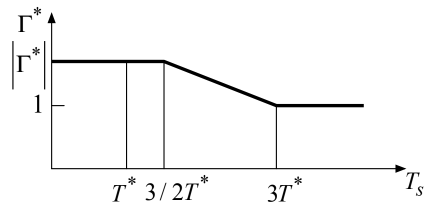

Figure 6 Period depending effective participation factor

A further difference between equations (1) and (8) concerns the damping matrix and the stiffness matrix: In (8) the off-diagonal terms are multiplied by ϕ_k, and the second part of the upper diagonal terms by ϕ_k^2. If the secondary structure is not mounted on the roof ($\phi_k \neq \phi_N (=1)$), the application of 2DOF spectra may be another source of inaccuracy for the prediction of the secondary peak response.

3.3. Application and assessment

In the following example problems generic planar multi-story single-bay frames of 12 stories as shown in Figure 5(a) serve as load-bearing structure. They are composed of elastic columns, rigid beams, and rotational springs at both ends of the beams (Medina et al., 2006). Identical point masses are assigned to each joint of the particular frame. The fundamental mode shape of the MDOF frames follows a straight line. Global cyclic response under earthquake excitation is represented by non-degrading bilinear hysteretic behavior of the rotational springs, see Figure 5(b). Strain hardening ratio α is 0.03 for all springs. Viscous damping is considered by means of 5% percent modal damping for all modes of the frame structure with 12 dynamic degrees-of-freedom. The fundamental period of vibration T_1 is 1.2 s for a stiff, and 2.4 s for a more flexible frame structure. On top of the frame a vibratory secondary structure is attached, which is modeled as an elastic SDOF oscillator with $\zeta_s = 0.005$. For the ESDOF system of the frame structure the fundamental mode shape is utilized as shape vector. Thus, the fundamental period of the frame structure and the period of the ESDOF primary structure are identical. The effective equivalent modal mass ratio $\bar{\bar{m}}^*$ according to equation (11) is 0.05. The target ductility μ of the frame is 4. For calibration of the yield moments M_{iy} the yield strength f_{py} of the “real” 2DOF system (which leads to a ductility of 4) and the base shear of the ESDOF system at onset of yield are equated. Subsequently, the yield rotations of the springs are tuned in such a way that a pushover analysis under a linear design load pattern leads to a simultaneous onset of yielding at all springs.

In Figures 7 and 8 median displacement and acceleration floor spectra, respectively, are presented for the stiff frame with a fundamental period of $T_1 = 1.2s$. The black full line corresponds to the results utilizing the complete set of equations (6), i.e. interaction between the primary and secondary substructures is considered. Additionally, the results of the simplified analysis based on an ESDOF system and 2DOF floor spectra for a period of $T_p = 1.2s$ are displayed. These spectra are presented in Figures 3 and 4, since their underlying mass ratio and the effective mass ratio of the ESDOF system, and the substructure damping coefficients are identical. Multiplication of the 2DOF floor spectra by the period dependent effective participation coefficient according to Figure 6 leads to the simplified solution depicted in Figures 7 and 8 by thick gray lines. It can be observed that

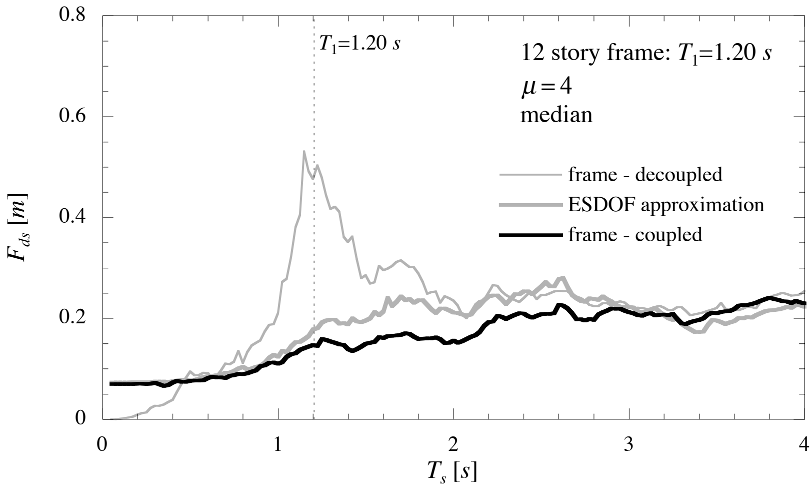

Figure 7 Median displacement floor spectra for a SDOF oscillator on a ductile 12 story frame structure

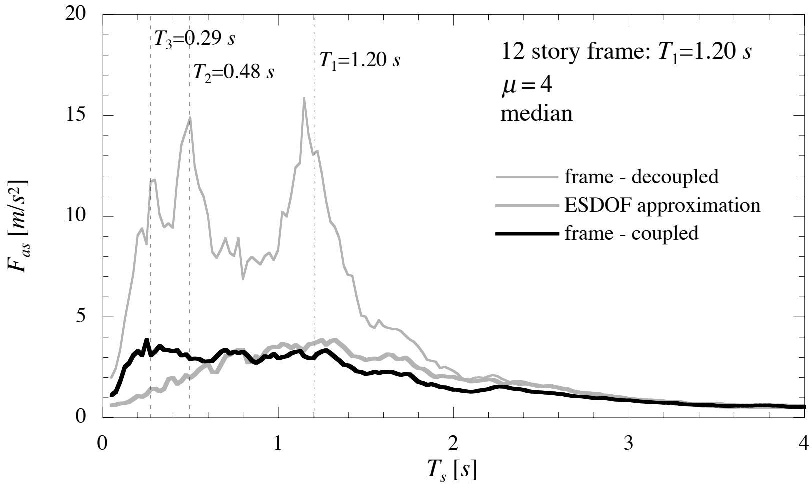

Figure 8 Median acceleration floor spectra for a SDOF oscillator on a ductile 12 story frame structure

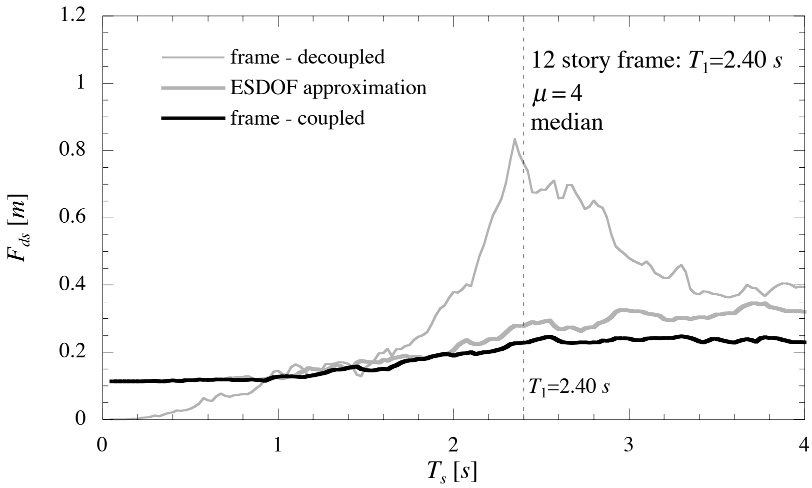

Figure 9 Median displacement floor spectra for a SDOF oscillator on a ductile 12 story frame structure

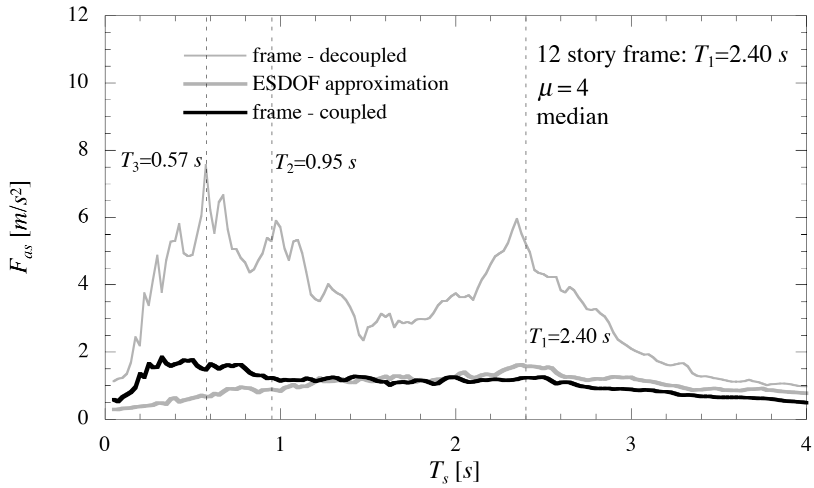

Figure 10 Median acceleration floor spectra for a SDOF oscillator on a ductile 12 story frame structure

the displacement floor spectrum is approximated in the entire period range with sufficient accuracy by modified 2DOF floor spectra. For periods T_S of the secondary structure larger than $0.6s$ the simplified methodology renders a satisfactory approximation of the acceleration floor spectrum shown in Figure 8. However, in the short period range the deviation from the "exact" peak acceleration is essential. Note, that ductile deformations of the primary structure lead to energy dissipation and period elongation, and thus, floor spectra of Figure 7 and 8 do not exhibit pronounced peaks at primary structure periods. Furthermore, the effect of interaction between the primary and secondary structure is examined. In Figures 7 and 8 floor spectra are shown, which are generated omitting coupling between the substructures. In such a decoupled analysis the input of the secondary structure is the seismic response of the stand-alone primary structure at its attachment point. The results verify that this simplified approach overestimates the actual response by a large amount when substructure periods become tuned. Thus, a decoupled analysis, which is common engineering practice, may lead to an over-conservative response prediction. Subsequently, floor spectra based on a more flexible 12 story frame structure with a fundamental vibration period of $T_1 = 2.40s$ are shown in Figures 9 and 10. These floor spectra confirm the previous findings for the $T_1 = 1.20s$ structure.

REFERENCES

Adam, C. and Fotiu, P.A. (2000). Dynamic analysis of inelastic primary-secondary systems. *Engineering Structures* **22**, pp. 58-71.

Adam, C. and Furtmüller, T. (2008). Seismic response characteristics of nonstructural elements attached to inelastic buildings. *Proceedings of 7th European Conference on Structural Dynamics*, University of Southampton, Southampton, UK (Brennan, M.J., ed.), CD-ROM paper, paper no. 120, 12 pp.

Fajfar, P. (2002). Structural analysis in earthquake engineering - a breakthrough of simplified non-linear methods. *Proceedings of the 12th European Conference on Earthquake Engineering*, CD-ROM paper, paper ref. 843, 20 pp., Elsevier.

Medina, R.A., Sankaranarayanan, R. and Kingston, K.M. (2006). Floor response spectra for light components mounted on regular moment resisting frame structures. *Engineering Structures* **28**, 1972-1940.

Medina, R.A. and Krawinkler, H. (2003). *Seismic demands for nondeteriorating frame structures and their dependence on ground motions*, Report No. 144. The John A. Blume Earthquake Engineering Center, Stanford University.

Villaverde, R. (2004). Seismic analysis and design of nonstructural components. *Earthquake engineering: From engineering seismology to performance-based engineering*, CRC Press, Boca Raton, Florida.
Building Seismic Safety Council. NEHRP recommended provisions for seismic regulations for new buildings and other structures, FEMA-450 (2003). Washington DC: Federal Emergency Management Agency.