# DYNAMIC ANALYSIS OF CONCRETE RECTANGULAR LIQUID STORAGE TANKS

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**ABSTRACT :**
A structural model using the generalized single degree of freedom (SDF) system is proposed for seismic design of concrete rectangular Liquid Containing Structures (LCS). The proposed model considers the effect of flexibility of tank wall on hydrodynamic pressures and uses the consistent mass approach. The proposed model is compared with the results obtained using the current practice as well as the finite element method. It is concluded that the current approach in design codes and standards does not truly represent the behavior of LCS. The proposed model using the generalized SDF system can be simply used in seismic design of LCS.

**KEYWORDS:**
Reinforced concrete, liquid containing, rectangular tank, seismic, dynamic analysis

## 1. INTRODUCTION

Liquid containing structures (LCS) as part of environmental engineering facilities are primarily used for water and sewage treatment plants and other industrial wastes. Normally, they are constructed of reinforced concrete in the form of rectangular or circular configurations. Currently there are few codes and standards available for seismic design of LCS in North America. In almost all of codes and standards, the Housner's model (Housner, 1963) has been adopted for dynamic analysis of LCS. This model approximates the effect of hydrodynamic pressure for a two fold-symmetric-fluid container subjected to horizontal acceleration as shown in Figure 1(a). The hydrodynamic pressures induced by earthquakes are separated into two parts of impulsive and convective components which are approximated by the lumped added masses. The added mass in terms of impulsive pressure is assumed rigidly connected to the tank wall and the added mass in terms of convective pressure is assumed connected to the tank wall using flexible springs to simulate the effect of sloshing motion. In this model, the boundary condition in the calculation of hydrodynamic pressures is treated as rigid.

Although the Housner's model has been applied in the seismic design of LCS in the past, recent studies show that due to the assumption of the lumped added mass and the rigid tank wall, this method leads to overly conservative results. Chen and Kianoush (2005) developed a procedure referred to as the sequential method for computing hydrodynamic pressures based on a two-dimensional model for rectangular tanks in which the effect of flexibility of tank wall was taken into consideration. Later Kianoush et al. (2006) and Ghaemian et al. (2005) applied the staggered method to solve the coupled liquid storage tank problems in three-dimensional space. Compared to the Housner's model, these results show that in most cases the lumped mass approach overestimates the base shear and base moment significantly.

Chen and Kinaoush (2007) proposed a generalized single degree of freedom (SDF) system for dynamic analysis of LCS. The consistent mass approach and the effect of flexibility of tank wall on hydrodynamic pressures were considered. The prescribed vibration shape functions representing the mode shapes for the cantilever wall boundary condition were validated.

In this paper, the proposed structural model using the generalized SDF system is compared with the Housner's Model adopted in the current design codes and standards. The design charts for the added mass of liquid due to impulsive hydrodynamic pressure and the corresponding effective height are presented. The contribution of higher modes to the dynamic response of LCS is included in the proposed model. The square root of sum of square (SRSS) method is used for the combination of the first two modes. A case study representing a tall tank is presented. The results are compared with those obtained using the Housner's model as well as the finite element method. It is recommended that the current design approach need to be modified. The proposed structural model using the generalized SDF system can be considered as simple model to overcome the current deficiencies in design of LCS.

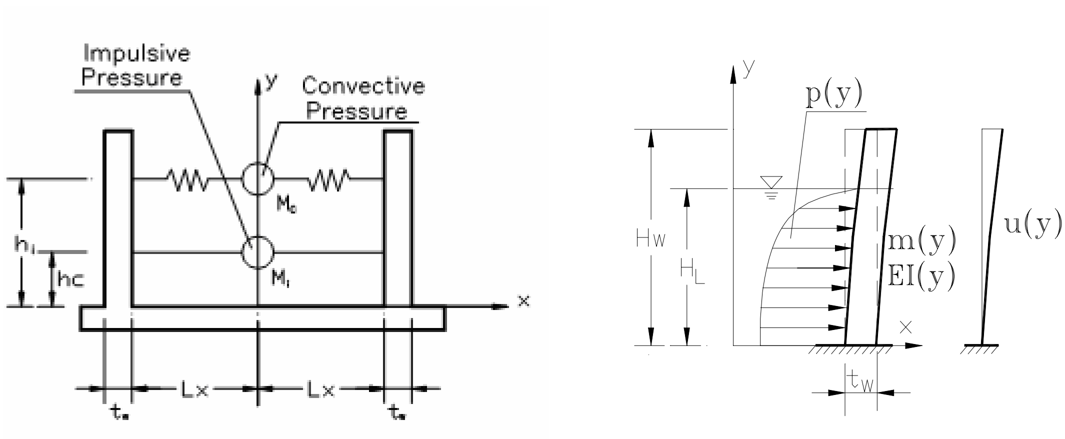
## 2. GENERALIZED SDF SYSTEM FOR DYNAMIC ANALYSIS OF LCS

### *2.1 Analysis Model and Equation of Motion*

The Housner's model (1963) is shown in Figure 1(a). Figure 1(b) shows a cantilever tank wall with the distributed mass m(y) and stiffness EI(y) per unit height subjected to the earthquake ground acceleration $\ddot{u}_g(t)$. The wall exhibits an infinite number of degrees of freedom for flexural mode of response. If there are some predetermined shapes to approximate the vibration of the system, then the motion of the system can be described by a single variable, or generalized coordinate in which only one DOF exists. The system idealized in this manner is referred to as generalized SDF systems. In this study, the generalized SDF system is applied to solve the dynamic response of liquid storage tanks subjected to earthquakes. The equation of motion for a generalized SDF system is that:

$$\tilde{m}\cdot\ddot{u}+\tilde{c}\cdot\dot{u}+\tilde{k}\cdot u=\tilde{p} \tag{2.1}$$

Where $\tilde{m}$, $\tilde{c}$, $\tilde{k}$, $\tilde{p}$ are defined as the generalized system of mass, damping, stiffness and force respectively.




(a) Housner's Model (b) Generalized SDF System

Figure 1 Analysis Model

For simplicity, the prescribed vibration shape function SF1 representing the first mode shape for the cantilever wall boundary condition can be used in dynamic analysis as follows:

$$\mathrm{SF1(y)}=\psi(y)=\frac{3}{2}\frac{y^2}{H_W^2}-\frac{1}{2}\frac{y^3}{H_W^3} \tag{2.2}$$

The validity of the shape function SF1 was verified and discussed in the previous study (Chen and Kianoush, 2007).

The direct coupling method is used in the dynamic analysis. The interaction between liquid and tank wall is solved directly in the equation of motion using the added mass method.

### *2.2 Added Mass of Liquid*

The hydrodynamic pressure can be solved using the separation of variables method which satisfies the boundary conditions. The hydrodynamic pressure distribution on the flexible wall condition can be expressed as follows:

$$p = \sum_{i=1}^{\infty} \frac{2 \cdot \rho_l \cdot \tanh(\lambda_i \cdot L_x)}{\lambda_i \cdot H_L} \cdot \cos(\lambda_i \cdot y) \cdot \int_0^{H_L} \cos(\lambda_i \cdot y) \cdot \ddot{u}(t) dy \tag{2.3}$$

Where $\lambda_i = (2i-1)\pi/2H_L$. As the series in the above equation convergence very fast, only the first three terms of the series are used for practical applications.

When using the generalized SDF system in the dynamic analysis of LCS, the hydrodynamic pressure is incorporated into the coupling analysis through the added mass. The generalized and effective added mass of liquid due to impulsive hydrodynamic pressure, $\widetilde{m}_L$ and $m_L$, can be calculated using Eqns. 2.4 and 2.5 respectively.

$$\widetilde{m}_L = \sum_{i=1}^{\infty} \frac{2 \cdot \rho_l}{\lambda_{i,n} \cdot H_L} \tanh(\lambda_{i,n} L_x) [\int_0^{H_L} \cos(\lambda_{i,n} y) \cdot \psi(y) dy]^2 \tag{2.4}$$

$$m_L = \sum_{i=1}^{\infty} \frac{2 \cdot (-1)^{i+1} \rho_l}{\lambda_{i,n}{}^2 \cdot H_L} \tanh(\lambda_{i,n} L_x) \int_0^{H_L} \cos(\lambda_{i,n} y) \cdot \psi(y) dy \tag{2.5}$$

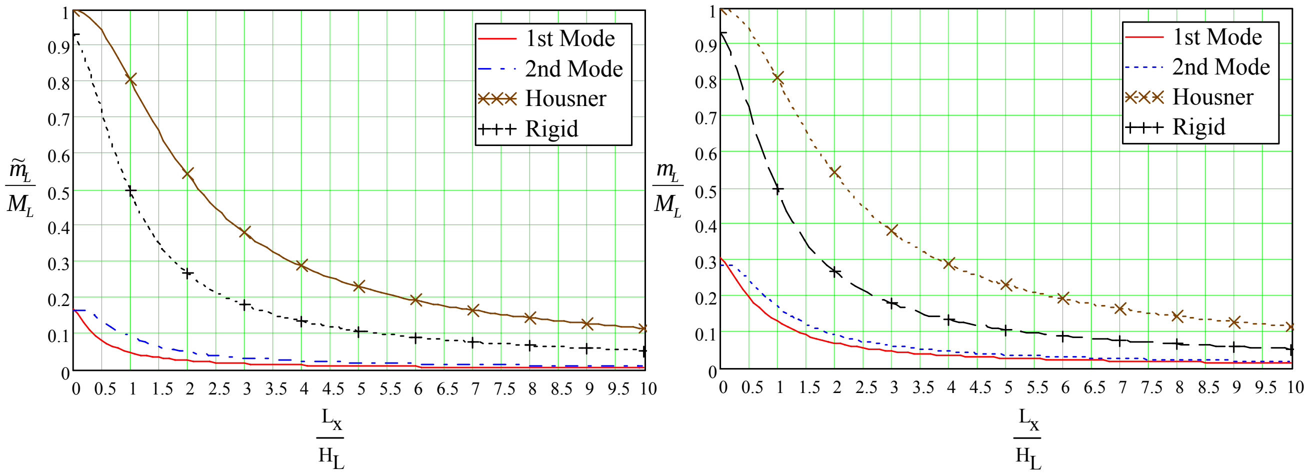
Based on the Housner's model, the ratio of the effective added mass of liquid due to impulsive hydrodynamic pressure $M_i$ to the total mass of liquid in the containment $M_L$ is expressed as:

$$\frac{M_i}{M_L} = \frac{\tanh[0.866(L_x / H_L)]}{0.866(L_x / H_L)} \tag{2.6}$$

Similarly for the generalized SDF system, the ratio of generalized and effective added mass of liquid due to hydrodynamic pressure for the prescribed mode shape to the half mass of liquid in LCS, i.e. $\widetilde{m}_L / M_L$ and $m_L / M_L$, can be calculated. It is worth noting that compared to the total mass of liquid in the Housner's model, only half the mass of liquid is considered in the generalized SDF system based on the two-fold symmetric fluid structural model.

Figures 2(a) and 2(b) show the added mass of liquid due to impulsive pressure based on the Hounser's model and the generalized SDF system using shape function $\psi(y) = 1$ which are both corresponding to a rigid tank wall, and the ratios of $\widetilde{m}_L / M_L$ and $m_L / M_L$ as functions of the ratio of width of tank to depth of liquid $L_x/H_L$. The shape function SF1 is used for the first mode considering the flexibility of tank wall in dynamic analysis. It is worth noting that Figure 2 is only for the full tank condition, i.e. $H_L = H_W$.

Figure 2 shows that the trend of curves for the Housner's model and Shape function SF1 for the first mode shape using the generalized SDF system is similar. However, the results obtained using the Housner's model are more than two times of those obtained using $\psi(y) = 1$ for the generalized SDF system considering the fold-symmetric-fluid structural model. The reason for the difference in response is due to the different methods used in the calculation of hydrodynamic pressure. In this study, the hydrodynamic pressure is calculated using the velocity potential method. It is assumed that the liquid is ideal, which is incompressible and inviscid. However, a simple Newtonian viscous shear model is used in the Housner model which may also result in stiff response.




(a) Generalized Added Mass

(b) Effective Added Mass

Figure 2 Ratio of Added Mass of Liquid due to Impulsive Hydrodynamic Pressure vs. $L_x$ / $H_L$ Ratio ($H_L = H_W$)

### *2.3 Effective Height*

In the current design practice, the inertial mass of concrete wall and the added mass of liquid due to hydrodynamic pressure are lumped at defined effective heights. The inertial mass of concrete tank wall is lumped at the center of gravity of the tank wall. If the tank wall is uniform, the total inertial mass of tank wall is lumped at the mid-height of the wall. The added mass of liquid due to impulsive hydrodynamic pressure is lumped at the centroid of the impulsive lateral force. This height can be calculated using Eqns. 2.7 and 2.8 as follows (ACI 350.3, 2006):

For tanks with $\dfrac{L_x}{H_L} < 1.333$ , $\quad \dfrac{h_i}{H_L} = 0.5 - 0.09375\left(\dfrac{L_x}{H_L}\right)$ (2.7)

For tanks with $\dfrac{L_x}{H_L} \geq 1.333$ , $\quad \dfrac{h_i}{H_L} = 0.375$ (2.8)

In the generalized SDF system, the effective heights at which the effective added mass of liquid due to hydrodynamic pressure is applied, $h_i$, can be calculated as follows:

$$h_i = \frac{\int_0^{H_L} \sum_{n=1}^{\infty} \frac{2 \cdot \rho_l}{\lambda_{i,n} \cdot H_L} \tanh(\lambda_{i,n} L_x) \cos(\lambda_{i,n} y) \int_0^{H_L} \cos(\lambda_{i,n} y)\, \psi(y) \cdot dy \cdot y \cdot dy}{\int_0^{H_L} \sum_{n=1}^{\infty} \frac{2 \cdot \rho_l}{\lambda_{i,n} \cdot H_L} \tanh(\lambda_{i,n} L_x) \cos(\lambda_{i,n} y) \int_0^{H_L} \cos(\lambda_{i,n} y)\, \psi(y) \cdot dy \cdot dy} \tag{2.9}$$

Figure 3 shows the normalized effective height at which the hydrodynamic pressure is applied as function of the ratio of half width of tank to liquid depth $L_x/H_L$ for the full tank condition i.e. $H_L = H_W$. The figure shows the first two modes, the rigid wall boundary condition $\psi(y) = 1$ and the Housner's model. It can be found that the effective heights $h_i$ obtained from the Hounser's model and the rigid wall boundary condition $\psi(y) = 1$ are similar.

For liquid containing structures, the effective height at which the total dynamic lateral force is applied can be calculated using Eqn. 2.10. This expression includes both the effects of inertial mass of tank wall and the added mass of liquid due to hydrodynamic pressure.

$$h = \frac{m_w \cdot h_w + m_L \cdot h_i}{m_w + m_L} \tag{2.10}$$

It is worth noting that considering the flexibility of tank wall, the effective height h at which the overall lateral dynamic force is applied is higher than that obtained from the rigid wall condition.

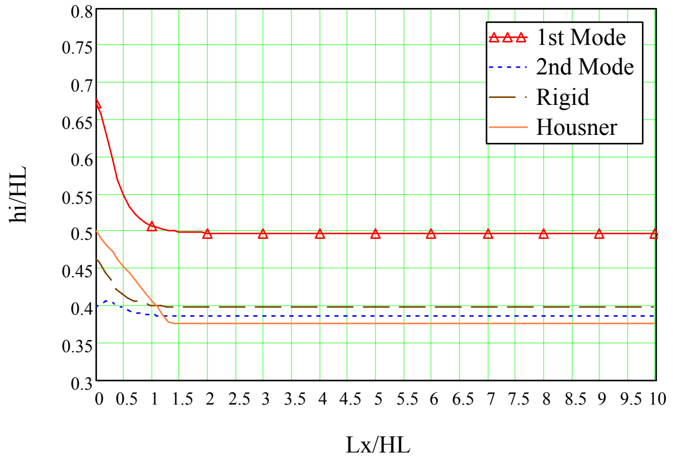



Figure 3 Effective Height Factors for Impulsive Hydrodynamic Pressure vs. $L_x/H_L$ Ratio ($H_L = H_W$)

### *2.4 Effect of Higher Modes*

The similar method used in dynamic analysis of LCS for first mode can be applied on the dynamic analysis for higher modes. The square root of sum of square (SRSS) method can be used for the combination of higher modes. Generally, the inclusion of the first two modes should provide sufficiently accurate results for design purposes.

## 3. FINITE ELEMENT IMPLEMENTATION FOR DYNAMIC ANALYSIS OF LCS

In this study, a different FEM procedure based on the fact that hydrodynamic pressure distribution is governed by wave equation in liquid domain is used to verify results. Assuming that water is incompressible and neglecting its viscosity, the small-amplitude irrotational motion of water is governed by the two-dimensional wave equation:

$$\nabla^2 P(x, y, t) = 0 \tag{3.1}$$

In the coupling system of liquid – structure the pressures are applied to the structure surface as the loads on the container walls. The general equation of fluid – structure can be written in the following form:

$$\begin{aligned}[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}&=\{f_1\}-[M]\{\ddot{U}_g\}+[Q]\{P\}=\{F_1\}+[Q]\{P\}\\ [G]\{\ddot{P}\}+[C']\{\dot{P}\}+[H]\{P\}&=\{F\}-\rho[Q]^T(\{\ddot{U}\}+\{\ddot{U}_g\})=\{F_2\}-\rho[Q]^T\{\ddot{U}\}\end{aligned} \quad (3.2)$$

In which, $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices of structure while $[H]$ and $[G]$ are representing stiffness and mass for liquid domain. The term $[C']$ is the damping matrix of liquid which is dependent on the viscosity of liquid and wave absorption in liquid domain and boundaries. The matrix $[Q]$ transfers the liquid pressure to the structure as well as structural acceleration to the liquid domain.

An 8-node isoparametric element with two translations degree of freedom in each node is used to model the tank walls and foundation. The liquid domain is modeled using four-node isoparametric fluid elements with pressure degree of freedom in each node. The finite element model is used to investigate the behavior of a tall tank as discussed in the following design example.

## 4. DESIGN EXAMPLE

In this investigation, a tall tank studied previously is used as a design example. The dimensions and properties of the tank are as follows:

$L_x$=9.8m, $H_w$=12.3m, $H_L$=11.2m, $t_w$=1.2m, $E_c$=20.776x$10^3$MPa, $\rho_w$= 2300 kg/$m^3$, $\rho_l$=1000 kg/$m^3$, $\nu$= 0.17

The design response spectrum based on ASCE 7-05 is used to obtain the response spectral acceleration. The site is assumed to be located in the West Coast of US in Washington State and the parameters for the design response spectrum are that:

(1) Short period maximum spectral response acceleration: $S_s$=1.25
(2) 1-second maximum spectral response acceleration: $S_1$=0.60
(3) Site class B

The calculations using the finite element method (FEM) and the ACI 350.3 code are also presented in this study. The finite element method proposed in the previous study (Chen and Kianoush, 2005) for Model 4 is used for verification. The consistent mass for both tank wall and added mass of liquid due to impulsive hydrodynamic pressure are considered in the FEM.

Also, the results obtained using the proposed FEM procedure are compared to those associated with added mass FEM method and ACI code. The horizontal component recorded for 1940 El-Centro is used as excitation of the system. The horizontal component was scaled in such a way that peak ground acceleration reaches 0.4g. The model configuration is depicted in Figure 4.

ACI 350.3 (2006) Code considers the effect of ductility through the response modification factor R. It is worth noting that the response modification factor R and the importance factor I, are not considered in this study (i.e. R and I are assumed as unity). Therefore, the comparison between the proposed model and ACI 350.3 Code is on the basis of elastic analysis.

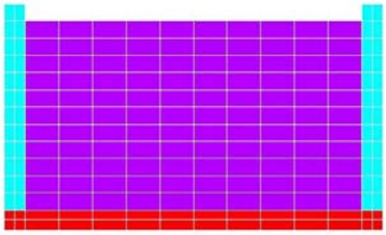


Figure 4 Finite Element Mesh Configuration

The calculation results are summarized in Table 1 for the tall tank. The comparison of the results obtained using both FEM procedures and the proposed model shows good agreement. However, the base shear obtained using ACI 350.3 Code is about 1.85 times higher than that obtained using the proposed generalized SDF system. The base moment for ACI 350.3 Code as compared to the proposed generalized SDF system is about 1.36 times higher. It is concluded that the design using the Hounser's model adopted in the current design standards and codes is overly conservative.

Table 1 Summaries of Dynamic Response

| Method | FEM (Proposed Method) | FEM (added mass) | Proposed Model | | | ACI 350.3 |
|---|---|---|---|---|---|---|
| | | | 1st Mode | 2nd mode | SRSS | |
| $\widetilde{m}_W$ ($10^3$kg) | - | - | 8.487 | 8.487 | - | - |
| $m_W$ ($10^3$kg) | - | - | 13.24 | 7.463 | - | 33.95 |
| $h_w$ (m) | - | - | 9.225 | 2.583 | - | 6.15 |
| $\widetilde{m}_L$ ($10^3$kg) | - | - | 4.32 | 10.02 | - | - |
| $m_L$ ($10^3$kg) | - | 59.8 | 13.46 | 23.16 | - | 92.67 |
| $h_i$ (m) | - | - | 5.744 | 4.726 | - | 4.681 |
| h (m) | - | - | 8.051 | 3.744 | - | 5.075 |
| $\widetilde{k}$ ($10^3$kN/m) | - | - | 4.823 | 192.9 | - | 68.66 |
| T (sec) | - | 0.344 | 0.324 | 0.062 | - | 0.27 |
| $A_a$(m/sec$^2$) | - | - | 0.833g | 0.651g | - | 0.833g |
| $d_{max}$ (mm) | 52 | 45.0 | 45.2 | 1.01 | 45.2 | - |
| $V_B$ (kN) | 631 | 437.4 | 454.8 | 323.6 | 558.2 | 1034 |
| $M_B$ (kNm) | 3958 | 3465.4 | 3661 | 1211 | 3856 | 5249 |
| $P_i$ (kN) | - | - | 229.3 | 244.7 | 335.3 | 757.0 |
| $M_i$ (kNm) | - | - | 1317.1 | 1156 | 1752.5 | 3544 |

## 5. CONCLUSIONS

In this paper, a structural analysis model using the generalized SDF system is proposed for seismic design of LCS. The proposed model can consider the consistent mass and the effect of flexibility of tank wall in design. The conceptual procedure for this methodology is similar to that of the Housner's model adopted in the current design codes and standards. However, the generalized and effective added mass of liquid due to impulsive hydrodynamic pressure and the corresponding effective height are introduced in the proposed model.

The curves for the added mass of liquid due to impulsive hydrodynamic pressure and the corresponding effective height are presented and compared with those adopted in the current design codes and standards.

The calculation for a tall tank is presented and compared with the results obtained using the current practice and the finite element method. The comparison shows that the results obtained from FEM and the proposed model are in good agreement. However, the results obtained using the current practice are overly conservative. It is recommended to use the generalized SDF system for seismic design of concrete rectangular LCS. The proposed model can provide fairly accurate results for the structural design while still maintaining the simplicity.

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