GLOBAL SEISMIC RELIABILITY ANALYSIS OF BUILDING STRUCTURES BASED ON SYSTEM-LEVEL LIMIT STATES

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ABSTRACT

In this paper, a global limit state function for load-carrying capacity of structural system is firstly set up, in which the margin of safety is the difference between the limit base shear of structural system and the total horizontal seismic action. To probabilistically assess the global seismic capacity of structure, a new point estimation method (PEM) for analyzing statistical moments of complex random function is put forward, and then it is combined with deterministic finite element analysis to produce the so-called “random pushover analysis (RPA)”. On the basis of the above new methodologies, a semi-analytical approach which integrates the improved point estimation method, pushover analysis and first order reliability method (FORM) is developed to analyze the nonlinear seismic reliability of structure as a global system. By applying the proposed approach in R.C. frame structure, the changing rules of the global seismic reliability of the structure with the coefficient of variation of the total seismic action and correlation coefficient of storey-level seismic forces are derived. It is demonstrated by a numerical example that the newly developed method in this paper is simple, practical and efficient compared with MCS, and that it has the same accuracy as MCS.

KEYWORDS: Global Reliability, Seismic Reliability, Random Pushover Analysis (RPA), Point Estimation Method (PEM), First Order Reliability Method (FORM), Semi-Analytical Approach

1. INTODUCTION

As a conventional method in structural system reliability theory (Moses, 1982), the failure mode approach (FMA) is difficult to apply in civil engineering practice, since it has many disadvantages: First, the constitutive relations of materials are assumed to be perfect rigid-plastic, however, this is not the real case of many civil engineering materials, such as concrete, structural steel, soil, etc.; Second, it is hardly to identify the significant failure modes and determine their corresponding failure mode equations of large-scale and complex structures, because the number of possible failure modes of realistic complexity can be extremely large; Third, the correlation between failure modes is another important problem not easy to deal with; Forth, the overall failure probability of structural systems cannot be evaluated accurately even though the dominant failure modes and limit state equations are known a prior.

On the other hand, a new trend in which structural systems reliability is approximately calculated by using global limit states based on nonlinear structural analysis techniques recently has been increasingly of interest in many different communities. This novel approach encompass the following items: (1) the integrated nonlinear analysis methods of structural systems considering real constitutive relations of materials, e.g., first order inelastic analysis, second order inelastic analysis, etc., are utilized to search for the dominant modes of failure; (2) the statistics of structural global load-carrying capacity are obtained by Monte Carlo simulations; (3) the probability density function (PDF) of the global load-carrying capacity is fitted by its first few moments; (4) a global limit state equation is set up, which comprises the global load-carrying capacity and structural load effects; (5) the classic structural component reliability theory, such as first order reliability method (FORM), is applied in the global limit state equation, the system reliability is then obtained approximately. This approach has two advantages: first, it can directly bypass the difficulties in searching the significant modes of failure in FMA; second, it can consider the real constitutive relations of structural materials. Therefore, it is a practical and efficient approximate method to solve the systems reliability problems of structures. In this paper, we call this approach as *global reliability theory* of structures.

To the author's knowledge, Gorman and Moses (1979) perhaps are the first researchers who presented the idea of directly computing the system reliability by structural system resistance. Grigoriu (1983) proposed a control variable approach to approximate the reliability of complex problems from estimators developed for the distribution of the control variable. Nowak and Zhou (1988, 1990) developed a numerical integration method to calculate the first few moments of complex random function, and then applied the approach in system reliability of highway bridges. Sigursdon et al. (1994) proposed a probabilistic collapse analysis method to assess the system reliability of jacket platforms. Zhao and Ono (1998) developed a failure mode independent performance function using load factor obtained by limit analysis, and then used response surface approach to approximate the performance function and FORM to evaluate the system failure probability of ductile frames. Onoufriou and Forbes (2001) reviewed and critically examined the recent developments in system reliability methods for fixed steel offshore platforms. Moreover, they paid special attentions to pushover analysis, simplified models and "component-based approach". Ou et al. (2001) developed a probabilistic pushover analysis to approximately evaluate the system reliability of buildings by randomizing both the capacity spectrum and demand spectrum. Ou et al. (2003) also established a global limit state function considering the limit base shear of the whole structure based on limit analysis to evaluate the system reliability of existing fixed jacket platforms. Li and Cheng (2004) employed pushover analysis together with Monte Carlo simulation to check the probability distribution types that the global resistance of steel and R.C. frames satisfies with K-S testing. Li et al. (2002, 2004, 2006) presented a reliability-based integrated design (RID) methodology of steel frames based on nonlinear structural analysis, and gave a system-level RID format like as the LRFD formulations of structural members to directly checks the structural system limit states and the corresponding system reliability. To compute the system reliability, they proposed a semi-analytical simulation method to assess the reliability of structural systems, which combines variance-reduction techniques including systematic sampling and antithetic variates simulations to obtain the moments of system resistance, the procedure of fitting the PDF of system resistance by exponential polynomial method (EPM), and first order reliability method (FORM). They have succeeded in applying their methodology in advanced design of steel portal frames with tapered members in industrial buildings and plane steel frames in high-rise buildings.

The theory of global reliability provides a practical and operational means of moving from member design level toward system design level for reliability-based probability design of structures, and bridges the gap between the two design levels. Moreover, the main ideas of this theory are consistent with those of performance-based design theory now prevailing in the community of earthquake engineering. Therefore, it has a broader prospect of applications. However, the research on global reliability theory of structures is still insufficient, and has not been paid much attention to. Furthermore, there is a little study on the global seismic reliability of structural systems, and nearly all the existing research employed Monte Carlo simulation to get the moments of system resistance. Since the probability of failure of structures due to strong earthquakes is usually very small, the number of nonlinear finite element analysis required by MCS is usually around 10^5-10^7, and so the computational cost may be prohibitively large.

In this paper, a global limit state function for global seismic reliability of structures is provided, which is the difference of the limit base shear of structural system minus the total horizontal seismic action. A new semi-analytical approach combing point estimation method (PEM), pushover analysis with FORM is developed for analyzing the global seismic reliability of structures. The developed methodology is applied in R.C. frame structures considering the nonlinear effects. The applied method is also compared with MCS. A numerical example demonstrates that the approach put forward by this paper can significantly reduce the number of finite element simulations, and has the same accuracy as that of MCS.

2. GLOBAL LOAD-CARRYING CAPACITY LIMIT STATE FUNCTION OF STRUCTURAL SYSTEM AND A NEW SEMI-ANALYTICAL METHOD FOR SEISMIC RELIABILITY ANALYSIS

2.1. Global Load-Carrying Capacity Limit State Function of Structural Systems

In Chinese seismic design code of buildings (GB50011-2001), the *base shear method* is a prevailing approach to obtain the seismic action for low to medium-rise buildings. In this paper, we take the limit base shear of structure as the global seismic capacity of structural system, and take the total horizontal seismic action as the seismic demand of structures. Based on these considerations, we propose the following global seismic capacity limit state function for structural systems:

$$g(V_S, F_E) = V_S - F_E \tag{2.1}$$

where V_S = limit base shear of structures, F_E = the total horizontal seismic action of structures in the base. They are all random variables, so we can use static reliability theory to conduct the analysis of seismic reliability, which is a dynamic reliability problem in nature.

The formulation of Eq. (2.1) is the same as the performance function of structural members in the format, in other words, the limit base shear V_S and the total horizontal seismic action F_E correspond with the resistance and load effect of structural members, respectively. Therefore, the classical component reliability methods such as FORM and SORM (Ditlevsen and Madsen, 1996) can be used to approximately compute the system reliability of structures.

There is another advantage in applying Eq. (2.1) in seismic reliability analysis: the real constitutive relations of structural materials and nonlinear effects of structural systems can be considered through the limit base shear V_S. As such, although Eq. (2.1) is linear in the format, the approach based on this formulation is a nonlinear reliability analysis method in nature.

There are two causes for Eq. (2.1) just to include one load effect, i.e. seismic action. One reason is that both the mean value and the variability in the seismic intensity are much larger than those of the live loads and wind load, so the randomness in the live loads and wind load should not be considered, and their characteristic values are taken, when analyzing the seismic reliability of structures. Therefore, only one single random load, i.e., seismic action, is included in Eq. (2.1). The second reason is that, when the total horizontal seismic action is computed, the equivalent total gravity load has included the combination effects of dead loads, live loads and wind load, so the total horizontal seismic action F_E is equivalent to the comprehensive load effect.

2.2. The Limit Base Shear of Structures

The limit base shear of structures V_S depends on not only the limit load-carrying capacity of structural members, but also the constitutive relations of materials, the correlation relationships among structural members, the correlation relationships between member resistance and loads, load path, system redundancy, structural types, loading cases of structures, etc. Due to the complexity of the problem, the traditional limit load analysis method is not suitable to the limit base shear analysis of structures. Instead, we take pushover analysis as the basic tool to evaluate the limit base shear of structures. Pushover analysis is an incremental static elastoplastic analysis method under the increasing monotonic load. The advantage of applying pushover analysis in system reliability evaluation is that it can trace the developing sequences of plastic hinges and so identify the significant failure modes. Actually, it is an extension of the incremental load approach proposed by Moses (1982) in system reliability theory of structures.

Much research has proven that the global load-carrying capacity of structures can be approximately modeled by log-normal distribution. Therefore, in this paper, the limit base shear of structures is also assumed to satisfy log-normal distribution:

$$F_{V_S}(v) = \Phi\left(\frac{\ln v - \lambda_{V_S}}{\zeta_{V_S}}\right) \tag{2.2}$$

where λ_{V_S} and ζ_{V_S} are logarithmic mean and logarithmic standard deviation of V_S respectively, their relationships with the mean value and coefficient of variation (COV) of V_S are

$$\lambda_{V_S} = \ln\left(\frac{\mu_{V_S}}{\sqrt{1+\delta_{V_S}^2}}\right) \tag{2.3}$$

$$\zeta_{V_S} = \sqrt{\ln\left(1+\delta_{V_S}^2\right)} \tag{2.4}$$

in which μ_{V_S} and δ_{V_S} are mean value and COV of V_S respectively.

2.3. Equivalent Static Random Seismic Action

Based on response spectrum of single-degree-of-freedom (SDOF) oscillator, the equivalent static random seismic action can be described as

$$F_E = \alpha GD = \frac{A_m}{g}\beta(T,\xi)GD \tag{2.5}$$

where, G is the equivalent total gravity load, D is an appending factor considering the modeling uncertainty from, $\alpha = \frac{A_m}{g}\beta(T,\xi)$ is the earthquake effect coefficient, g is the gravity acceleration, A_m is the peak ground acceleration (PGA), $\beta(T,\xi)$ is the dynamic amplification factor, in which T and ξ are the vibration period and damping ratio of the oscillator respectively.

Ou et al. (1994) has proven that the random seismic action under deterministic earthquake intensity satisfies type I extreme value distribution. When the seismic intensity I takes J, the CDF of F_E is

$$F_{F_E}(f \mid I = J) = \exp\{-\exp[-\alpha(f-u)]\} \tag{2.6}$$

where the distribution parameters take the forms of

$$\alpha = \frac{\pi}{\sqrt{6}\mu_{F_J}V_{F_J}} \tag{2.7}$$

$$u = \mu_{F_J}(1-0.45V_{F_J}) \tag{2.8}$$

where μ_{F_J} and V_{F_J} are the mean value and COV of F_E under the jth intensity respectively. Based on the research of Ou et al. (1994, 1995), $\mu_{F_J} = 0.75F_{JK}$, in which F_{JK} is the characteristic value of the horizontal seismic action under the jth intensity, $V_{F_J} = 0.73$. Put the above results in Eqs. (2.7) and (2.8), we can obtain the final results: $\alpha = 2.34/F_{JK}$, $u = 0.5F_{JK}$.

2.4. A New Semi-Analytical Method for Global Seismic Reliability Analysis of Structures

From the above statements we can come to the conclusion that the central problem of applying Eq. (2.1) in analysis of global seismic reliability is how to get the moment information of limit base shear V_S, since the probability model and distribution parameters of random seismic action F_E have been certain. Most of the available research generally makes use of Monte Carlo simulation combined with pushover analysis to obtain the samples and then the estimators of statistical moments. To reduce the variance of simulation, many techniques have been introduced, such as importance sampling, systematic sampling, antithetic variates, etc. Unfortunately, the computation cost of these random simulation approaches based on MCS is still extremely large. In next section, we will propose a random pushover analysis approach based on point estimation method (PEM), which can significantly reduce the number of nonlinear finite element analysis while keeping the same accuracy as MCS.

After obtaining the statistical moments of V_S by numerical analysis techniques, the approximate analytical method such as FORM/SORM can then be applied to solve Eq. (2.1). For this purpose, we suggest herein a new semi-analytical method as follows:

(1) Obtaining the statistical moments of limit base shear V_S by using random pushover analysis based on point estimation method;

(2) Fitting the PDF of V_S according to its statistical moments. If the probability model of V_S can be decided a prior, then this step can be omitted;

(3) Using FORM and/or SORM to solve Eq. (2.1).

3. RANDOM PUSHOVER ANALYSIS BASED ON POINT ESTIMATION METHOD AND ITS APPLICATIONS IN PROBABILISTIC ANALYSIS OF STRUCTURAL LIMIT BASE SHEAR

3.1. Characteristics and Difficulties of Probabilistic Analysis of Structural Limit Base shear

Since there are randomness and uncertainties in many factors influencing structural limit base shear V_S, we should make use of probability theory to analyze and compute the statistical moments of V_S. Put all factors influencing V_S together into a basic random vector $\mathbf{X} = [X_1, X_2, \cdots, X_n]^T$, then V_S can be generally denoted as an implicit and nonlinear function of **X**:

$$V_S = h(\mathbf{X}) = h(X_1, X_2, \cdots, X_n) \quad (3.1)$$

There exist the following characteristics and difficulties in probabilistic analysis of limit base shear of structures: (1) Multiple scales. The factors influencing V_S can be classified as five scales, i.e. material scale, section scale, member scale, sub-structure scale and structural system scale. The random information propagates across the above scales from bottom to top, so as to make the analysis of uncertainty propagation very difficult. (2) Nonlinearity. The limit base shear of structures is a nonlinear function of the above influencing factors in nature, especially in the case that the structure goes into the severe damage or even collapse. (3) Correlation. There may be correlating relations to some degree between the factors in the same scale, or between the factors across the different scales. (4) Highly implicitness. The limit base shear is usually determined by numerical analysis techniques, such as FEM. Therefore, the limit base shear is a highly implicit function of basic random variables, generally takes the form of a black box.

Due to the above difficulties, the conventional methods for analysis of uncertainty propagation, such as mean-value first order second moment (MVFOSM) method and Monte Carlo simulation, have the disadvantages of low accuracy or too much computational cost. Therefore, we should look for some approaches whose accuracy and efficiency can all be accepted by engineering community, among which the point estimation method is such an approach.

3.2. Point Estimation Method Based on Nataf transformation

Point estimation method (PEM) was proposed by Rosenblueth (1975) to approximate the lower-order moments of functions of random variables. It is a special case of numerical quadrature based on orthogonal polynomials. For normal variables, it corresponds to Gauss-Hermite quadrature. While the point estimate method is popular in practice, it has many detractors. Numerous modifications or improvements have been made for the original PEM. However, the early developments of PEM are all undertaken in the original space of random variables, requiring the higher order moments of random variables without considering the distribution information. To overcome these shortcomings, Zhao and Ono (2000) introduced a new point estimation method based on Rosenblatt transformation in which the numerical quadrature is completed in standard normal space. Unfortunately, Rosenblatt transformation cannot deal with the case of random variables with given marginal distributions and correlation information. In this paper, we introduce Nataf transformation (Liu and Der Kiureghian, 1986) into Zhao-Ono point estimation method.

The forward Nataf transformation T_N can be denoted by

$$T_N : \mathbf{u} = \mathbf{L}_0^{-1}\mathbf{\Phi}^{-1}[\mathbf{F}_{\mathbf{X}}(\mathbf{x})] \quad (3.2)$$

where, $\mathbf{x}$ and $\mathbf{u}$ are the realizations of n dependent non-normal random variables $\mathbf{X}$ and independent standard normal random variables $\mathbf{U}$, respectively; $\mathbf{\Phi}^{-1}()$ represents the column vector composed of all inverse functions of standard normal random variables; $\mathbf{F}_{\mathbf{X}}(\mathbf{x})$ is the column vector comprised of CDFs of random variables X_i $(i = 1, \cdots, n)$; $\mathbf{L}_0$ is the lower triangle matrix of Choleski decomposition of correlation coefficients matrix $\mathbf{R}_0$ of dependent normal random vector $\mathbf{Y} = \mathbf{\Phi}^{-1}[\mathbf{F}_{\mathbf{X}}(\mathbf{x})]$, i.e. $\mathbf{R}_0 = \mathbf{L}_0\mathbf{L}_0^T$; the relationships between the elements $\rho_{0,ij}$ of $\mathbf{R}_0$ and the elements ρ_{ij} of $\mathbf{R}$, the correlation coefficients matrix of $\mathbf{X}$, are

$$\rho_{0,ij} = F_{ij}\rho_{ij} \quad (3.3)$$

where, the coefficient F_{ij} is function of correlation coefficient ρ_{ij} and marginal distributions $F_{X_i}(x_i)$ and $F_{X_j}(x_j)$ of random variables X_i and X_j. In general, $F_{ij} \geq 1$. Liu and Der Kiureghian (1986) gave the practical formula for computing coefficient F_{ij} corresponding to different probability distributions.

The inverse Nataf transformation T_N^{-1} can be denoted by

$$T_N^{-1} : \mathbf{x} = \mathbf{F}_{\mathbf{X}}^{-1}[\mathbf{\Phi}(\mathbf{L}_0\mathbf{u})] \quad (3.4)$$

where, $\mathbf{F}_{\mathbf{X}}^{-1}(\cdot)$ represents the column vector composed of all inverse functions of random variables X_i $(i=1,\cdots,n)$; $\mathbf{\Phi}(\cdot)$ denotes the column vector comprised of all CDFs of standard normal random variables.

The computation of the first two moments of random function $h(\mathbf{X})$ is undertaken in the standard normal space by using the inverse Nataf transformation,

$$\mu_h = \int h(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int h\left[T_N^{-1}(\mathbf{u})\right] \varphi_n(\mathbf{u}) d\mathbf{u} \tag{3.5a}$$

$$\sigma_h^2 = \int [h(\mathbf{x}) - \mu_Z]^2 f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int \left\{h\left[T_N^{-1}(\mathbf{u})\right] - \mu_Z\right\}^2 \varphi_n(\mathbf{u}) d\mathbf{u} \tag{3.5b}$$

where, μ_h and σ_h are mean value and standard deviation of random variable h respectively; $f_{\mathbf{X}}(\mathbf{x})$ is the joint PDF of random vector $\mathbf{X}$; $\varphi_n(\mathbf{u})$ is the joint PDF of n-dimension standard normal random variables.

For single-variable function $h(X)$, Nataf transformation reduces to iso-probability transformation $x = F_X^{-1}[\Phi(u)]$, and then Eq. (3.5) can be approximated by using Gauss-Hermite numerical quadrature in standard normal space:

$$\mu_h \approx \sum_{j=1}^{m} P_j h\left\{F_X^{-1}[\Phi(u_j)]\right\} \tag{3.6a}$$

$$\sigma_h^2 \approx \sum_{j=1}^{m} P_j \left[h\left\{F_X^{-1}[\Phi(u_j)]\right\} - \mu_h\right]^2 \tag{3.6b}$$

where, $u_j (j=1,\cdots,m)$ are estimation points; P_j are corresponding weights; m is the number of estimation points.

The abscissas x_j and weights w_j of Gauss-Hermite quadrature with weight function $\exp(-u^2)$ are listed in Table 3.1.

Table 3.1 Abscissas and weights for Gauss-Hermite integration

Order (m)	Abscissas (x_j)	Weights (w_j)	Order (m)	Abscissas (x_j)	Weights (w_j)
1	0	1.7724538509	5	0	0.9453087205
2	±0.707106781	0.8862269255		±2.020182871	0.0199532421
3	0	1.1816359006		±0.958572465	0.3936193232
	±1.224744871	0.2954089752	6	±2.350604974	0.0045300100
4	±1.650680124	0.0813128354		±1.335849074	0.1570673203
	±0.524647623	0.8049140900		±0.436077412	0.7246295952
7	0	0.8102646176	8	±2.930637420	0.0001996041
	±2.651961357	0.0009717812		±1.981656757	0.0170779830
	±1.673551629	0.0545155828		±1.157193712	0.2078023258
	±0.816287883	0.4256072526		±0.381186990	0.6611470126

The estimating points u_j and weights P_j in Eq. (3.6) can be obtained according to Table 3.1:

$$u_j = \sqrt{2} x_j, \ \ P_j = \frac{w_j}{\sqrt{\pi}} \tag{3.7}$$

For a function of random variables $h(\mathbf{X})$, it is approximated by a non-product function proposed by Zhao and Ono (2000):

$$h(\mathbf{X}) \approx h'(\mathbf{X}) = \sum_{i=1}^{n} (H_i - H_{\boldsymbol{\mu}}) + H_{\boldsymbol{\mu}} \tag{3.8}$$

in which,

$$H_{\boldsymbol{\mu}} = h(\boldsymbol{\mu}) = h(\mu_1, \cdots, \mu_i, \cdots, \mu_n) \tag{3.9}$$

$$H_i = h\left[T_N^{-1}(\mathbf{u}_i)\right] = H(\mathbf{u}_i) = H(u_{\mu 1}, u_{\mu 2}, \cdots, u_{\mu i-1}, u_i, u_{\mu i+1}, \cdots, u_{\mu n}) \tag{3.10}$$

where $\boldsymbol{\mu}$ represents the vector in which all the random variables take their mean values; $\mathbf{u}_i$ represents the vector in which only u_i is a random variable, while other variables take the corresponding transformed values of their mean values in standard normal space; $u_{\mu j}(j \neq i)$ is the *j*th element of the transformed vector $\mathbf{u}_\mu$ who corresponds the vector $\boldsymbol{\mu}$ in standard normal space $\mathbf{u}$; $H(\mathbf{u}) = h[T_N^{-1}(\mathbf{u})]$ is the formulation of random function $h(\mathbf{x})$ in standard normal space based on Nataf transformation.

Note that we have introduced Nataf transformation into Eq. (3.8), so it is different from that proposed by Zhao and Ono, although their forms are the same.

Based on Eq. (3.8), the first two moment of multi-variable random function $h(\mathbf{X})$ can be estimated by

$$\mu_h \approx \sum_{i=1}^{n}(\mu_i - H_{\boldsymbol{\mu}}) + H_{\boldsymbol{\mu}} \tag{3.11a}$$

$$\sigma_h^2 \approx \sum_{i=1}^{n}\sigma_i^2 \tag{3.11b}$$

where μ_i and σ_i are mean value and standard deviation of H_i by using point-estimation of single-variable function.

3.3. Application of Point-Estimation Based Random Pushover Analysis in Statistical Moments Computation of Structural Limit Base shear

The nature of point estimation method is that it is a kind of deterministic sampling in standard normal space according to Table 3.1 and Eq. (3.7), whose total sampling number is $m \times n$, in which m is the order of numerical quadrature, and n is the number of basic random variables. Compared with the huge sampling number of Monte Carlo simulation, obviously the sampling number of point estimation method reduces dramatically. On the other hand, from the viewpoints of experimental design, point estimation method belongs to a kind of deterministic experimental design.

Since point estimation method makes use of deterministic sampling or experimental design techniques, we can combine this method with deterministic finite element analysis, herein the pushover analysis, to compute the statistical moments of limit base shear of structures. We call this combination of pushover analysis with point estimation method as "random pushover analysis (RPA)", the detailed implementation steps are as follows:

(1) Building the finite element model of structure;

(2) Determining the probability distribution types and their distribution parameters of basic random variables $\mathbf{X}$ that influence the limit base shear V_S of structures;

(3) Generating structural samples by sampling of the basic random variables $\mathbf{X}$ according to Table 3.1 and Eq. (3.7);

(4) Conducting pushover analysis for each structural sample to derive its base shear-top displacement curve, from which the limit base shear is obtained;

(5) Computing the statistical moments of limit base shear V_S according to Eqs. (3.8) to (3.11).

4. Application of the methodology to a R.C. frame building

4.1. Basic Data of the Structure

The analyzed structure shown in Figure 1 is a three-bay and six-storey reinforced concrete frame building, the sizes of beams and columns are listed in Table 4.1. The uniformly distributed load on the top floor is 16.6 KN/m, the loads on other floors are all 20.06 KN/m.

4.2. Probability Models and Statistical Parameters of Basic Random Variables

According to the research of Ou et al. (1994, 1995), the random horizontal seismic forces acting on structural floors computed by base shear method all satisfy Type-I extreme value distribution. The statistical parameters of storey-level seismic forces according to Eqs. (2.7) and (2.8) are listed in Table 4.2. Ou et al. (1995) assumed that the storey-level seismic forces were perfect correlation. In order to investigate the effects of the variation of total horizontal seismic action and correlation relation of storey-level seismic forces on the limit base shear of structures, this paper assumes the variation of total horizontal seismic action changes from 0.1 to 1.0; while the correlation coefficient of storey-level seismic forces changes from 0 to 0.9. The randomness considered in structural resistance includes yielding strength f_c of concrete, yielding strength f_y, elasticity modulus E and the second stiffness factor α of steel, their probability models and distribution parameters are also listed in Table 4.2.

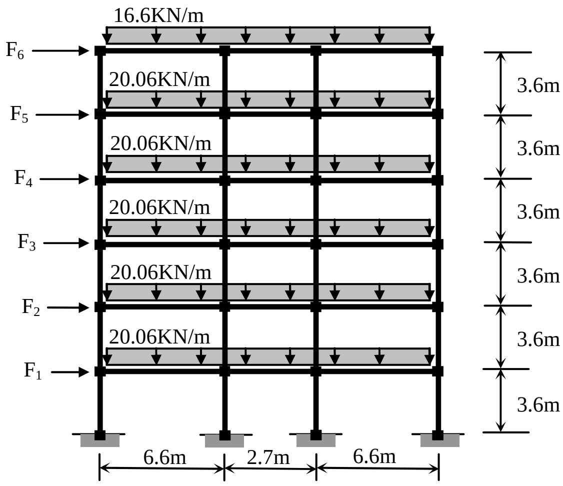


Figure 1 Three-bay and six-storey R.C. frame

Table 4.1 Sizes of structural members

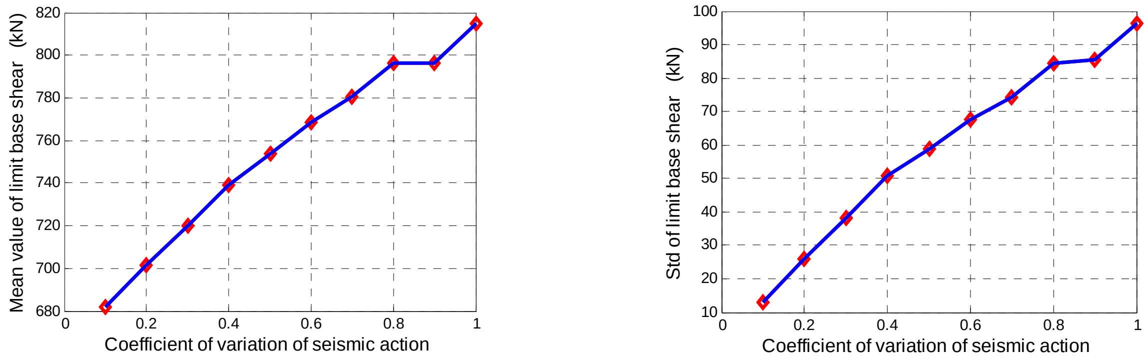
Members	Height (mm)	Width (mm)	Strength Grade of Concrete	Steel Grade of rebar	Steel Grade of hoops
Interior Columns	500	500	C30	HRB335	HPB235
Exterior Columns	500	500	C30	HRB335	HPB235
Main Beams	600	300	C30	HRB335	HPB235
Side Beams	500	200	C30	HRB335	HPB235

Table 5.2 Statistics and probability types of basic random variables

RVs	Mean value	Std	COV	Types	Correlation coefficient
f_c (N/mm^2)	14.3	2.86	0.2	Log-normal	0.3
f_y (N/mm^2)	363	72.6	0.2		
E (N/mm^2)	2×10^5	0.4×10^5	0.2		
α	0.05	0.001	0.2	normal	
F_1 (KN)	33.513	3.351~33.513	0.1~1.0	Type-I largest	0.0~0.9
F_2 (KN)	58.379	5.838~58.379	0.1~1.0		
F_3 (KN)	80.345	8.035~80.345	0.1~1.0		
F_4 (KN)	98.292	9.829~98.292	0.1~1.0		
F_5 (KN)	111.170	11.117~111.170	0.1~1.0		
F_6 (KN)	91.721	9.172~91.721	0.1~1.0		

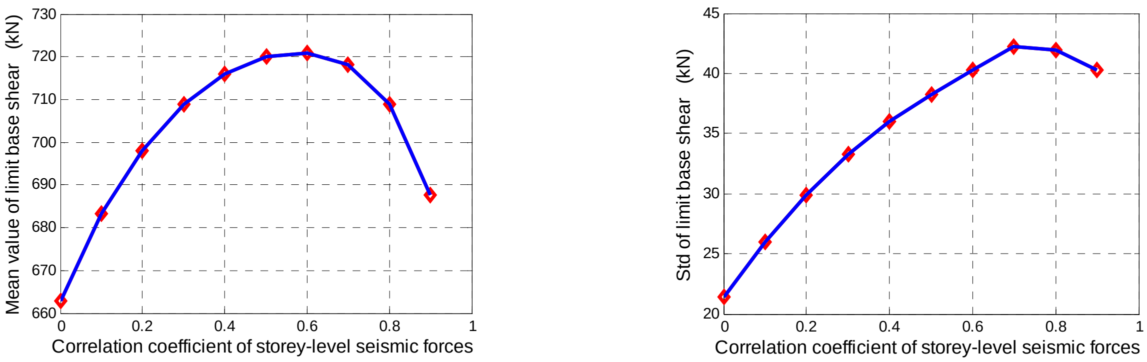
4.3. Probabilistic Analysis of Limit Base shear of the Structure

By applying the random pushover analysis based on Nataf transformation proposed in this paper in probabilistic analysis of limit base shear of the R.C. frame, the changing rules of mean value and standard deviation of limit base shear with COV of total horizontal seismic action and correlation coefficient of storey-level seismic forces, as shown in Figures 2 and 3.



(a) Mean value of limit base shear (b) Std of limit base shear

Figure 2 Changing of mean value and standard deviation of limit base shear with COV of seismic action



(a) Mean value of limit base shear (b) Std of limit base shear

Figure 3 Changing of mean value and standard deviation of limit base shear with correlation coefficient of storey-level seismic forces

From Figure 2 it is evident that the mean value and standard deviation of limit base shear tend to be large with the increasing of COV of total horizontal seismic action. This result is predictable since the pushover results of structures depend on the loading cases and lateral load patterns. Obviously, the more is the randomness in the total seismic action, the more are the statistics of limit base shear.
From Figure 3 we can see that the mean value and standard deviation of limit base shear do not necessarily increase with the correlation coefficient of storey-level seismic forces. When correlation coefficient $\rho > 0.6$, the mean value of limit base shear becomes smaller; while when $\rho > 0.7$, the standard deviation becomes smaller. If we assume the storey-level seismic forces are all perfect correlated, then the conservative results will be obtained. In other words, the storey-level seismic forces are not perfect correlated in nature, since the combinations of dead load, live load and wind loads have been considered when calculating the characteristic value of total seismic actions, thus it lead to the partial correlation between storey-level seismic forces.

4.4. Seismic Reliability of Global Load-Carrying Capacity of the Structure

We herein only analyze the seismic reliability of structures under major earthquakes by means of first order reliability method (FORM) based on the established system-level limit state. The first two moments of the total horizontal seismic action according to base shear method and Eqs. (2.7) and (2.8) are listed in Table 4.3.

Table 4.3 Statistics of total horizontal seismic action

Characteristic value (KN)	Mean value (KN)	Std (KN)	COV
631.224	473.418	236.709	0.5

By using the semi-analytical method proposed in this paper, the changing rules of the reliability index of global load-carrying capacity of the structure with the COV of total seismic actions and the correlation coefficient of storey-level seismic forces, as shown in Tables 4.4 and 4.5 as well as Figures 4 and 5, are obtained. To investigate the accuracy and efficiency of the proposed method, Monte Carlo simulations are also conducted with simulation number $N = 10^6$. The results of MCS are also listed in the corresponding tables and figures.

Table 4.4 Results of global seismic reliability index considering the variations of total seismic action

δ_{F_i}	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
β_{FORM}	2.8645	1.9306	1.5394	1.3209	1.1716	1.0667	0.9836	0.9248	0.8537	0.8220
β_{MCS}	2.8699	1.9335	1.5380	1.3216	1.1729	1.0684	0.9874	0.9300	0.8572	0.8280

Table 4.5 Results of global seismic reliability index considering the correlation of storey-level seismic forces

ρ_{F_i}	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
β_{FORM}	0.9032	0.9695	1.0161	1.0492	1.0704	1.0821	1.0838	1.0741	1.0447	0.9772
β_{MCS}	0.9003	0.9706	1.0189	1.0493	1.0710	1.0844	1.0824	1.0756	1.0453	0.9774

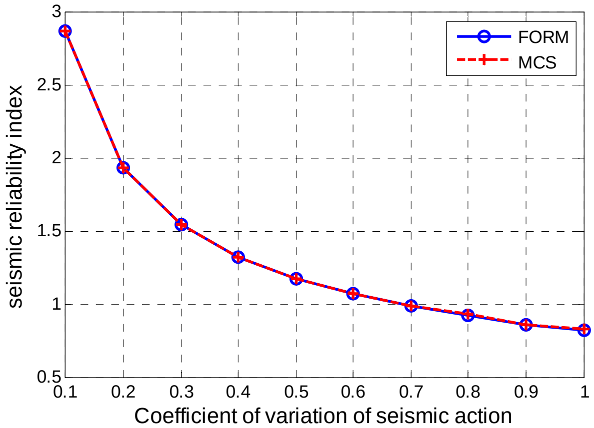


Figure 4 Changing of global seismic reliability index with the COV of seismic action

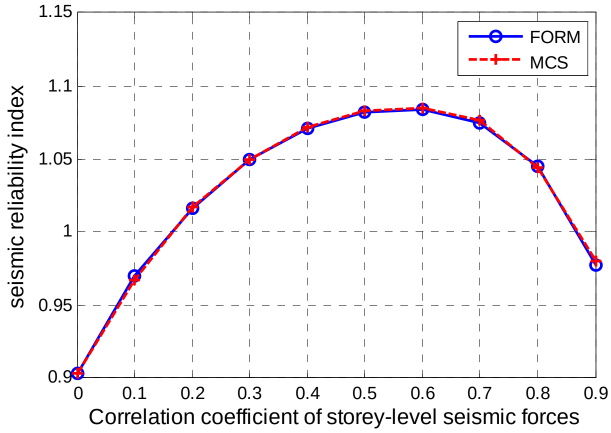


Figure 5 Changing of global seismic reliability index with the correlation coefficient of seismic forces

From Tables 4.4 and 4.5, it is evident that the results by using the method proposed in this paper has nearly the same accuracy as that of MCS, while the number of nonlinear finite element analysis in our method is only 50.
It is shown from Table 4.4 and Figure 4 that the global reliability index tend to become smaller with the

increasing of COV of total seismic action, and that the speed of decreasing becomes more rapidly. This illustrates that the more the randomness in total seismic action is, the smaller the reliability index is, and hence the larger the failure probability is, which makes the structure more unsafe.

It is shown from Table 4.5 and Figure 5 that the global reliability index tends to become larger with the increasing of correlation coefficient of storey-level seismic forces when $\rho < 0.6$. When $\rho = 0.6$, the global reliability index attains the largest value. Otherwise, when $\rho > 0.6$, the global reliability index becomes smaller. This result shows that not considering the correlations between storey-level seismic forces or assuming their perfect correlation will lead to conservative results in any case.

5. CONCLUSIONS

This paper built up a global load-carrying limit state function based on limit base shear, put forward a new semi-analytical method to analyze the nonlinear global seismic reliability of structures, which comprises point estimation method, pushover analysis and FORM. By applying the proposed methodology in reinforced concrete frame buildings, some changing rules of global seismic reliability of the structure with COV of total seismic action and correlation coefficient of storey-level seismic forces were obtained. Through the comprehensive study in this paper, some conclusions are derived as follows:

(1) The method of system reliability analysis based on global load-carrying capacity is simple, practical and efficient. On the one hand, this method can overcome many difficulties of conventional system reliability theory; on the other hand, it can be linked with the current design codes so that the static reliability method can solve the difficult dynamic seismic reliability problems.

(2) Semi-analytical method is a compound approach which combines numerical simulation or integration methods, deterministic finite element analysis and approximate analytical reliability methods such as FORM/SORM. This method is a practical and efficient approach to conduct the seismic reliability analysis of large-scale and complex structures. The practice shows that this method has the same accuracy as MCS.

(3) Random pushover analysis is a good alternative for probabilistic seismic capacity analysis (PSCA) of structures. Meanwhile, it is also an efficient tool of uncertainty propagation structural system.

(4) The variation of total seismic action and the correlation between the storey-level seismic forces have great effect on the limit base shear and the global seismic reliability of structures.

(5) The methodology proposed in this paper can be extended to probabilistic seismic performance assessment and design of structures based on global reliability.

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