Nonlinear Torsional Response of Asymmetric Structures

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ABSTRACT :

Torsional response of structures would be changed while some lateral load resisting members yield. The present article pertains to nonlinear vibration characteristics and load-carrying capacity of asymmetric structures, employing a single-story model. It is found that torsional response of structures is usually more evident while taking nonlinear deformation into account. Ratio of torsional response before and after members yield is influenced by arrangement, shape of structure, and eccentricity. This paper suggests that there would be three torsion collapse modes for asymmetric structures and gives methods to avoid torsion collapse for each case. The research demonstrates that it is important to take nonlinear torsional response into account while controlling torsional distortion of structures. And it is essential for asymmetric structures to maintain lateral stiffness and torsion stiffness in bi-directions uniformly.

KEYWORDS: Torsional response, Nonlinear analyses, Vibration behavior, Torsion collapse mode

1. INTROCTION

It is common that damage of structures is aggravated by torsional response in earthquake. For example, 15 percent of structures destroyed due to torsional response in the 1985 earthquake of Mexico(Esteva 1987)., and in the 2008 earthquake of Wenchuan in Sichuan province of China, many asymmetric structures destroyed heavily because of torsional response. So torsional response plays obvious role to result in the damage of asymmetric structures under earthquake action.

In order to reduce the impact of torsional response, Code for Seismic Design of Buildings of China (GB50011-2001) brings forward some criterions to control torsional distortion of structures. But these criterions do not take nonlinear behavior of lateral load resisting members into account while investigating torsional response, for calculation equation of torsional response in that code are based on elastic behavior of column and wall. Collapse of structures usually contains three successive processes, which are elastic behavior phrase, plastic behavior phrase, and collapse phrase. Torsional response of structures markedly influenced by displacement occurred in plastic behavior phrase. This paper investigates torsional distortion of structures while some lateral load resisting members yield, and studies the collapse mode of structures affected by torsional response. Conclusion can be used as the reference of torsion control for tall buildings.

A single story structure is utilized as study building shown in Figure 1. Distance of rigidity center (point O) and mass center (point C) indicates eccentricity distance, whose denotation is e. Denotation of gyration radius of the structure is r. Denotation of mass is m. And J_r ($J_r=mr^2$) stands for mass moment of inertia. Lateral stiffness of members in y-direction is D1, D2 respectively, and that in x-direction is D3, D4. For the study building, earthquake action is substituted by static load. Lateral load resisting members were considered as ideal nonlinear elements. As shown in Figure 2, displacement ductility is β for all lateral load resisting members.

Response control provisions of Chinese codes were putted forward through investigating on definition $\theta r/u$ which named relative torsion effect, so $\theta r/u$ is used to evaluate torsional response of structures in this paper.

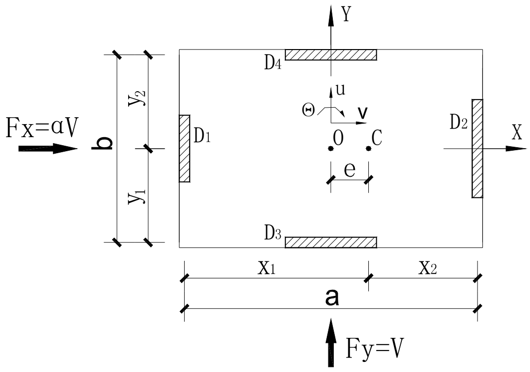


Figure 1 Study building

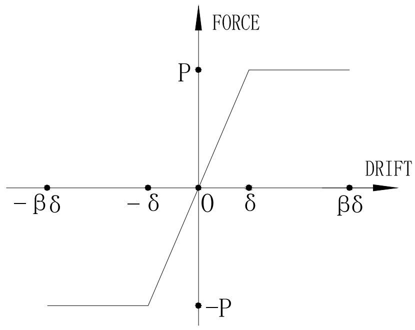


Figure 2 Relation of load and drift for lateral load resisting members

2. TORSIONAL RESPOSE OF STRUCTURES WHILE SOME LATERAL LOAD RESISTING MEMBERS YIELD

As a matter of convenience, it is assumed that earthquake acts only along y-direction(that means $F_x=0$) for study building in this section, so the building vibrates along y-direction (u freedom) and torsion direction (θ freedom) only. At that condition, lateral load resisting member D_1 yields at first for study building under earthquake action. After D_1 yields, increased lateral force in y-direction (named ΔV) is entirely beard by D_2, and increased torsion moment (named ΔM) is entirely beard by D_3, D_4. Only lateral load resisting members in x-direction can make up of internal moment while D_1 yields, so it is important to arrange lateral load resisting members in bi-directions. Neglect damp, eigenfunction of free vibration of study building before and after members yield are given by

$$\begin{bmatrix} K_y & -e\cdot K_y \\ -e\cdot K_y & K_\theta + K_y e^2 \end{bmatrix}\begin{Bmatrix} u \\ \theta \end{Bmatrix} - \lambda \begin{bmatrix} m & 0 \\ 0 & Jr \end{bmatrix}\begin{Bmatrix} u \\ \theta \end{Bmatrix} = 0 \tag{2.1}$$

$$\begin{bmatrix} D_2 & 0 \\ 0 & K_{\theta x} \end{bmatrix}\begin{Bmatrix} \Delta u \\ \Delta\theta \end{Bmatrix} - \lambda \begin{bmatrix} m & 0 \\ 0 & Jr \end{bmatrix}\begin{Bmatrix} \Delta u \\ \Delta\theta \end{Bmatrix} = 0 \tag{2.2}$$

Where

$K_\theta = D_1 x_1^2 + D_2 x_2^2 + D_3 y_1^2 + D_4 y_2^2$;

$K_{\theta x} = D_3 y_1^2 + D_4 y_2^2$;

$K_y = D_1 + D_2$;

λ = eigenvalue of the equations.

Neglect translation and torsion coupled vibration, relation of displacement and force before elements yield can be written as

$$\theta = \frac{Ve}{K_\theta} \tag{2.3}$$

$$u = \frac{V}{D_1 + D_2} + e\cdot\theta = \frac{V}{K_y} + \frac{V\cdot e^2}{K_\theta} \tag{2.4}$$

And relation of displacement and force after elements yield can be written as

$$\Delta V = D_2 \cdot \left[\Delta u - \left(x_2 + e \right) \Delta \theta \right] \tag{2.5}$$

$$\Delta V \cdot \left(x_2 + e \right) = K_{\theta x} \cdot \Delta \theta \tag{2.6}$$

For translation in y-direction and torsional distortion can be gained through above 4 equations, torsional response before and after elements yield can be given. The resulting equations are

$$\frac{\theta r}{u} = \frac{\left(D_1 + D_2 \right) \cdot e \cdot r}{K_\theta + \left(D_1 + D_2 \right) \cdot e^2} \tag{2.7}$$

$$\frac{\Delta \theta \cdot \mathrm{r}}{\Delta u} = \frac{D_2 \left(x_2 + e \right) r}{K_{\theta x} + D_2 \left(x_2 + e \right)^2} \tag{2.8}$$

Actually, vibration of study building is translation and torsion coupled. Translation in y-direction and torsional distortion influences each other. Definition $\theta' r / u'$ indicates torsional response considering translation and torsion coupled vibration. $\theta' r / u'$ can be written as

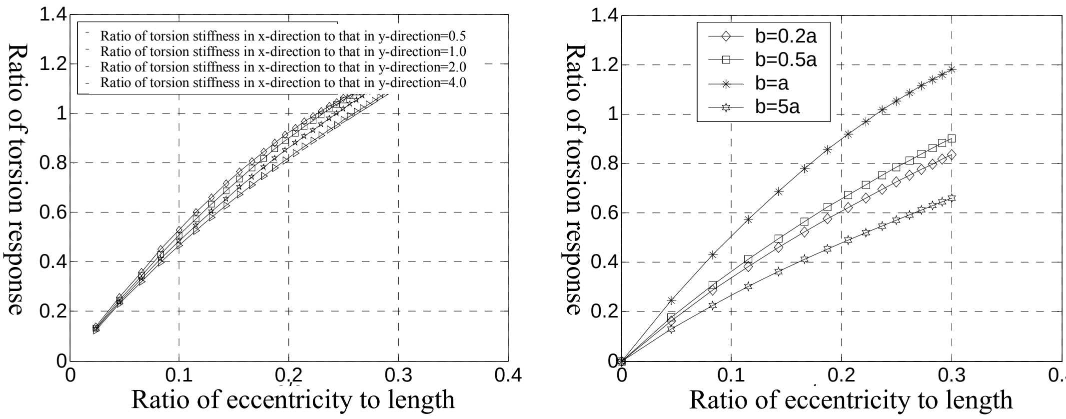
$$\frac{\theta' r}{u'} = \xi \frac{\theta r}{u} \tag{2.9}$$

Where the value of ξ related to e/r and K_y / K_θ. Equation for calculate ξ is too complicated to bring forth in this paper. For mot conditions, $1.0 \le \xi \le 2.0$ (Xu, Huang and Wei 2000). Ratio of torsional response before and after elements yield is given by

$$\varphi = \frac{\dfrac{\theta' r}{u'}}{\dfrac{\Delta \theta \cdot r}{\Delta u}} = \frac{\xi \dfrac{\left(D_1 + D_2 \right) \cdot er}{K_\theta + \left(D_1 + D_2 \right) \cdot e^2}}{\dfrac{D_2 \left(x_2 + e \right) r}{K_{\theta x} + D_2 \left(x_2 + e \right)^2}} = \xi \frac{\left(D_1 + D_2 \right) \cdot e}{D_2 \left(x_2 + e \right)} \cdot \frac{K_{\theta x} + D_2 \left(x_2 + e \right)^2}{K_\theta + \left(D_1 + D_2 \right) \cdot e^2} \tag{2.10}$$

Ratio of torsional response before and after lateral load resisting members yield is shown in Figure3. In most condition, torsional response of asymmetric building increased obviously after some elements yield, especially for short eccentricity distance structure. In addition, figure of the structures also makes an impact on the ratio of torsional response. If the length of a building is equal to the width, torsional response changes little after elements yield.

The change of torsional response after lateral load resisting yield makes a great impact on the collapse mode of structures. In order to control torsion behavior of structures effectively, it is significant to investigate torsional response after elements yield. Certainly, buildings in practice are provided with more lateral load resisting members than study building, and collapse mode of these buildings are not absolutely same with study building.



（a）Ratio of torsion stiffness in x-direction to that in y-direction changed

（b）Aspect ratio changed

Figure 3 Ratio of torsion response before and after some lateral load resisting members yield

3. TORSION COLLAPSE MODE OF ASYMMETRIC BUILDINGS

It is not common that a building makes torsional distortion wholly or collapses entirely by torsional response. For most cases, torsional response plays a role to aggravate damage of structures. There are mainly three torsion collapse modes for asymmetric structures. The first is that a certain element bears too great torsion to be destroyed. The elements generally damaged by the action of moment, shear, and torsion together. The second is that translations of some elements increase to maximal drift and be destroyed before all members yield. The third is that torsion resistance of the whole building or certain floor is little than external torsion. That produces wholly torsional distortion of the building, or makes the whole floor torsion collapse together. Because torsion collapse mode 1 mainly lies on the behavior of elements, so this section primarily study collapse mode 2 and collapse mode 3, which lie on arrangement of structures.

3.1. How to Avoid Torsion Collapse Mode 2

If structures collapse like mode 2, translation of some lateral load resisting members would be greatly larger than others. To avoid torsion collapse mode 2, D_1 should not reach maximal drift before D_2 yields for study building shown in Figure1. So two equations as follows should be satisfied

$$u+(x_1-e)\theta+\Delta u+(x_1-e)\Delta\theta\le\beta\delta \quad (3.1)$$

$$u-(x_2+e)\theta+\Delta u-(x_2+e)\Delta\theta=\delta \quad (3.2)$$

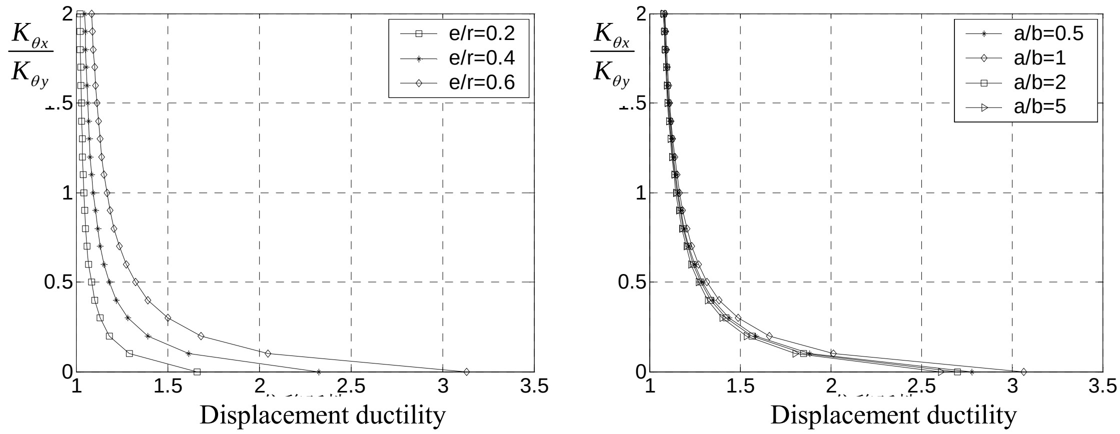
from which

$$(\beta-1)\left(1+\frac{\Delta u}{u}\right)\ge\left[\frac{(x_1-e)+\beta(x_2+e)}{r}\right]\left(\frac{\theta r}{u}+\frac{\Delta\theta\cdot r}{\Delta u}\cdot\frac{\Delta u}{u}\right) \quad (3.3)$$

Study building is symmetric in y-direction. For this structure $x_2+e=x_1-e=a/2$, so simple result can be given, as Eqn.3.4.

$$\left(1-\frac{2}{\beta+1}\right)\left(1+\frac{\Delta u}{u}\right)\geq\frac{a}{2r}\left(\frac{\theta r}{u}+\frac{\Delta\theta\cdot r}{\Delta u}\cdot\frac{\Delta u}{u}\right) \quad (3.4)$$

Combined Eqn.2.3~Eqn.2.8 with Eqn.3.4, the relation of displacement ductility of elements and ratio of torsion stiffness in bi-directions can be gained. The method how to avoid torsion collapse mode 2 is shown in Figure 4.



（a）Eccentricity distance changed （b）Aspect ratio changed

Figure 4 Minimal ratio of torsion stiffness in x-direction to that in y-direction to avoid torsion collapse mode 2

The little ratio of torsion stiffness in x-direction to that in y-direction is, the greater displacement ductility of elements required. Torsion stiffness in x-direction should be great enough to bear the torsion after resisted D_1 yields, for at that phrase elements in y-direction can't resist torsion. If eccentricity distance becomes longer, torsion stiffness in x-direction should increase. Because buildings usually suffer bi-directions earthquake action, torsion stiffness should not too little in any directions. Lateral load resisting members should be arranged on bi-directions uniformly.

3.2. How to Avoid Torsion Collapse Mode 3

If structures collapse like mode 3, it means that the torsion stiffness of whole building or certain floor is little than external torsion. To avoid torsion collapse mode 3, D_3 and D_4 should not yield before D_2 for study building shown in Figure1.In this section, it assumed that study building suffers bi-directions earthquake action and translation in x-direction is v. Earthquake action in x-direction is given by

$$F_x=\alpha\cdot F_y=\alpha\cdot V \qquad \left(0<\alpha\leq 1\right) \quad (3.5)$$

In order to avoid torsion collapse mode 3, two equations as follows should be satisfied

$$v+y_1\theta+y_1\Delta\theta\leq\delta \quad (3.6)$$

$$u-\left(x_2+e\right)\theta+\Delta u-\left(x_2+e\right)\Delta\theta=\delta \quad (3.7)$$

External force of D_3, D_4 are given

$$V_3=D_3\left(v+y_1\theta\right) \quad (3.8)$$

$$V_4 = D_4\left(v - y_2\theta\right) \tag{3.9}$$

For study building, $D_3 \cdot y_1 = D_4 \cdot y_2$ .The translation in x-direction is

$$v = \frac{\alpha \cdot V}{K_x} \tag{3.10}$$

Combined Eqn.3.10 and Eqn.3.6, Eqn.3.6, following equation can be given

$$1 + \frac{\Delta u}{u} - \frac{v}{u} \geq \left(\frac{x_2 + e + y_1}{r}\right)\left(\frac{\theta r}{u} + \frac{\Delta\theta r}{\Delta u}\frac{\Delta u}{u}\right) \tag{3.11}$$

In order to avoid torsion collapse mode 3, ratio of lateral stiffness in x-directions to that in y-direction is should not be too little, especially for the building that aspect ratio is big or eccentricity is great. Figure 5 shows the results.

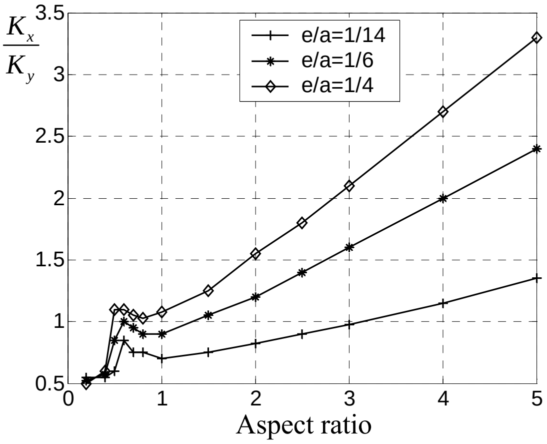


Figure 5 Minimal ratio of lateral stiffness in x-direction to that in y-direction
to avoid torsion collapse mode 3

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4. CONCLUSIONS

Torsional response of structures would be changed while some lateral load resisting members yield. That would play obvious role to result in the damage for some asymmetric structures, and would exert a clear influence on the collapse mode of structures.

Torsional response of structures varies while some lateral load resisting members yield. For most conditions, torsional response would increase and torsional distortion would be evident. Generally torsional response increases more obvisiously for little eccentricity structures.

There are mainly three torsion collapse modes for asymmetric structures. In order to avoid torsion collapse of structures, it is essential to arrange lateral load resisting members in bi-directions uniformly. Not only lateral stiffness but also torsion stiffness should not make great difference, especially for structures that eccentricity is great or aspect ratio is big.

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