MULTICRITERIA METHOD FOR AN OBJETIVE SELECTION OF TIME-FREQUENCY REPRESENTATIONS OF SIGNAL FROM CIVIL ENGINEERING STRUCTURES

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ABSTRACT :

The main part of this paper is devoted to propose a new multicriteria method for an objective selection scheme of Time-Frequency Representations (TFRs) of signals from Earthquake Engineering and Civil Engineering (EE-CE) applications. The proposed method, consider four criteria, the first two are specifically oriented to EE-CE signals, while the last two has been proposed previously in the literature by other authors for general applications. An evaluation of the proposed multicriteria method to select a TFR from several Linear and Quadratic TFRs is shown using synthetic ambient vibration signals and real strong motion records.

KEYWORDS: Time-Frequency, Structural Damage Detection, Signal Analysis, Ambient Vibration, Strong Motion

1 INTRODUCTION

It is well known that traditional signal analysis (like standard Fourier based analysis) is unable to analyze signals when frequency is changing on time, such as signals from a structure submitted to a strong motion event (Cano and Martinez, 2007). For this reason many techniques has been developed to deal with this kind of signals, the most recent of them includes time-frequency analysis, Hilbert Huang Transform HHT (Huang. et. al. 1998), and time-scale techniques (wavelets).

Disregarding the selected main type of analysis (TFR, HHT, or Wavelets), it is necessary to select a subtype of analysis tool to be use. The amount of different TFRs, HHT variants, and wavelets actually allowed is enormous, and the selection process is not a straightforward procedure. In general, selection of specific TFR has been based on visual criteria or trial-error parameters adjusted to some specific TFR (Bonato et. al. 1997a,b). In Civil Engineering applications, no work has been yet reported on systematic and objective selection of TFRs. This article proposed a systematic and objective method in order to answer the question of what type of TFR have the best performance for a given specific EE-CE signal.

2 PROPOSED MULTICRITERIA METHOD

It is proposed a weighting multicriteria measure to select the best performance TFR for structural civil engineering applications. The method uses two specific structural oriented criterias, and two general criterias proposed by others. The Structural Multicriteria Quality factor can be evaluated using:

$$SMQ = \sum_{i=1}^{n} W_i EF_i \;; \quad 0 < SMQ \le 1 \;; \quad \sum_{i=1}^{n} W_i = 1 \;; \quad W_i \ge 0 \;; \quad EF_i \le 1 \tag{2.1}$$

Where:
SMQ: Structural Multicriteria Quality factor for the TFR, W_i: Weighting factor, EF_i: Evaluation Factor.
The Evaluation Factors (*EFi*) are: Desiderable Mathematic Properties (*DMP*), Performance in typical signal analysis from structures, namely here Structural Performace Indicator (*SPI*), Resolution and Concentration Measure (*RCM*) (Boashash and Sucic, 2003), and Information Measure (*IM*) (Sang and Williams, 1995).

2.1 Weighting Factors (W_i)

To avoid a subjective decision, (W_i) will be taken 0.25 for all factors, this mean a 25% of importance of the factor in the selection procedure of TFRs.

2.2 Evaluation Factors (EFi)

2.2.1 Desiderable Mathematic Properties

Although proposed methods for general application have many strengths, none of them have taken into account the desiderable mathematics properties that in an ideal TFR should be stablished (Gröchening 2001, Cohen 1989, Hlawatsch and Boudreaux-Bartels 1992). To solve this problem it is propose to include a quality factor for the TFRs selected according to the desiderable mathematic properties that it fulfills (Table 2.1), and that are important from a structural point of view. In table 2.1 $x(t)$ is the analytical signal, and $P_x(t,f)$ is a general TFR.

Table 2.1 Mathematic Properties and recommended weighting factors.

Mathematic Property that TFR fulfill	Simplify Expression	Quality factor
Linearity (P_1)	$x(t)=\left(k_i x_i(t)\right) \Rightarrow P_x(t,f)=\sum_{i=1}^{n}\left(k_i P_{x_i}(t,f)\right)$	0.1-0.3
Multiplication (P_2)	$x(t)=x_1(t)x_2(t) \Rightarrow P_x(t,f)=\int_{-\infty}^{+\infty} P_{x_1}(t,f-f')P_{x_2}(t,f')df'$	0.05-0.2
Convolution (P_3)	$x(t)=\int_{-\infty}^{\infty} x_1(t-t')x_2(t')dt' \Rightarrow P_x(t,f)=\int_{-\infty}^{+\infty} P_{x_1}(t-t',f)P_{x_2}(t',f)dt'$	0.1-0.3
Real Valued (P_4)	$P_x^*(t,f)=P_x(t,f)$	0.1-0.3
Time-Shift (P_5)	$x_1(t)=x(t-t_0) \Rightarrow P_{x_1}(t,f)=P_x(t-t_0,f)$	0.1-0.2
Frequency-Shift (P_6)	$x_1(t)=x(t)e^{j2\pi f_0 t} \Rightarrow P_{x_1}(t,f)=P_x(t,f-f_0)$	0.1-0.2
Time-Marginal (P_7)	$\int_{-\infty}^{+\infty} P_x(t,f)df=\lvert x(t)\rvert^2$	0.1-0.3
Frequency Marginal (P_8)	$\int_{-\infty}^{+\infty} P_x(t,f)dt=\lvert X(f)\rvert^2$	0.1-0.3
Instantaneous Frequency (P_9)	$IF_x(t)=f_x(t)=\dfrac{\int_{-\infty}^{+\infty} fP_x(t,f)df}{\int_{-\infty}^{+\infty} P_x(t,f)df}$	0.1-0.35
Group Delay (P_{10})	$GD_x(f)=t_x(f)=\dfrac{\int_{-\infty}^{+\infty} tP_x(t,f)dt}{\int_{-\infty}^{+\infty} P_x(t,f)dt}$	0.1-0.2
		$\sum_{i=1}^{10}\mathcal{Q}_i=1.0$, $\mathcal{Q}_i\geq 0$

	Mathematic Property										
TFR	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	$\sum_{i=1}^{10}$
Born-Jordan	×	×	×	√	√	√	√	√	√	√	0.7
Zhao-Atlas-Marks	×	×	×	√	√	√	√	√	√	√	0.7
Choi-Williams	×	×	×	√	√	√	√	√	√	√	0.7
Margenau-Hill	√	×	√	×	√	√	√	√	×	×	0.6
Rihaczek	√	×	√	×	√	√	√	√	×	×	0.6
Levin	×	√	×	√	√	√	√	√	×	×	0.6
Wigner-Ville Distribution	×	√	√	√	√	√	√	√	√	√	0.9
Pseudo-Wigner-Ville	×	√	√	√	√	√	×	×	√	√	0.7
Spectrogram	×	×	×	√	√	√	×	×	×	×	0.3
sinc	×	√	×	√	√	√	√	√	×	×	0.6

2.2.2 Structural Performance Indicator (SPI)

The structural response of civil engineering structures has some particular aspect that important to have into account. To mention some of them:

-Frequency Bandwidth: In common structures the frequency range of interest is between 0.05 and 30 Hz.

-Signal Duration: In operation conditions (ambient vibration) the signal is in general very long, from hours to months and some cases years, like in long-time structural health monitoring projects. In strong events the duration of the signal varies from few milliseconds like in blast and explosions, to several hours like in high-wind speed events.

-Frequency Laws: In the linear response the signal frequency might varies slightly, but in non-linear response regime the signal frequency is clearly time dependent (Cano 2008).

In order to have into account the specific-signal characteristics, the structural performance indicator evaluate the behavior of each TFR from two type of signals: Ambient Vibration and Strong Events.

Therefore, the Structural Performance Indicator of a TFR is obtained using:

$$SPI_{TFR} = 1 - \left(c_1 * AVP_{TFR} + c_2 * SEP_{TFR}\right) \tag{2.2}$$

Where: AVP_{TFR} **:** Ambient Vibration Performance of TFR, SEP_{TFR}**:** Strong Event Performance of TFR, c_1 , c_2**:** Weighting factors for *AVP* and *SEP*, suggested values c_1=0.3 , c_2=0.7.

Usually, in ambient vibration conditions the frequency change are becomes very smooth, the Signal to Noise Ratio (*SNR*) is low, and the duration is long. In this context, the best-qualified TFR is obtained from those that better identified the structural frequencies in high-noise signals. This procedure is done using only the principal values of the time-frequency map.

To evaluate the ambient vibration performance (*AVP*) of one particular TFR, a multicomponent synthetical signal containing *k* number of frequencies is selected using:

$$x(t) = \sum_{i=1}^{k} \left(A_i \cos\left(2\pi f_i t\right)\right) + n(t) \tag{2.3}$$

Where:

A_i: Constant amplitude to frequency f_i, f_i **:** Frequency (Hz), *n(t)*: Gaussian noise added, *k*: Number of the frequencies in the signal. The constant amplitude (A_i) is controlled to have an *SNR*<<<1.

The *AVP* qualification procedure begins by applying it to a sort of time-frequency map and selecting the first k^{th} maximum values in the time-frequency.

Because the theoretical frequency values are constant in time, a time qualify factor for each k^{th} frequency value can be evaluated using:

$$f_{TFRke}(t) = \frac{\left|f_T - f_{TFR}(t)\right|}{f_T} \tag{2.4}$$

Where f_{TFRke}(t)**:** Instantaneous frequency error in decimal form, f_T**:** Theoretical frequency (Hz), and f_{TFR}**:** Frequency value for time instant *'t'* obtained from TFR map (Hz).

The above frequency error values could varies a lot, being sometime close to zero or sometime getting high values. For this reason an average frequency error is obtained for each frequency. These average frequency errors are added up and divided by the number of frequencies (*k*). The TFR with the minimum *AVP* is considered as the one with the best performance.

$$f_{i_{mAVTFRe}} = \frac{\sum_{j=1}^{N} f_{TFRke}\left(t_j\right)}{N} \quad ; \quad AVP_{TFR} = \overline{f}_{TFR_{AVe}} = \frac{\sum_{i=1}^{k} f_{i_{mAVTFRe}}}{k} \tag{2.5}$$

Where: $f_{imAVTFRke}$ **:** Mean Ambient Vibration instantaneous frequency error (in decimal form) for f_i , *N:*Number of signal samplings.

The second part of the Structural Performance Indicator is the assessment of the TFR performance with respect to the strong event. It is called: Strong Event Performance (*SEP*) of TFR.

A data base of signals recollected from structures subjected to strong motion was developed. The signals are classified according to the pattern frequency laws obtained of its TFRs (Cano 2008): (1) Linear response (without frequency changes), (2) Linear Response - linear stiffness degradation, (3)Linear Response - exponential stiffness degradation, (4) Linear Response, instantaneous stiffness loss - linear stiffness degradation, (5) Linear Response, instantaneous stiffness loss - exponential stiffness degradation, (6) Linear Response - exponential stiffness degradation - sudden instantaneous stiffness loss - exponential stiffness degradation or linear degradation, (7) Any precedent behavior and some frequency recovery to end of signal.

For modeling these types of signals the piecewise monocomponent signals are selected. In early works Cohen 1994 and Cohen et. al. 2002, defined some signal with similar characteristic. Using the same notation in Cohen's article the following general form can be defined:

$$x(t)=\left\{\begin{matrix} A(t)\cos\left(2\pi f_0 t\right) & if & 0\le t<t_1 & ; & & A(t)=A_0 e^{\frac{a_1 t}{2}} \\ A(t)\cos\left(2\pi f_0 e^{-b_1 t}\right) & if & t_1\le t<t_2 & ; & A(t)\text{ with} & p\left(A(t)\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{A(t)} e^{\frac{-\xi^2}{2}}d\xi \\ A(t)\cos\left(2\pi f_0 e^{-b_2 t}\right) & if & t_2\le t & ; & & A(t)=A_0 e^{-\frac{a_2 t}{2}} \end{matrix}\right\}+n(t) \tag{2.6}$$

Where: *x(t):* Time history (Acceleration, velocity or displacement), *A(t)*: Amplitude (i.e. time function), f_0**:** Initial structural frequency (Hz), a_i , b_i :Parameters greater than zero to control the time variable rate of amplitude or frequency, respectively, t_i :Time instant when a change in amplitude and/or frequency begin or end, *n(t)*: Gaussian noise added.

Therefore by changing the parameters of Eqn. 2.6, many signals with desiderables like-look structural response can be founded. Two typical examples of aforementioned structural signals are shown in figure 2.1.

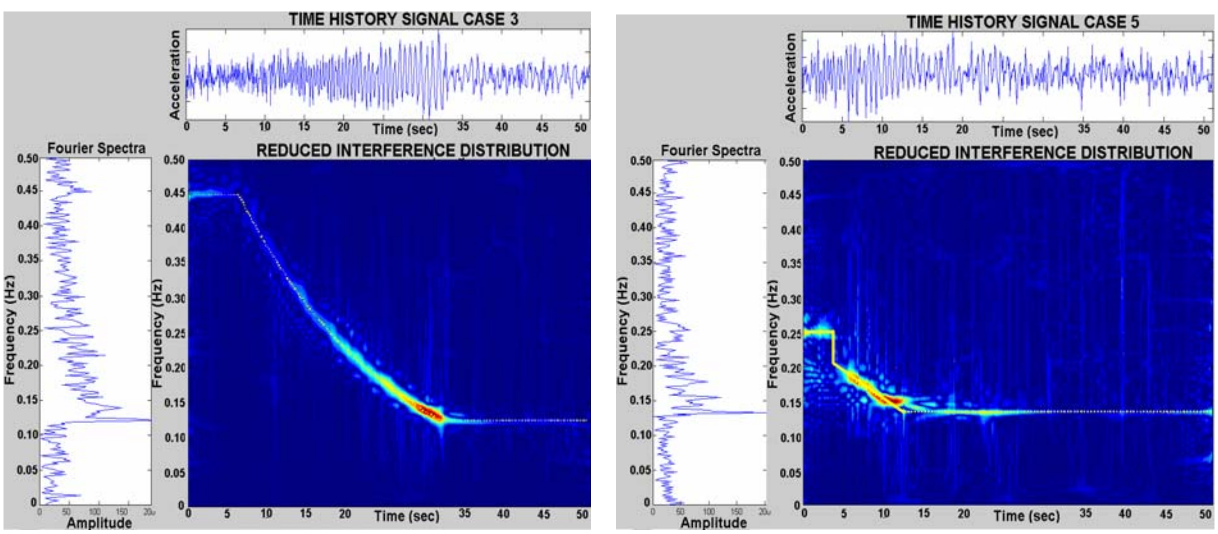

Figure 2-1 Reduced Interference Distribution of Typical Structures (Left: Signal # 3 Slowing Stiffness Decrease as flexural damage, Right: Signal #5, Sudden Stiffness Decrease as shear damage)

According to the Eqn. 2.6, several general syntectic signals can be developed . These signals can be used to test the Strong Event Performance (*SEP*) of TFRs, the basic criteria is evaluated for each specific signal using:

$$f_e(t)=\frac{\left|f_T(t)-f_{TFR}(t)\right|}{f_T(t)} \quad ; \quad f_{i_{mTFRSEe}}=\frac{\sum_{j=1}^{N} f_e\left(t_j\right)}{N} \quad ; \quad SEP_{TFR}=\overline{f}_{TFRSE_e}=\frac{\sum_{i=1}^{k} f_{i_{mTFRSEe}}}{k} \tag{2.7}$$

Where $f_e(t)$: Instantaneous frequency error, $f_T(t)$ Theoretical frequency obtained from the instantaneous frequency law (Eqn. 2-6, Thus time derivative of the theoretical phase), $f_{TFR}(t)$**:** Frequency value obtained from TFR map, f_{imTFRe} **:** Mean instantaneous frequency error (*MIFE*) for a signal type of Strong Event in decimal form. *N***:** Number of signal samplings, *k*: Number of analyzed signals.

2.2.3 Resolution and Concetration Measure (RCM)

The resolution and concentration method proposed by Boashash and Sucic 2003 is incorporated in the suggested multicriteria measure. The theoretical background of the method and practical applications has been presented by the authors in several technical publications. Therefore, only the general equation necessary to obtained the resolution performance measure is here presented.

From Boashash and Sucic 2003, the Resolution Performance Measure can be evaluate using:

$$P_i(t) = 1 - \frac{1}{3}\left(\frac{A_s(t)}{A_M(t)} + \frac{1}{2}\frac{A_X(t)}{A_M(t)} + (1 - D(t))\right) \qquad RCM = P_{TFR_{overall}} = \frac{\sum_{i=1}^{N} P_i(t_i)}{N} \tag{2.8}$$

$$0 < P_i(t) < 1$$

Where: $P_i(t)$: Resolution Performance Measure, $A_s(t)$, $A_x(t)$: Are the magnitudes of sidelobe of auto-term and cross-term, respectively, $A_M(t)$: Magnitude of the mainlobe, $D(t)$: Component separation measure.

Therefore, in order to apply the procedure, the $P_i(t)$ is evaluated "*N*" time instants. Then the overall P_{TFR} is evaluated for each TFR. This value is taken as the Resolution and Concentration Measure (*RCM*) (see Eqn.2.8):

2.2.4 Information Measure (IM)

It has been mentioned that visual inspection is a popular method for selecting a TFR. There exist a strong link between the Renyi entropy measure and the visually based notion of the TFR complexity (Flandrin et. al. 1994). Thus it is possible to have an objective selection of TFR using the Renyi entropy measure, which relates the complexity content of the TFR. The third order Renyi information measure, proposed by Sang and Williams 1995, will be use as another useful criteria.

For discrete cases, the Renyi third order entropy measure (R_3) defined by Sang and Williams 1995 is adopted:

$$R_3(P_x(t,f)) = \frac{1}{2}\log_2\left(\sum_{l=-L}^{L}\sum_{k=-K}^{K}(P_{Nx}(l,k))^3\right); \quad IM(P_x(t,f)) = 1 - R_3(P_x(t,f)) \tag{2.9}$$

3 APPLICATION OF MULTICRITERIA METHOD

In this section an example of a real case of the multicriteria method, developed to select a best-performance TFR, is shown.

The used signal is the East-West component of the acceleration record obtained on the roof of the Robert Millikan Library of the California Institute of Technology, during the San Fernando Earthquake of February 9, 1971, with M_L=6.4, focal depth: 13 Km, epicentral distance = 30 Km., Peak Acceleration: 0.35g (roof). This earthquake produced some structural damage in the Library.

The best-performance TFR for this signal has to be selected for example, among the following fixed kernel TFRs: Spectrogram, Wigner-Ville Distribution (WVD), Smoothed Pseudo-Wigner-Ville Distribution (PSWVD), Choi-Williams Distribution (CWD), Margenau-Hill (MH), and Reduced Interference Distribution (RID).

First of all the weighting factors (W_i) are all set equal to 0.25. Then the qualification of the Desiderable Mathematic Properties (*DMP*) can be evaluated according to section 2.2.1. The results are presented in the Table 3.1:

Table 3.1 Evaluation of qualification for Desiderable Mathematic Properties (*DMP*)

	Mathematic Property										
TFR	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	**DMP**
Spectrogram	×	×	×	√	√	√	×	×	×	×	0.3
WVD	×	√	√	√	√	√	√	√	√	√	0.9
SPWVD	×	√	√	√	√	√	×	×	√	√	0.7
CWD	×	×	×	√	√	√	√	√	√	√	0.7
MH	√	×	√	×	√	√	√	√	×	×	0.6
RID	×	√	√	√	√	√	√	√	√	√	0.9

The next step is the evaluation of the Structural Performance Indicator (*SPI*). First, for ambient vibration conditions the global mean error in the detection of "*k*" frequencies from a severe noisy signal is calculated. These signals are multicomponent and frequency modulated and are obtained from Eqn. 2.3. For this particular case, the following parameters are selected: Number of frequencies k=5, Sampling rate = 10 sps, Number of samplings (N) : 1024, t=102.3 sec, Structural frequencies: f_i = 0.1 Hz, 0.3 Hz, 0.5 Hz, 1.0 Hz, and 2 Hz, A_i: Equal to all frequencies, $x(t)$ max = +/- 2 gals (2 cm/sec^2=0.002g), $n(t)$= Gaussian added noise, with a *SNR* maximum of 1/20.

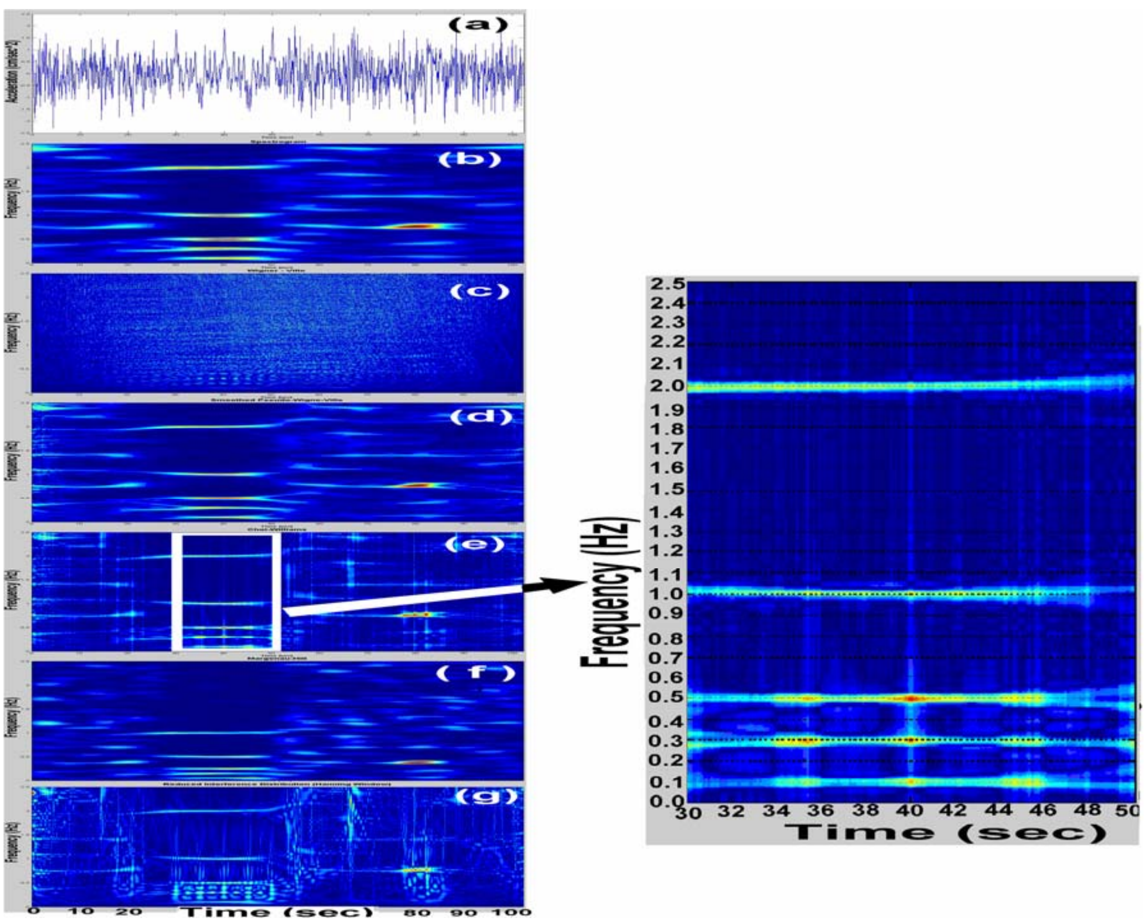

Figure 3-1 TFR for Ambient Vibration (a) Time History, (b) Spectrogram, (c) WVD (d) PSWVD, (e) CWD, (f) MH, (g) RID. Right side: Zoom in CWD.

The results of applying the Eqn. 2.5 to the TFRs of Figure 3-1 is shown in the Table 3.2:

Table 3.2 Ambient Vibration Instantaneous Frequency Error and Ambient Vibration Performance (*AVP*)

TFR	$f_1=0.1$	$f_{1_{mAVTFRe}}$ (%)	$f_2=0.3$	$f_{2_{mAVTFRe}}$ (%)	$f_3=0.5$	$f_{3_{mAVTFRe}}$ (%)	$f_4=1.0$	$f_{4_{mAVTFRe}}$ (%)	$f_5=2.0$	$f_{5_{mAVTFRe}}$ (%)	$AVP_{TFR}=\bar{f}_{TFRAV_e}$
Spectrogram	0.13	34.10	0.21	29.60	0.61	21.38	1.32	32.18	2.06	2.79	24.0
WVD	0.09	14.90	0.28	6.43	0.49	2.28	1.23	22.52	2.09	4.27	10.1
SPSWVD	0.11	12.30	0.23	24.13	0.44	11.88	0.95	5.43	1.94	3.12	11.4
CWD	0.12	22.10	0.19	35.70	0.44	11.18	0.94	5.55	1.94	3.03	15.5
MH	0.09	12.10	0.35	16.03	0.57	13.92	0.99	0.94	1.55	22.74	13.1
RID	0.12	17.40	0.28	6.37	0.56	12.12	1.01	0.68	1.92	3.86	8.1

The next step is the assessment of the TFR performance to deal with strong events (*SEP*):

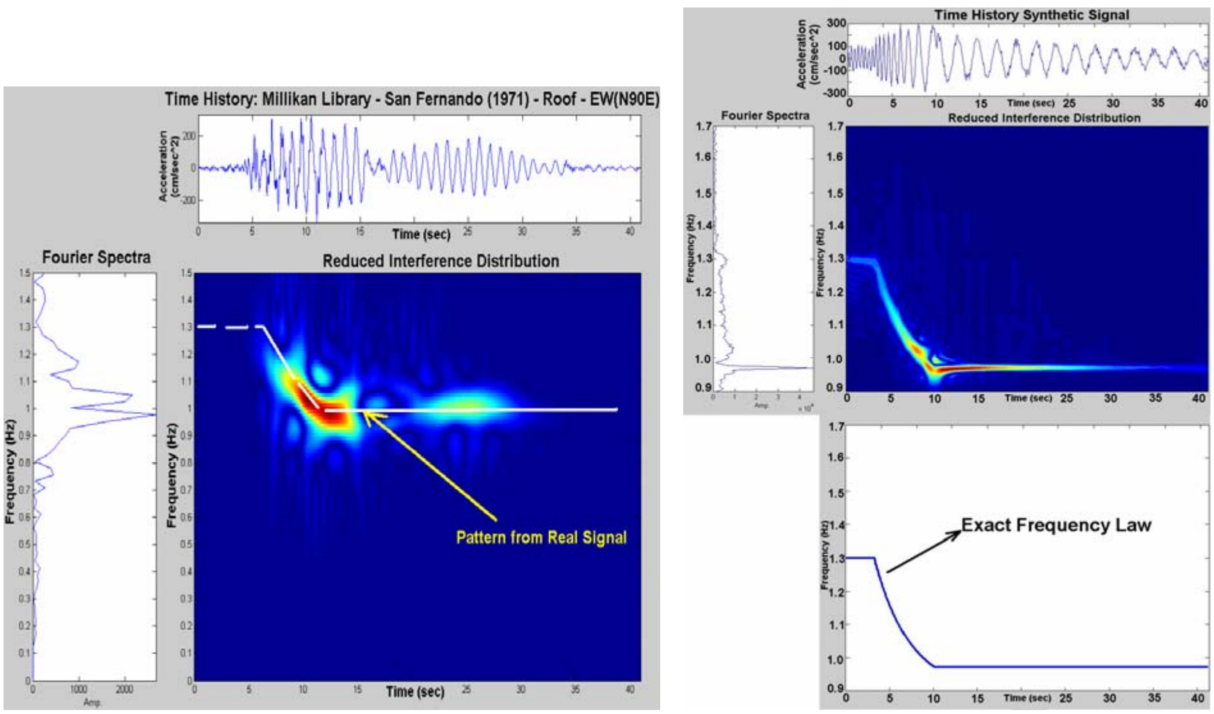

Figure 3-2 Left side: TFRs of San Fernando Earthquake (1971) recorded in Millikan Library (EW-Roof Component), Right side: TFR of Syntethic Signal and its theoretical frequency law to this record

A TFR of the San Fernando Earthquake is shown in the Figure 3-2. It can be noted that the pattern of the time-frequency plane look like the typical pattern #3, shown previously in the of section 2.2.2., (linear - exponential decay and linear with in permanent frequency shift). The synthetic signal from the select pattern is shown at the top of the right side of the Figure 3-2 followed by its TFR and its exact frequency law.

In order to acquire these selected patterns, the following values are set into the Eqn. 2.6:

$$x(t)=200*\left[\begin{cases} A(t)\cos\left(2\pi*1.3t\right) & if \quad 0\le t<40 \quad ; \qquad A(t)=0.5*e^{\frac{0.001t}{2}} \\ A(t)\cos\left(2\pi*1.3e^{-0.02t}\right) & if \quad 40\le t<128 \quad ; \quad A(t)\text{ with } \quad p\left(A(t)\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{A(t)} e^{\frac{-\xi^2}{2}}d\xi \\ A(t)\cos\left(2\pi*1.3e^{-0.006t}\right) & if \quad 128\le t \quad ; \qquad A(t)=e^{-\frac{0.001t}{2}} \end{cases} + n(t)\right]$$

The ability of any TFR to adjust or track instant by instant this type of frequency law can be evaluated with its theoretical frequency law. The pattern could be associated to a typical mechanism of structural damage like flexural or shear damage. The *MIFE* and the Structural Performance Indicator (*SPI*) are shown in the Table 3.3:

Table 3.3 MIFE and Structural Performance Indicator (*SPI*)

TFR	$f_{i_{m\,TFRSBe}} = \frac{\sum_{i=1}^{N} f_e(t)}{N}$ (%)
Spectrogram	4.8
WVD	4.7
SPSWVD	2.9
CWD	2.6
MH	3.5
RID	2.1

TFR	$SPI_{TFR} = 1-(c_1 * AVP_{TFR} + c_2 * SEP_{TFR})$
Spectrogram	0.8947
WVD	0.9369
SPWVD	0.9455
CWD	0.9351
MH	0.9358
RID	0.9607

The next step is the evaluation of the *RCM* criteria according to Boashash and Sucic 2003 and the Information Measure (*IM*) according to Sang and Williams 1995. Finally the Structural Multicriteria Quality factor (*SMQ*) can be evaluate using the Eqn. 2.1:

Table 3.4 Structural Multicriteria Quality Factor (*SMQ*)

TFR	Desiderable Mathematic Properties DMP	Structural Performance Indicator SPI	Resolution and Concentration RCM	Information Measure IM	Structural Multicriteria Quality Factor SMQ
Spectrogram	0.3000	0.8947	0.7608	0.8658	0.7053
WVD	0.9000	0.9369	0.7115	0.8701	**0.8546**
SPWVD	0.7000	0.9455	0.8011	0.8618	0.8271
CWD	0.7000	0.9351	0.7457	0.8619	0.8107
MH	0.6000	0.9358	0.8363	0.8649	0.8092
RID	0.9000	0.9607	0.7707	0.8627	**0.8735**

From the Table 3.4. the best perform TFR of the selected TFRs with fixed kernel is the Reduced Interference Distribution, the traditional Wigner-Ville Distribution closely follows.

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